Factor Models in Economics Research

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Economic Intuition

- A life cycle model of youth and adult decision making over horizon $\bar{T}$:
  - *Agent maximizes*

$$\int_0^{\overline{T}} \exp(-\rho t) U(c(t), \ell(t); \eta)\, dt$$

subject to dynamic constraints:

$$\begin{aligned}
\overset{\bullet}{A}(t) &= Y(t)h(t)\ell(t) - P(t)'c(t) + rA(t), \\
\overset{\bullet}{h}(t) &= \varphi(h(t), I(t), \tau), \\
Y(t) &= R(h(t); \gamma),
\end{aligned}$$

and initial conditions $h(0)$, $A(0)$.

- Latent dimensions might play a critical role:

$$\begin{aligned} \text{preferences } \eta &= \eta(\theta), \rho = \rho(\theta), \\ \text{human capital productivity } \tau &= \tau(\theta), \\ \text{direct market productivity } \gamma &= \gamma(\theta), \end{aligned}$$

$$\begin{aligned} h(0) &= h_0(\theta), \\ A(0) &= A_0(\theta). \end{aligned}$$

- Thus, these factors (θ), unobserved heterogeneity, should explain a variety of outcomes.
- Candidates for θ?

Latent Skills and Outcomes:

The evidence demonstrates independent and important roles of latent cognitive and socio-emotional skills. They determine schooling attainment, labor market outcomes and social behavior.

*Hernestein and Murray (1994), Neal and Johnson (1996), Gottfredson (1997), Harting and Wigdor (1998), Cunha et. al. (2006), Cawley, Heckman, Vytlacil (2001), Conti et al (2010), Fergusson et al (2005), Carneiro and Heckman (2003), Armor and Roll (1994), Glewwe (2002), Kuncel et al (2004), Chown (1959), Heckman et al (2006), Urzua (2008), Conti et al (2010), Murnane et al (2000), Lazear (2003), Bowles and Gintis (1976), Klein et al (1991), Barrick and Mount (1991), Tett et al (1991), Borghans et al (2008), Maxwell (2007), Duncan and Dunifon (1998), Moss and Tilly (2000), Maxwell (2007), Duckworth et al (2007), Diaz. et. al (2010), Carneiro et at. (2007), Lindquist and Westman (2010), Van Praag et al. (2009), Brockhaus (1980), Hasemark (2003), Kaufman, et al (1995), Caliendo et al. (2007), Ekelund et al. (2005) and Stewart et al. (1999), Fairlie (2002), Dunn and Holtz-Eakin (2000), Burke et al (2001), Taylor (1996), Hamilton (2000), Eren and Sula (2012), Spector and O.Connell (1994), De Mel et al. (2008 and 2009), Benz and Frey (2008), Caliendo and Kritikos (2012), Hartog and Sluis (2010), Bassi et al (2012), Tambunlerchai(2011), Yamaguchi (2012), Boehm (2013), Sarzosa and Urzua (2014), Prada (2014), and **many others.***

Static Roy Model Framework

- Potential outcomes, two regimes (treatment and control)

$$\begin{aligned} Y_1 &= \mu_1(X) + U_1 \\ Y_0 &= \mu_0(X) + U_0 \end{aligned}$$

- Decision rule

$$D = \mathbf{1}[\mu_D(Z) - V \geq 0]$$

- Factor structure: U_1, U_0, V might information on unobserved dimensions. Thus,

$$\begin{aligned} U_1 &= \alpha_1 \theta + \varepsilon_1 \\ U_0 &= \alpha_0 \theta + \varepsilon_0 \\ V &= \alpha_V \theta + \varepsilon_v \end{aligned}$$

where $\varepsilon_V \perp\!\!\!\perp \varepsilon_1 \perp\!\!\!\perp \varepsilon_0$, and $\theta \perp\!\!\!\perp (\varepsilon_V, \varepsilon_1, \varepsilon_0)$

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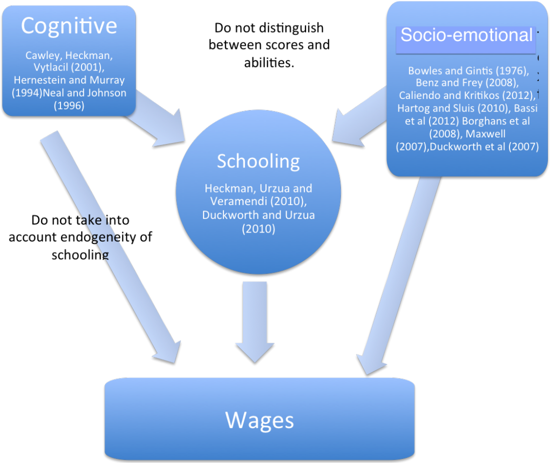
Roy Model Framework



The Roy Model provides a simple framework to analyze the effects of latent skills on outcomes.

Factor models can be integrated into this economic setting.

