## The Math-mystery of the Egyptian Pyramids

Secondary: (ages 11-14) Mathematics

One of the greatest mysteries of mankind is how the ancient Egyptians were able to construct the pyramids of Giza. Scores of experts have studied these historical relics but, even with the advancement of modern technology, it remains an enigma. In this task, students use their knowledge of geometry, algebra, formulas related to polygons, and interpretation of diagrams to achieve a feat that rivals the ancient Egyptians: crafting plans and methods to create their own pyramids.
Time allocation 3-4 lesson periods

Subject content $\quad$| Calculate area, length, and other geometrical properties |
| :--- |
| Use metric tables to reproduce scales and diagrams |

Creative and This unit has a creativity and critical thinking focus:
critical thinking

- Generate and play with a range of strategies and unusual ideas to envision how to solve a real-life maths problem
- Consider different perspectives and reflect on strengths and limitations of proposals

Other skills Collaboration, Communication, Persistence/Perseverance

Key words Egypt, geometry, trigonometry; arithmetic; design; polygons; angles; pyramids; diagrams

## Products and processes to assess

In this exercise, students discuss and create construction plans for a pyramid and use creative and critical thinking to identify ways they can use mathematics to support this. They respond to mathematical challenges and respond critically and creatively to a series of questions about real-life applications of mathematics. At the highest levels of achievement, they consider several ways of formulating and answering problems, some of which are unusual, novel or original, and are able to explain and justify their positions. Their work process displays willingness to explore a range of ideas, give and receive feedback, and demonstrates an ability to see connections between ideas and domain.

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## Teaching and Learning plan

This plan suggests potential steps for implementing the activity. Teachers can introduce as many modifications as they see fit to adapt the activity to their teaching context.

| Step | Duration | Teacher and student roles | Subject content | Creativity and critical thinking |
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| 1 |  | The teacher_introduces the lesson with a brief history of the Egyptian civilization and the various challenges historians, archaeologists, and others have faced to understand how the pyramids were constructed. <br> One possibility is that the students now make a list of all of the questions they can think of about how the pyramids were built <br> As appropriate, this can be followed with an initial introductory discussion of possible ways in which different branches of mathematics can help to plan the design and construction of buildings. | Relating mathematical concepts to real life: geometry, trigonometry, etc., as appropriate to teaching context | Building understanding of the context of the problem <br> Posing questions to identify gaps in knowledge |
| 2 | Lesson period 1 | The teacher explains that students should make a building plan for a new pyramid and makes students aware of the resources they will have to guide their construction <br> *Workers have granite hammers and copper chisels. These are hard enough to chip out the stone blocks for the pyramids. <br> * Each block weights one to two tons. You will need to cut out and move two million of these to build your pyramid. Modern archaeologists believe most of the blocks came from quarries at the base of the pyramids so they only had to be moved a kilometer or two to the building site. <br> * You can have as many workers as you need. Ancient Egypt was a rich farming area with <br> a population of at least ten million, so two to three million adult males are potential <br> labourers. (Remember to leave a few people to do the farming!) <br> * You have a plentiful supply of rope - made from cat tails along the Nile's banks. You also have all the water, sand, and clay you need. You have a lot of short, soft timber pieces from the Nile palm trees, and you can import a small supply of good hardwood from the Lebanon. <br> Students organize themselves into teams of 4-5 and begin discussing the challenge. What do they need to work out in order to come up with a construction plan? (e.g. How will they know where to place materials and how will they transport them etc.) Teacher can visit each group to encourage students to be as imaginative in their thinking as possible, | Calculating area, length, and other geometrical properties <br> Using arithmetic to create work plans, etc. <br> Drawing on knowledge of angles and using geometry on a very large scale | Generating questions and playing with unusual ideas to work out how to use mathematics to approach a real-life problem. <br> Envisioning and planning how to meaningfully solve a problem <br> Considering different perspectives <br> Making connections between mathematical ideas, and historical architectural feats |


|  |  | help them organize their ideas, and prompt them to consider the challenges listed in the appendix, as appropriate. <br> Students then begin to work through the challenges, where necessary drawing on their knowledge of mathematics to generate possible solutions. Students should keep a record of their thinking, calculations, and diagrams used or produced at each step of the process | Producing calculations and diagrams to help in mathematical problem-solving |  |
| :---: | :---: | :---: | :---: | :---: |
| 3 | Lesson period 2 (this may continue into a third period) | The teacher checks on progress of groups thus far and provides guidance to continue with their work. Students can be encouraged to assess and revise their thinking as appropriate and the teacher may help them become aware of any assumptions they are making (e.g. regarding materials, people, time, resources etc.). If the teacher has a particular related area of mathematical knowledge they want to reinforce, they may choose to present this to the class as a whole at this point. <br> Students continue to work on their plans within their groups until they have finished, when they can be asked to present their work to the rest of the class if desired. | Opportunities to reinforce particular areas of mathematical knowledge as related to the local curriculum | Reflecting on steps taken so far, identifying and challenging own assumptions, and reviewing alternatives |
| 4 | Lesson period 3/4 | The teacher provides challenge questions for students to answer independently about the activity, i.e.: <br> 1. <br> a) What was the key function of a pyramid in ancient Egypt? <br> b) What was the significance of the pyramid shape in ancient Egyptian religion? <br> c) Why did pharaohs build such massive pyramids? What's the point in making them so large? <br> d) How did mathematics help you meet this challenge? <br> 2. What part of your work are you most proud of and why? What were the most puzzling parts of the pyramid-building problem for your team? Which parts of the exercise were the most difficult for your group? Why? <br> 3. Write an argumentative paragraph in response to the following question: <br> "From all the evidence we have looked at, which is more probable: that the ancient <br> Egyptians built the pyramids on their own or that they had help from extra-terrestrial beings" <br> Be sure to include references to actual physical evidence and pyramid facts in your answer. <br> 4. After looking at all the evidence we have studied in class, what remains the most puzzling or mysterious aspects of the pyramids for you? <br> - Overall what did you learn about ancient Egyptians from our pyramid building exercise? <br> - What did you learn about mathematics? | Summarizing how and why they used mathematics (arithmetic, geometry, trigonometry etc.) during the challenge | Reflecting on steps taken to pose and solve a problem using mathematics <br> Explaining strengths and acknowledging uncertainty and limits of proposed plans <br> Identifying and questioning convention, checking accuracy, and justifying opinion based on sound reasoning |

