Geometrical architecture and artwork

Secondary: (ages 11 – 14)	Mathematics

Students are asked to design their own architectural plans or artwork using geometric figures. Students then need to apply their mathematical knowledge to calculate perimeters and areas that could be constructed with different materials or covered by paint of different colours. The focus on mathematical creativity comes from tasks that require students to devise different strategies for adding areas and for calculating changes in the areas as a function of changes in the sides of the figures. Students use their basic understanding of geometric shapes, perimeters and areas in an uncommon but appealing situation. The activity further invites students to dive into a real collaborative problem-solving activity.

Time allocation	2 or 3 lesson periods
Subject content	Geometry (identifying geometrical figures, calculating areas and perimeters of geometrical figures with straight lines and/or circles) Basic algebra
	This unit has a **creativity** focus: • Play with unusual and radical ideas • Generate ideas, make connections, and find several perspectives on a mathematical problem • Produce, perform or envision something personal
Other Skills	Collaboration
Key words	geometry; area; perimeter; volume; figures; shapes; algebra

Products and processes to assess

Students produce a geometric design and then use this to complete a number of mathematical challenges. Their work process demonstrates a willingness to explore and the ability to see real-life and creative applications of simple mathematical principles. They show good awareness of areas of personal novelty and risk and reflect well on why they made their final choices.

Teaching and Learning plan

This plan suggests potential steps for implementing the activity. Teachers can introduce as many modifications as they see fit to adapt the activity to their teaching context.

Step	Duration	Teacher and student roles	Subject content	Creativity and critical thinking
1	Lesson period 1: Introducing the activity to students	Students are presented with an authentic challenge which will later be used to develop mathematical knowledge of how to calculate perimeters and areas. Teachers may consider different alternatives for this challenge: a) an architectural design competition which requires teams to come up with aerial plans of adjacent geometrical buildings (designs by architects such as Le Corbusier or Oscar Niemeyer may be shown); b) a painting competition which requires teams to come up with artwork designs using geometrical figures (works from artists such as Piet Mondrian, Ilya Bolotowsky, Josef Albers, Maxime Defert may be shown); or c) a competition to design stained glass with geometric and colourful stained glass patterns for their own school (examples of geometric stained glass may be shown: several patterns can be found on the internet). If the context chosen is an architectural plan, ensuing questions should refer to 2-D rather than referencing 3-D volumes. Students will be provided with a set of geometric figures (suggested number: 20) with standard sizes. Teachers can decide whether they want students to use a minimum number of figures or all 20, and whether to have 20 different figures or some figures repeated. Teachers should also establish a set of basic rules for the designs, for instance: designs should include some (but not all) figures that overlap fully, as well as some partially overlapping figures; the designs must fit within a frame of a predetermined size (this will assist in calculations in later steps), and other conditions that teachers may consider. Students should work in small groups.		Making connections between real-world problems, geometrical shapes, and mathematical techniques
2	Lesson period 1: Creating artwork	Students are given the geometric figures and are asked to identify them and make a design (e.g. architectural plan, painting, stained glass) working in groups for around 20 minutes. Students are allowed to combine figures freely and make them overlap as in a collage. When students finish making their pictures, they can be invited to give them a title, and to fill in a simple report template about "How we made our picture" that specifies what they agreed on as a group and their justifications for their decisions.	Identifying geometric figures and relationships between them (e.g. how one figure can be decomposed in others)	Doing: Producing, performing or envisioning something personally novel

		The teacher monitors the work of the groups, for instance by filling an observation checklist on students' opinions, suggestions and overall engagement.		
3	Lesson period 1: Calculating perimeters	The teacher introduces or refreshes students' memory on what "perimeter" is and how it can be calculated. Students are also given (access to) a ruler, a measuring grid, and other objects that may serve for measuring purposes such as ropes or thread of known lengths. Each group should come up with at least 2-3 different strategies to measure the perimeter of their design (see Appendix 2). The teacher should put the emphasis on the use of different calculation strategies. If there is no time to implement all of these strategies, students should at least describe how else they could have done it. One example is to calculate the perimeter of the "external" lines. For example, students may put a rope or thread around the external lines. What is the length of the rope? Students go on to calculate the perimeter of their designs. Students can also be asked to add or remove one or several figures: what would be the change in the design and its perimeter? All designs could be put on display at the end of the lesson.	Identifying geometric figures Concept of perimeter Algebra involved in calculating perimeters	Imagining: Generating ideas and making connections in order to approach a mathematical problem creatively Inquiring: Playing with unusual and radical ideas (strategies for calculation) Reflecting on steps taken to pose and solve maths problems
4	Lesson period 2: Preparations for main task	Students take their designs back and put them on their desks in front of them.		
5	Lesson period 2: Calculating areas	The teacher introduces or refreshes students' memory on what "area" is and how it can be calculated. Students go on to calculate the areas of their designs. Each group should spend 20-25 minutes coming up with at least 3 different strategies to calculate the area of their design (see Appendix 2). The teacher should put the emphasis on the use of different calculation strategies. If there is no time to implement all strategies, students should at least describe how else they could have done it. The possibility of adding lines can be specifically mentioned to students. Students can add lines in order to calculate the areas of figures for which they do not know how to calculate the area; by adding lines, they can either transform or decompose those figures into other figures whose area they can then calculate. For instance, if students know how to calculate the areas of a triangle, rectangle and square, they could add	Identifying geometric figures Concept of area Algebra involved in calculating areas	Inquiring: Playing with unusual and radical ideas (strategies for calculation)