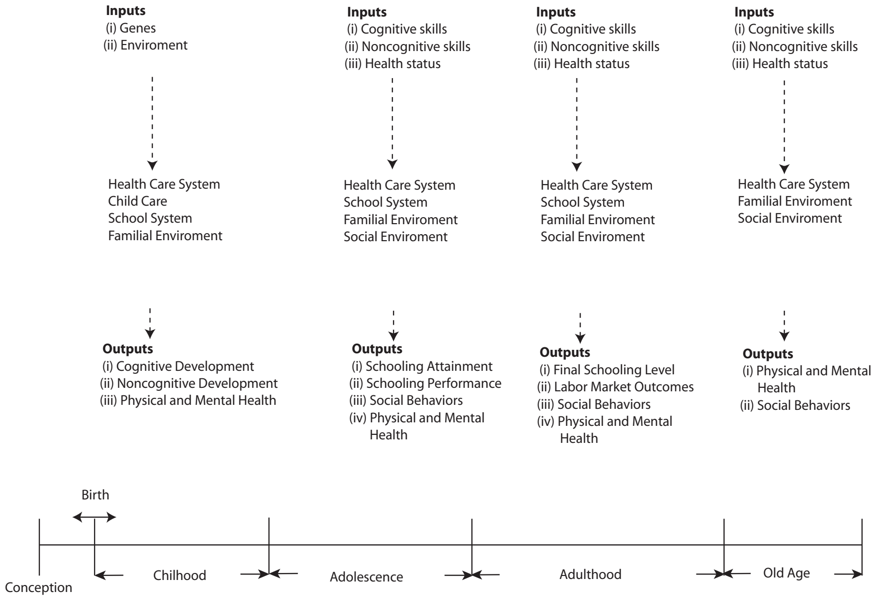
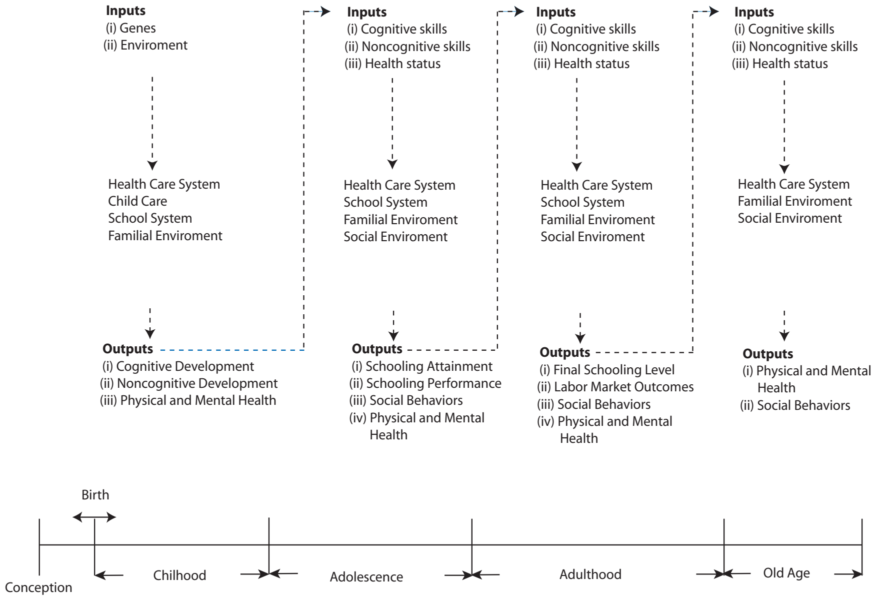


Figure 2. Human Development at Each Stage (inputs/ouputs)



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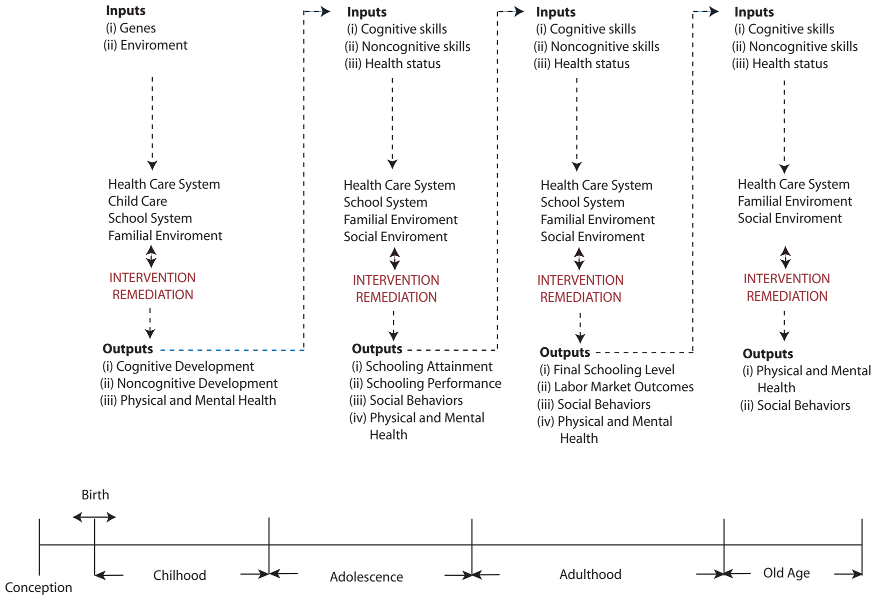