- What, if anything, are you still unsure or uncertain about?
- What was your personal contribution to the group's success? How did your group effectively work together?


## nowledge gained and remaining

 challenges
## Resources and examples for inspiration

## Web and print

> Blueprints, history, and picture of the Pyramids of Giza: https://en.wikipedia.org/wiki/Egyptian pyramids
Original Lesson Plan: http://schools.yrdsb.ca/markville.ss/history/16th/MysteriesofthePyramids.html
Other
> Projector, poster board, markers/pencils, lined/graph paper
Opportunities to adapt, extend, and enrich
> Links could be made to history, geography, visual arts, physical science, and intercultural understanding.

| Creativity and critical thinking rubric for mathematics | - Mapping of the different steps of the lesson plan against the OECD rubric to identify the creative and/or critical thinking skills the different parts of the lesson aim to develop |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | CREATIVITY <br> Coming up with new ideas and solutions | Steps | CRITICAL THINKING <br> Questioning and evaluating ideas and solutions | Steps |
| INQUIRING | Make connections to other maths concepts or to ideas from other disciplines | 1 | Identify and question assumptions and generally accepted ways to pose or solve a maths problem | 1,3,4 |
| IMAGINING | Generate and play with several approaches to pose or solve a maths problem | 2 | Consider several perspectives on approaching a maths problem | 2,3 |
| DOING | Pose and envision how to solve meaningfully a maths problem in a personally novel way | 2 | Explain both strengths and limitations of different ways of posing or solving a math problem based on logical and possibly other criteria | 4 |
| REFLECTING | Reflect on steps taken to pose and solve a maths problem | 3,4 | Reflect on the chosen maths approach and solution relative to possible alternatives | 3,4 |

## POSSIBLE CHALLENGES THAT STUDENTS CAN BE SET

1. It's easy to use chisels and granite hammers to bash out the top and sides of a pyramid block, but how do you cut out the base to free the block from the quarry wall? If you tunnel under the block won't it fall on you?
2. Design a simple, labour efficient way to move blocks from the quarry to the base of the pyramid. What materials do you need to move each block? How many people or animals will you need to move each block? How long will it take to move one block two kilometres to the pyramid site?
3. To ensure that the pyramid gets off to a good start you need to make sure the 65 hectare $(250 \mathrm{~m} \times 250 \mathrm{~m})$ base is perfectly smooth and level with no bumps, undulations or hollows more than .3 of a metre above or below a perfectly level surface. How can you make sure the site is this level? Devise a survey and inspection system to achieve this.
4. You need to make sure the initial floor plan of the pyramid is correct. This means the four base lines must be perfectly straight (the pyramid is over 250 m on each side) and each corner must be exactly 90 degrees - not 89.5 or 90.5 degrees. Devise a survey and inspection system to achieve this.

Note: you cannot use a protractor - it's not accurate enough.
Hint: use geometry on a very large scale!
5. The sides of the pyramid must rise up at exactly 51 or 53 degrees, with no deviation anywhere from bottom to top. Devise a system to make sure every outer block is at exactly the correct angle. For this problem you do have a protractor that can show you a 51 or 53 degree angle.
6. It's easy to get the first million blocks into place, but it becomes much harder to move the blocks up for the top half of the pyramid. Devise a system to move blocks up to the top half of the pyramid in a labour efficient manner. You need to raise them 60 to 140 m (the block at the very top is 140 m above ground level).

## Notes:

a) You cannot use pulleys - the Ancient Egyptians did not have them.
b) You cannot stand on one "step" and raise the block to the next "step". The edge for each step is too narrow and the slope is too steep. Your workers would fall off!
c) You cannot build a straight ramp up the side of the pyramid. This would require another 5 million blocks!
7. Work out the logistics of pyramid building:
a) List all the jobs directly necessary in building a pyramid.
b) Estimate how many people you need doing each job each day (on average), in order to cut out, move, raise, and correctly position two million blocks in only 25 years. For instance, how many blocks need to be cut each day and how many block cutters will you need each day.
c) What materials, tools, and supplies do you need to do the job? How much of each item do you need per year? (Assume each item will wear out after one year).
d) List the support materials and calculate the number of workers that will be needed per year. For example, how much food, what sort of pottery, what materials must be made by someone else for the workers' use. Prepare an estimate of everything and every job needed to work on the pyramid for one year.

Create an organizational structure showing who will oversee what, and who will report to whom.


[^0]:    This work was adapted by the OECD for the CERI project Fostering and assessing creativity and critical thinking skills from the activity "Mysteries of the Pyramids". It is available under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 IGO licence (CC BY-NC-SA 3.0 IGO). © OECD