		lines to or within other figures in order to find out how to calculate areas, so as to decompose a parallelogram (whose area may be difficult to calculate in these grades). Students are then asked to add or remove one or several figures: what would be the change in the design and its area? The teacher should give students some extra figures, for example 4 squares 2x2 cm, if there are no spare figures.		
6	Lesson period 2: Changes in perimeter and area when sides change	Students should then spend 20-25 minutes exploring how perimeters and area change differently as a function of changes in the sides of the figures. Students should realize that enlarging the sides of figures by X changes the area by X-squared. Students can also find out that the changes in the perimeter are different than the changes in the area. For instance, one suggestion to compare changes in perimeter and area in a rectangular or a square (easiest case) is to formulate the task as follows: due to limited availability of building materials, or to changes in the spec for the design (i.e. a wall that must be painted, a window frame), we need to double or divide the length of some sides. What would be the change in the perimeter and area? Students may intuitively say that when the sides double in length, so does the area. They then have to calculate it and see that this is not the case. The same activity should be repeated with enlarging or dividing the sides by three, four, five, etc. Each time students would have to calculate changes in the perimeters and areas and compare them. This can be formulated as follows: we need to enlarge each side three times more, what would be the change in the area? And so on, if each side was enlarged four times, what would be the change in the area? Can you generalize your discovery?	Concepts of perimeter and area; relationship between the two Algebra involved in calculating perimeters and areas	Imagining: Generating ideas and making connections to come up with different ideas regarding changes in perimeters
7	Lesson period 3: Preparations for the main task	The teacher introduces a problem-solving task by explaining to students that, in order to produce and/or publish their designs, they need to paint all or some of the figures, or use different construction materials. The teacher then provides students with a list of construction materials, glass types or paint colours and respective prices (in area units). The aim is to get students to make calculations based on the price of these materials and taking into account budget limitations.		
8	Lesson period 3: Areas turn into materials/paint	The calculation of the areas allows students to find out what area of the whole sheet would be built or covered by paint. Students are then asked to suggests changes in the colours of all or some of the figures, and calculate how much that would cost them, based on the list of prices they are	Algebra involved in calculating areas Algebra involved in calculating total prices and changes therein as	Imagining: Finding several perspectives on the problem

		provided with. Examples of questions to ask students: How much material/paint is needed? It turns out that concrete/red paint is very expensive. If you want to keep it in the design while reducing the cost, what could you do? Your budged has been cut in half: how many figures in your picture would have to be built/painted in different, cheaper materials/ colours? What would be the cheapest and the most expensive designs if you had to use one/two/three different materials/colours?	a function of changes in area or paint prices	
9	Lesson period 3: Reflection and presentations	Students should give a short presentation that will answer some of these questions: 1. What is the title of your design? What is it about? Why do you find it novel? Answer in 2-3 sentences. 2. In how many different ways did you calculate (or could you have calculated) the perimeter? Answer in 2-3 sentences. 3. In how many different ways did you calculate (or could you have calculated) the area? Answer in 2-3 sentences. 4. What's the conclusion? Can you generalize your findings about perimeters and areas? 5. What difficulties did you encounter? What were the limitations of the techniques used? Answer in 2-3 sentences. (See Appendix for further guidance). Teachers should record the work: picture of the figure and collection of the measures and justifications.	Communicating mathematical knowledge	Reflecting: Appreciating the novelty of solution and/or possible consequences Reflecting: Acknowledging uncertainty/limits of chosen solution/position

Resources and examples for inspiration

Web and print

- Designs by architects such as Le Corbusier or Oscar Niemeyer
- Works from artists such as Piet Mondrian, Ilya Bolotowsky, Josef Albers, Maxime Defert may be shown
- Examples of geometric stained glass

Other (materials for each group)