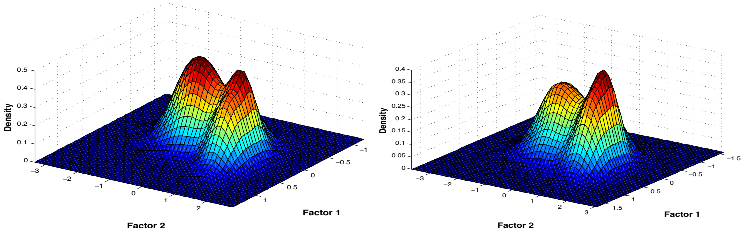
Figure 2. Human Development at Each Stage (inputs/ouputs)

Where do skills come from?

- θ_t is (latent) skill and I_t represents investments. Thus,

$$\begin{aligned} \theta_t &= g(\theta_{t-1}, I_{t-1}) + v_t \\ I_{t-1} &= \iota(\theta_{t-1}) \\ \theta_0 &= \text{initial condition} \end{aligned}$$

- Do skills evolve over time? Sarzosa and Urzua (2013) for South Korea:



(a) Age 14 (b) Age17

Implications?

Endowments, skills, labor market outcomes, and inequality

- Consider, for instance, earnings:

$$W_t(i,j) = W\left[\underbrace{H_0(i), S_t(i)}_{\text{Efforts/Circumstances}}, \underbrace{Z_t(j)}_{\text{Firms}}, \underbrace{T_t(i,j)}_{\text{Match}}\right]$$

- Income inequality?

$$\text{Gini}_t = G\left[\left[\left[W_t(i,j)\right]_{i=1}^{N(j)}\right]_{j=1}^{J}\right]$$

- Thus, the evolution of latent skills should shape the income distribution.

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How to identify the distribution of unobserved factors?

- Strong functional form assumptions are usually imposed.
- Additional information might provide a flexible alternative. .

- Measurement system: Let T denote a vector of test score (e.g. math score, IQ test)

$$T = \mu_T(X) + U_T = \mu_T(X) + \alpha_T \theta + \varepsilon_T$$

where $(X_T, \varepsilon_T) \perp\!\!\!\perp \theta$.

  - We can link/anchor latent factors to, for example, test scores .
  - The set up recognizes that T is not a direct measure of θ.

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Identification

- Information on at least three test scores (T_1, T_2, T_3).
- For simplicity, θ is scalar and omit X. Thus,

$$\begin{aligned} T_1 &= \alpha_{T_1}\theta + \varepsilon_{T_1} \\ T_2 &= \alpha_{T_2}\theta + \varepsilon_{T_2} \\ T_3 &= \alpha_{T_3}\theta + \varepsilon_{T_3} \end{aligned}$$

- We can compute RHS from $\mathrm{Cov}(T_i, T_j) = \alpha_{T_i}\alpha_{T_j}\sigma_\theta^2$, and

$$\frac{\mathrm{Cov}(T_1, T_2)}{\mathrm{Cov}(T_2, T_3)} = \frac{\alpha_{T_1}}{\alpha_{T_3}} \text{ and } \frac{\mathrm{Cov}(T_1, T_2)}{\mathrm{Cov}(T_1, T_3)} = \frac{\alpha_{T_2}}{\alpha_{T_3}}$$

- By normalizing $\alpha_{T_3} = 1$, we get α_{T_1} and α_{T_2}.

- Finally, we can rewrite the system as:

$$
\begin{aligned}
\frac{T_1}{\alpha_{T_1}} &= \theta + \frac{\varepsilon_{T_1}}{\alpha_{T_1}} = \theta + \varepsilon_{T_1}^{'} \\
\frac{T_2}{\alpha_{T_2}} &= \theta + \frac{\varepsilon_{T_2}}{\alpha_{T_2}} = \theta + \varepsilon_{T_2}^{'}
\end{aligned}
$$

and we can apply Kotlarski's Theorem (Kotlarski, 1967) to identify

$$
f_{\varepsilon_{T_1}}(\cdot), f_{\varepsilon_{T_2}}(\cdot), f_{\theta}(\cdot)
$$

- We can identify the whole model applying this logic.

Estimation: MLE

- We observe Y_j, T_j, D_j for $j = 1, ..., N$, with

$$Y_j = D_j Y_{1,j} + (1 - D_j) Y_{0,j}$$

- Conditional on unobserved abilities, U_1, U_0, V and U_T are mutually independent. Thus,

$$\prod_{j=1}^{N} f(Y_j, T_j, D_j | X, X_T, Z) = \prod_{j=1}^{N} \int f(Y_j, T_j, D_j | X, X_T, Z, \theta) dF(\theta)$$

where we can write

$$f(Y_j, T_j, D_j | X, X_T, Z, \theta) = f(Y_j, D_j | X, Z, \theta) f(T_j | X_T, \theta)$$

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Three Economic Applications

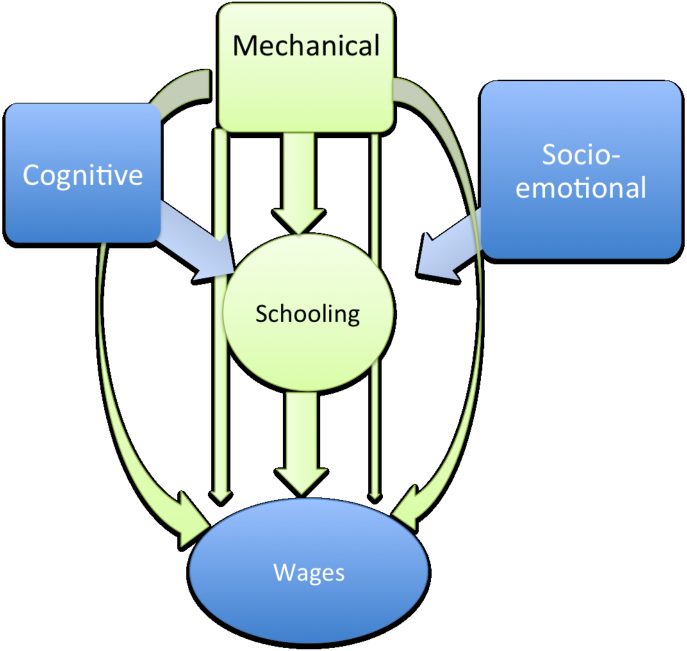
1. Multidimensional ability.
2. Social interactions.
3. Dynamic effects of Training.

Prada & Urzua (2017): The role of **"Mechanical Ability"**

The multidimensionality of skills, ability and knowledge must be at the "center stage of the theoretical and empirical research on child development, educational attainment and labor market careers". (Altonji, 2010)

Unlike standard constructs, it reduces the probability of attending a four-year college, while presenting positive reward on the labor market.

For individuals with very high levels of mechanical ability but low levels of cognitive and socio-emotional ability, <u>not going to college</u> is associated with higher expected hourly wage.


Mechanical
Cognitive
Socio-emotional
Schooling
Wages

Basic Idea and Approach

- USA & NLSY79. Armed Services Vocational Aptitude Battery (ASVAB) and the Armed Forces Qualification Test (AFQT). ASVAB: arithmetic reasoning, word knowledge, paragraph comprehension, mathematics knowledge, numerical operations, coding speed, general science, auto and shop information, electronics information, and mechanical comprehension.

Mechanical comprehension section

Ability to solve simple mechanics problems and understand basic mechanical principles

- Roy Model, three correlated factors, college and earnings, MCMC.

Roy Framework

$$Y = \begin{cases} Y(0) = \mu_0 + \varepsilon_0 & \text{if } D = 0 \\ Y(1) = \mu_1 + \varepsilon_1 & \text{if} D = 1 \end{cases}$$

$$D = \mathbf{1}\{Z\gamma + \varepsilon_D \geq 0\}$$

- Not assuming normality on the unobserved components $\varepsilon_0, \varepsilon_1, \varepsilon_D$, imposing instead the following structure:

$$\varepsilon_0 = \lambda_0^C \theta_C + \lambda_0^M \theta_M + \lambda_0^S \theta_S + e_0$$
$$\varepsilon_1 = \lambda_1^C \theta_C + \lambda_1^M \theta_M + \lambda_1^S \theta_S + e_1$$
$$\varepsilon_D = \lambda_D^C \theta_C + \lambda_D^M \theta_M + \lambda_D^S \theta_S + e_D$$

- Assuming

$$e_0 \perp e_1 \perp e_D \text{ and } (\theta_C, \theta_M, \theta_S) \perp (e_0, e_1, e_D)$$

Latent Factors: Flexible Distributions

$$\theta_{c,i} \sim \textstyle\sum_{k=1}^{K} p_k N\left(\mu_c^k, \left(\sigma_c^k\right)^2\right)$$

$$\theta_{s,i} \sim \textstyle\sum_{l=1}^{L} p_l S\left(\mu_s^l, \left(\sigma_s^l\right)^2\right)$$

$$\theta_{m,i} = \alpha_1 \theta_{c,i} + \alpha_2 \theta_{2,i}$$

with

$$\theta_{2,i} \sim \textstyle\sum_{j=1}^{J} p_j N\left(\mu_2^j, \left(\sigma_2^j\right)^2\right)$$

$$\sum_{k=1}^{K} p_k = \sum_{j=1}^{J} p_j = \sum_{l=1}^{L} p_l = 1$$

In this case, we use mixtures of 2 normals so $K = J = L = 2$ and

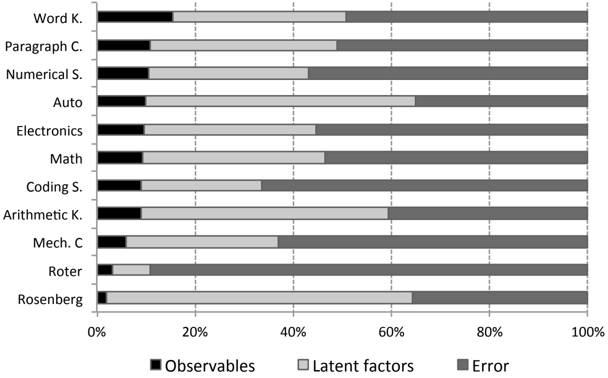
$$E[\theta_c] = E[\theta_m] = E[\theta_s] = 0$$

Finally, $(\theta_c, \theta_s) \perp \theta_2$ and $(\theta_c, \theta_m) \perp \theta_s$.