- 20 geometric figures of any colour. Figures should be chosen on the basis of the age and the level of the students (i.e. avoid figures whose area may be too difficult to calculate for students at this age, such as ellipses or circles, depending on the context).
- If possible, each group should get two sets, one that they use for their figure and one they can keep to analyse, measure, use on another piece of paper, etc.
- Teachers should adjust the set to their own teaching context, but a possible set includes: 3 big triangles (catheti 2x2 cm), 3 small triangles (catheti 1x1 cm), 2 big circles (R= 2 cm), 2 small circles (R=1), 1 big rectangle (2x4 cm), 1 small rectangle (1x2 cm), 4 big circles (2x2 cm), 4 small circles (1x1 cm). All figures should be put in an envelope along with 4 extra figures (for instance, squares 1x1 cm, of any colour).
- A glue stick if the figures are made of paper.
- 1 white paper sheet as a background for a picture (A3).
- Material to do a measuring grid. For instance 1 plastic sleeve or tracing paper and 1 black marker pen to make the grid.
- Ropes or threads of known lengths.
- A ruler or other objects that may serve to measure.
- Paper and pencils.
- Tracing paper.

Opportunities to adapt, extend, and enrich

- Cross-curricular opportunities could be developed in art (e.g. studying geometric shapes in the art of Picasso, Kandinsky or Rothko), in science (e.g. looking at geometric shapes in biology), in history (e.g. looking at the construction of the Pyramids or history of architecture)
- Students could create challenges for other groups to solve (i.e. create geometrical shapes and designs for which they think it will be challenging to calculate area and perimeter)

Creativity and critical thinking rubric for mathematics

- Mapping of the different steps of the lesson plan against the OECD rubric to identify the creative and/or critical thinking skills the different parts of the lesson aim to develop

	CREATIVITY Coming up with new ideas and solutions	Steps	CRITICAL THINKING Questioning and evaluating ideas and solutions	Steps
INQUIRING	Make connections to other maths concepts or to ideas from other disciplines	1-9	Identify and question assumptions and generally accepted ways to pose or solve a maths problem	
IMAGINING	Generate and play with several approaches to pose or solve a maths problem	3,5,6	Consider several perspectives on approaching a maths problem	8
DOING	Pose and envision how to solve meaningfully a maths problem in a personally novel way	1-9	Explain both strengths and limitations of different ways of posing or solving a math problem based on logical and possibly other criteria	
REFLECTING	Reflect on steps taken to pose and solve a maths problem	3,9	Reflect on the chosen maths approach and solution relative to possible alternatives	9

Appendix

Appendix 1: Presentations (example of one group)
➢ Students show their picture, maybe with the use of a video-projector or one student can come closer to the rest of their classmates to show the picture. ➢ It is important that students give a title to their artwork and explain to everyone what message they want to share. Note that sometimes students just list everything they put on the picture. This might be a sign of insufficient collaboration (maybe the group didn't manage to agree one idea, or everyone did what s\he wanted without any discussion, or the speaker did not do a lot during the group work). The teacher should not interrupt the presentation, but can put this information into the observation checklist.
Appendix 2: Calculations
➢ Note that calculation strategies can differ from group to group. The aim is to develop several strategies to solve the problem. ➢ Students should be encouraged to calculate perimeters/areas by using different strategies; for instance, after they calculate the total area through one strategy the teacher should ask them to calculate it again by using another strategy. ➢ It is also important to compare calculation strategies e.g. whether they yield different answers or not, which one is shorter, original, common, etc. ➢ Finding different strategies for adding the areas and realizing that the final answer is the same could also enhance mathematical creativity. ➢ Answers can also differ as figures can overlap, or one part of a figure can be covered with another, e.g. a mushroom, which is a square and part of a circle. ➢ Students should design their own strategies to deal with overlapping figures. The strategies are just about getting an <u>estimated</u> perimeter or area – it does not always have to be the exact perimeter or area, but a good estimate. Students should realise when it is just an estimate, and get a sense of the scope of error of their measures. **Some common strategies:** ➢ Add up the areas of all figures. An error can be no more than two units or 8 cm^2. ➢ The use of spare squares (4 squares will be just right to find the area). 1 square = "a unit". ➢ An evaluation of the entire white paper sheet area and subtraction of the uncovered area. ➢ The use of a self-made measuring grid. ➢ Count the area of the set and calculate the area of what was unused. ➢ Use tracing paper and draw a new figure whose perimeter or area is easier to calculate. ➢ Base calculations on the overlap of figures. ➢ If the figures are given to the students and they have to use all 20 figures without overlapping the area has to be the same, no matter which strategy they follow.