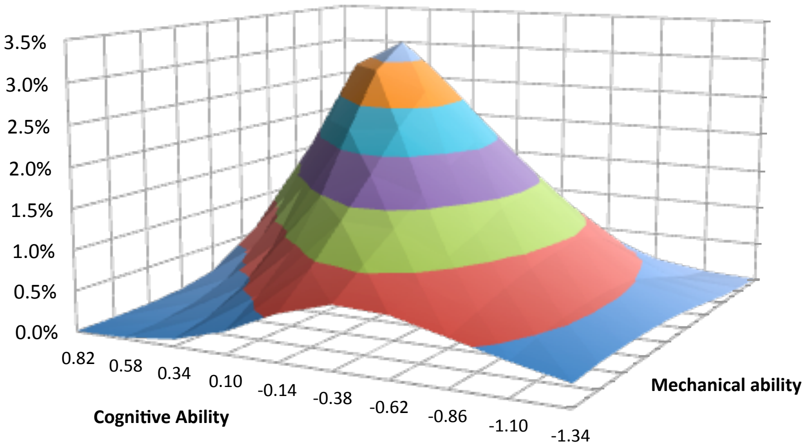
Results

Variance Decomposition

After controlling for the latent variables, we are able to explain between 34 and 65 percent of the total variance, except for the Rotter Scale.



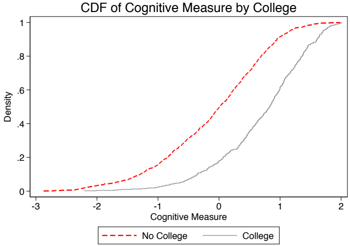
Joint Distribution Cognitive and Mechanical Ability



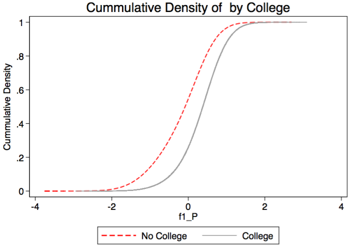
$$\sigma_{\theta^M} = 0.58,\ \sigma_{\theta^c} = 0.73,\ \sigma_{\theta^S} = 0.89,\ COV(\theta^c, \theta^m) = 0.21,\ \rho_{\theta^c\theta^m} = 0.52$$

Distribution Measurements vs Estimated Cognitive Ability: Marginal CDF

Same sorting by Schooling



(a) Measurement

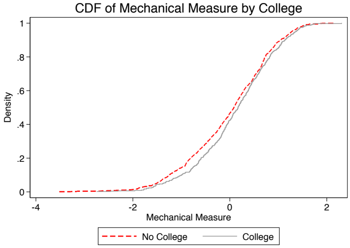


(b) Estimated Ability

$$\sigma_{\theta^c} = 0.73, \; COV(\theta^c, \theta^m) = 0.21, \; \rho_{\theta^c \theta^m} = 0.52$$

Distribution Measurements vs Estimated Mechanical Ability: Marginal CDF

Distributions and Sorting Differ Between Measurements and Estimated Mechanical ability



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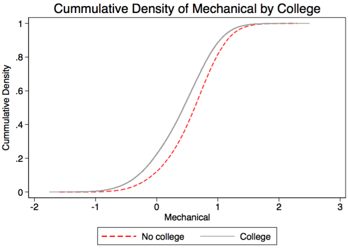


(b) Estimated Ability

$$\sigma_{\theta^M} = 0.58,\ COV(\theta^C, \theta^M) = 0.21,\ \rho_{\theta^C\theta^M} = 0.52$$

Schooling Choices

Effect of 1 SD of each ability: Mechanical Predicts Lower Schooling

Table: College Attendance: Estimated Marginal Effects

	Cognitive	**Mechanical**	**Socio-emotional**
College Decision	0.229***	-0.095***	0.024***

Note: Standard errors in parenthesis. College Decision equation includes family background controls, cohort dummies and geographical controls for region and urban residence at the age of 14. For hourly wages we control for cohort dummies as well as geographical controls for region and urban residence at age 25.

Hourly Wages

Effect of 1 SD of each ability: Mechanical has Positive Returns

Table: College Attendance and Hourly Wages: Estimated Marginal Effects

	Cognitive	Mechanical	Socio-emotional
Log hourly wages	0.098***	0.014***	0.041***
w0	0.047***	0.044***	0.033***
w1	0.108***	-0.031***	0.047***

Note: Standard errors in parenthesis. College Decision equation includes family background controls, cohort dummies and geographical controls for region and urban residence at the age of 14. For hourly wages we control for cohort dummies as well as geographical controls for region and urban residence at age 25

Conclusions

- We analyze the importance of mechanical, cognitive and socio-emotional ability.
- We show that like standard measures of ability:
  - Mechanical ability is positively rewarded by the labor market,
  - But that in contrast to these other measures, it predicts the choice of low levels of schooling: reduces the likelihood of attending college.
- For individuals with very high levels of mechanical ability but low levels of standard ability (cognitive and socio-emotional), not going to college is associated with the highest expected hourly wage, despite the high returns associated with college.

Three Economic Applications

1. Multidimensional ability.
2. Social interactions: Bullying.
3. Dynamic effects of Training.

Sarzosa & Urzua (2019): Bullying

60,000 children miss school every day in the US because of fear of being bullied. Bully victims are between 2 to 9 times more likely to consider suicide than non-victims in the US. In the UK at least half of suicides among young people are related to bullying.

Examines longitudinal data on teenagers to assess the effects of skills on several outcomes in the context of bullying. Unobserved cognitive and non-cognitive ability as drivers of bullying.

Bulling victims are more likely to be depressed, feel sick, have mental health issues and feel stressed that non victims. Bullies are more likely to be depressed, smoke, and feel stressed that non bullies. Non-cognitive skills signiÞcantly reduce the chances of being bullied during high school.

Basic Idea and Approach

- S. Korea & Junior High School Panel (JHSP) of the Korean Youth Panel Survey (KYP). Cognitive (CS) and Non-Cognitive (NCS) measures, self-reported bullying, the proportion of peers that report being bullies in the class and the proportion of peers in the classroom that come from a violent family.
- Roy Model, two factors, bullying and different outcomes, MLE.

Setting

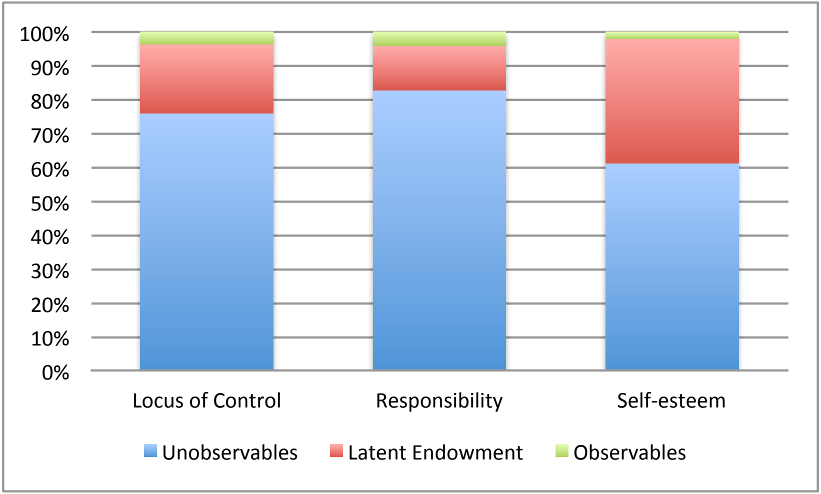
- Structural model implemented is a set of measurement systems that are linked by a factor structure

$$
\begin{aligned}
Y_1 &= \mathbf{X}_Y \beta^{Y_1} + \alpha^{Y_1,C}\theta^C + \alpha^{Y_1,N}\theta^N + e^{Y_1} \qquad \text{if } D=1 \\
Y_0 &= \mathbf{X}_Y \beta^{Y_0} + \alpha^{Y_0,C}\theta^C + \alpha^{Y_0,N}\theta^N + e^{Y_0} \qquad \text{if } D=0 \\
D &= 1\left[\mathbf{X}_D \beta^{Y_D} + \alpha^{Y_D,C}\theta^C + \alpha^{Y_D,N}\theta^N + e^D > 0\right] \\
\mathbf{T} &= \mathbf{X}_T \beta^{T} + \alpha^{T,C}\theta^C + \alpha^{T,N}\theta^N + \mathbf{e}^{\mathbf{T}}
\end{aligned}
$$

- D denotes bullying (at age 15), potential outcomes (at age 18) include depression, life satisfaction, sickness, smoking, mental health. The model is estimated using MLE.

Results

Decomposing Variances of Non-Cognitive Measures



Determinants of Bullying

Table: Non-Cognitive and Cognitive Skills at age 14 and the Probability of Being Bullied at age 15 (τ_1)

	(1)		(2)		(3)	
VARIABLES	Coeff.	Std. Err	Coeff.	Std. Err	Coeff.	Std. Err
Age in Months	0.0034	(0.010)	0.0021	(0.010)	0.0009	(0.010)
Male	0.3234***	(0.068)	0.2900***	(0.069)	0.3076***	(0.068)
Youngsiblings	-0.1153*	(0.067)	-0.1117*	(0.067)	-0.1159*	(0.067)
% Peer Bullies			0.8107**	(0.340)		
% Peer Vlnt Fam					-3.8408**	(1.711)
% Peer Vlnt Fam^2					4.8220**	(2.147)
Non-Cogn. Skills	-0.2810**	(0.132)	-0.2986**	(0.133)	-0.3004**	(0.133)
Cognitive Skills	0.0759	(0.058)	0.0784	(0.059)	0.0804	(0.059)
Observations	2,690		2,690		2,690	

Outcomes: $D = 0$ no bullying, $D = 1$ bullying

Table: Outcome Equations (age 18, τ_2) by Bullying Status D (age 15, τ_1)

	(1) Depression		(2) Drink		(3) Smoke	
Bullied	D =0	D =1	D =0	D =1	D =0	D =1
Non-Cogn	-0.425***	-0.729***	-0.108***	-0.219*	-0.093***	-0.159*
	(0.070)	(0.192)	(0.038)	(0.118)	(0.024)	(0.084)
Cognitive	0.108***	0.107	-0.005	-0.042	-0.034***	-0.138***
	(0.033)	(0.089)	(0.018)	(0.052)	(0.012)	(0.037)
Obs	2,395		2,690		2,690	

	(4) Life Satisfaction		(5) Sick		(6) Mental Health	
	D =0	D =1	D =0	D =1	D =0	D =1
Non-Cogn	0.235***	0.477***	-0.043**	-0.090	-0.022*	-0.099*
	(0.035)	(0.098)	(0.017)	(0.068)	(0.013)	(0.059)
Cognitive	0.007	0.046	-0.001	0.011	0.008	0.016
	(0.017)	(0.044)	(0.008)	(0.031)	(0.006)	(0.026)
Obs	2,690		2,514		2,514	

What is the effect of Bullying (ATE, TT)?

- ATE=$E[Y_1 - Y_0]$ and TT=$E[Y_1 - Y_0 | D = 1]$
- We can estimate these as a function of latent factors.

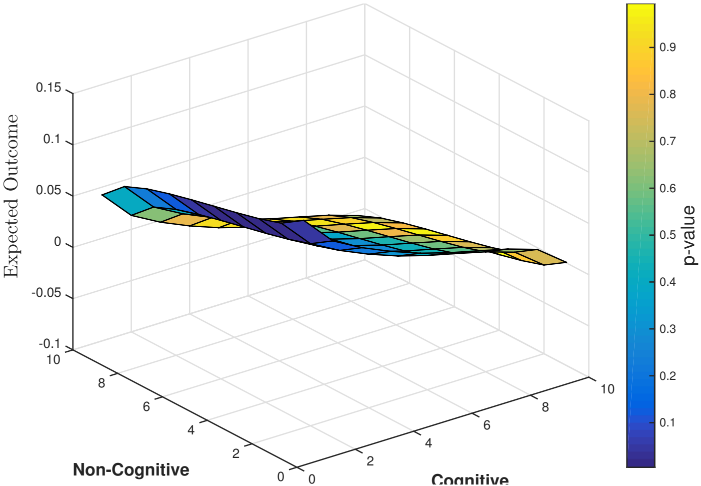
Table: Treatment Effects: Outcomes at Age 18 (τ_2) of Being Bullied at Age 15 (τ_1)

	Depression	Smoking	Drinking	Sick	Mental Hlth	Life Satisfact
ATE	0.0416	0.0183	0.0001	0.0552**	0.0298	-0.0040
	(0.0755)	(0.0261)	(0.0347)	(0.0244)	(0.0210)	(0.0355)
TTE	0.0520	0.0174	0.0230	0.0498**	0.0410**	-0.0268
	(0.0590)	(0.0237)	(0.0307)	(0.0213)	(0.0179)	(0.0311)

	inCollege†	Str:Friends	Str:Parent	Str:School	Str:Poverty	Str:Total
ATE	-0.0257	0.1939**	0.1508**	0.0513	0.0242	0.1467*
	(0.0337)	(0.0924)	(0.0752)	(0.0818)	(0.0783)	(0.0814)
TTE	-0.0396	0.2735***	0.1450**	0.1422**	0.0825	0.2191***
	(0.0307)	(0.0725)	(0.0684)	(0.0635)	(0.0646)	(0.0642)

ATE on Smoking as a function of θ

Figure: ATE on Smoking



Three Economic Applications

1. Multidimensional ability.
2. Social interactions: Bullying.
3. Dynamic effects of Training.

Rodriguez, Saltiel & Urzua (2019): Dynamic Treatment Effects

Technological progress +
Mismatch in the labor market
=
Complexity and uncertainty in the demand for skills.

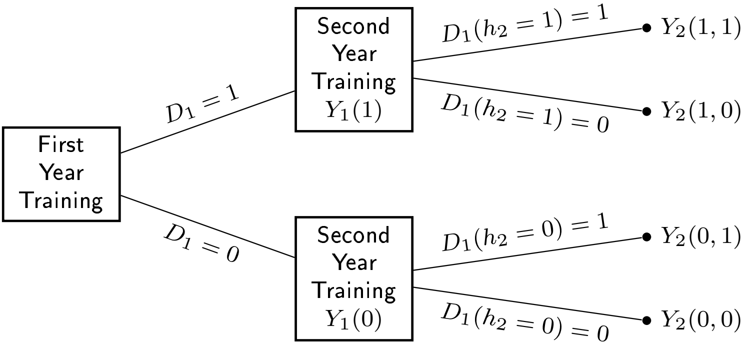
Workers invest in their human capital multiple times in their careers. Thus, what is the impact of human capital investments (training) on earnings in a dynamic world?

Effects of training differ by past training choices, dynamic substitutability, policy-relevant treatment effects differ by compliers type. Policy-makers should consider impact on labor market trajectories and how are these affected by training policies dinamically.

Basic Idea and Approach

- Chile & Administrative Information. Training during 1998-2010, unemployment Insurance database (workers' monthly earnings in the quarter following a training period), college-entry standardized exam (test scores).
- Roy Model, dynamic decisions, one factor, MLE.

Decision Tree



Dynamic-discrete choice model of training choices and earnings

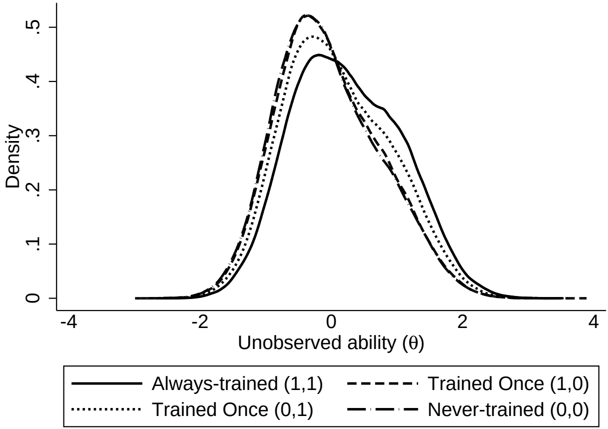
- Individual chooses training for many periods, evaluating benefits and costs. Period-t earnings depend on current and past training choices. Unobserved ability.

- Setting:
  - h_t: state of training decision in previous periods $(t-1, t-2, ...)$.
  - $D_t(h_t) \in \{0,1\}$: training choice, given history h_t.
  - $Y_t(h_t, j)$: outcomes in period t given a training history h_t and current decision $j \in \{0,1\}$.

- Formally:
  - Choices: $D_t(h_t) = \mathbf{1}[\mu^I(h_t) + \theta\lambda^I(h_t) + \varepsilon_t^I(h_t) \geq 0]$.
  - Earnings: $Y_t(h_t, j) = \delta^Y(h_t, j) + \lambda^Y(h_t, j)\theta + \varepsilon_t^Y(h_t, j)$.
  - θ is unobserved heterogeneity known only to the agent.

Results

Distribution of Unobserved Ability by Training History



Dynamic Treatment Effects

Let $\widetilde{Y}_1(j)$ be the present value of (observed) earnings associated with choosing training option j in period $t = 1$. In two periods:

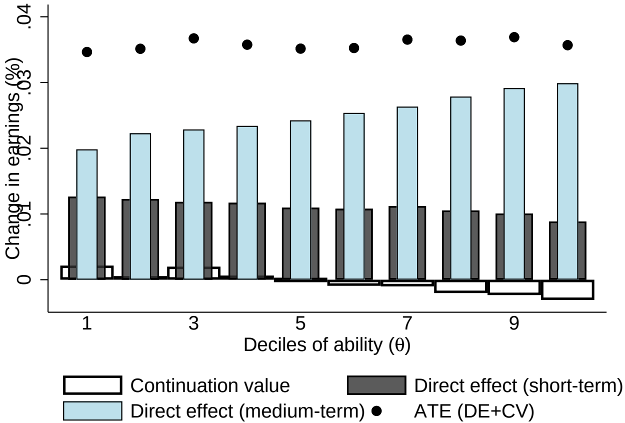
$$\widetilde{Y}_1(1) \equiv Y_1(1) + \rho\,(D_2(j)\,Y_{i2}(j=1,1) + (1 - D_2(j))\,Y_{i2}(j=1,0))$$

Dynamic Treatment Effects (DATE): $\widetilde{Y}(1) - \widetilde{Y}(0)$.

- DATE can be decomposed as:

$$DATE = \underbrace{(Y_{i1}(1) - Y_{i1}(0))}_{\text{Direct effect (short-term)}} + \underbrace{\rho[Y_{i2}(1,0) - Y_{i2}(0,0)]}_{\text{Direct effect (medium-term)}}$$
$$+ \underbrace{\rho[D_{i2}(1)(Y_{i2}(1,1) - Y_{i2}(1,0)) - D_{i2}(0)(Y_{i2}(0,1) - Y_{i2}(0,0))]}_{\text{Continuation value}}$$

Dynamic Treatment Effects: Direct Effects and Continuation Value



Conclusions

Large-scale, government-subsided training program to show novel evidence on dynamic treatment effects of job training.

Effects of training differ by past training choices, dynamic substitutability, policy-relevant treatment effects differ by compliers type.

Factor model allows the estimation of the impact on labor market trajectories and how are these affected by training policies dynamically.