OECD Skills Studies

The Assessment Frameworks for Cycle 2 of the Programme for the International Assessment of Adult Competencies

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Foreword

The Programme of the International Assessment of Adult Competencies (PIAAC) is an international assessment of the literacy, numeracy and problem solving skills of adults. The first cycle of the study involved three rounds of data collection involving a total of 39 countries and regions. Preparations for Cycle 2 of PIAAC started in 2018. As part of this process, the assessments of literacy, numeracy and problem solving have been redeveloped to reflect contemporary understandings of these skills and to make use of the opportunities offered by advances in testing technologies since Cycle 1 of the study. This document contains the frameworks that define and describe the skills assessed in Cycle 2 of PIAAC.

PIAAC is a collaboration between the countries participating in the study, the international contractor (a consortium led by Education Testing Service) responsible for the development of the study instruments, survey procedures, quality control and data preparation and the Organisation for Economic Co-operation and Development (OECD) responsible for the management of the project on behalf of participating countries.

The project is steered by the PIAAC Board of Participating Countries. Over the course of the development of Cycle 2 of PIAAC, the Board has been chaired by Ted Reininga (the Netherlands), together with Aviana Bulgarelli of Italy (until September 2020) and, from October 2020, Katalin Zoltán (Hungary).

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The literacy framework was developed by the PIAAC Cycle 2 literacy expert group (LEG) under the leadership of Jean-François Rouet (Centre national de la recherche scientifique [CNRS], University of Poitiers, France). The members of the expert group contributing to the development of the literacy framework were Mary Anne Britt (Northern Illinois University, USA), Egil Gabrielsen (University of Stavanger, Norway), Johanna Kaakinen (University of Turku, Finland) and Tobias Richter (University of Würzburg, Germany).

The numeracy framework was developed by the PIAAC Cycle 2 numeracy expert group (NEG), chaired by Dave Tout (Australian Council for Educational Research, Australia). The members of the expert group contributing to the development of the numeracy framework were Isabelle Demonty (University of Liège, Belgium), Javier Díez-Palomar (University of Barcelona, Spain), Kees Hoogland (HU University of Applied Sciences Utrecht, the Netherlands), Terry Maguire (National Forum for the Enhancement of Teaching and Learning, Ireland) and Vince Geiger (Australian Catholic University, Australia).

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Table of contents

Foreword 3

Acknowledgements 5

Executive summary 11

1 The assessment frameworks for Cycle 2 of PIAAC: An introduction and overview 13
- Introduction 14
- The PIAAC assessment 14
- Assessment frameworks 19
- The evolution of assessment frameworks in international adult assessments 21
- Relationship of the PIAAC and PISA assessments 27
- The framework documents 28
- References 29
- Annex 1.A. Summary of the evolution of assessment frameworks – from IALS to PIAAC Cycle 2 32
- Notes 37

2 PIAAC Cycle 2 assessment framework: Literacy 39
- Introduction 40
- Definition of literacy 42
- Core dimensions of the literacy domain 45
- Assessing literacy 53
- Recommended distribution of items 54
- The role of fluent reading, engagement and metacognition 55
- Factors driving task difficulty 56
- References 58
- Note 64

3 PIAAC Cycle 2 assessment framework: Numeracy 65
- Introduction 66
- The assessment of numeracy in PIAAC 67
- Conceptual and theoretical foundations 71
- Numeracy assessment construct in PIAAC Cycle 2 91
- Operationalisation of the PIAAC numeracy assessment 110
- Relationship between PIAAC and PISA 124
- Numeracy components 127
- References 139
- Notes 153

4 PIAAC Cycle 2 assessment framework: Adaptive problem solving 155

Introduction 156
Definition of adaptive problem solving 158
Core dimensions of the APS domain 166
Assessing adaptive problem solving 188
Adaptive problem solving in the nexus of related constructs and implications for PIAAC Cycle 2 194
References 197
Annex 4.A. Description of difficulty drivers 201

FIGURES

Figure 1.1. Assessment design: PIAAC Cycle 2 17
Figure 1.2. Sample PIAAC test item (Numeracy) 18
Figure 2.1. Three core cognitive processes supporting literacy 45
Figure 3.1. Use of the term numeracy versus other terms in the books published between 1950 and 2008, included in Google Books 73
Figure 3.2. The PISA 2012 model of mathematical literacy in practice 84
Figure 3.3. The PISA 2021 model of mathematical literacy: the relationship between mathematical reasoning and the problem solving (modelling) cycle 85
Figure 3.4. Model for numeracy in the twenty-first century 86
Figure 3.5. Components of number sense 134
Figure 4.1. The nexus of task dimensions, metacognitive and cognitive processes, and proficiency levels 167
Figure 4.2. Adaptive problem solving 170

TABLES

Table 1.1. Countries and regions participating in PIAAC 15
Table 1.2. Key features of the PIAAC study design 16
Table 1.3. Main features of the assessment frameworks for PIAAC Cycle 2 19
Table 1.4. Domains assessed in IALS, ALL and PIAAC 22
Table 2.1. Correspondence between the processes in PIAAC Cycle 2, PIAAC Cycle 1 and PISA 2018 45
Table 2.2. Summary of different types of evaluation processes 49
Table 2.3. Main dimensions of texts considered in the PIAAC literacy framework 52
Table 2.4. Recommended distribution of items as a function of text size and cognitive task demands 55
Table 2.5. Text, task, and text-by-task factors driving difficulty as a function of cognitive task demands 57
Table 3.1. Four categories of numeracy use and their connections with technology 83
Table 3.2. Cognitive processes labels in PIAAC and PISA 100
Table 3.3. Mathematical content labels in PIAAC and PISA 104
Table 3.4. Context labels in PIAAC and PISA 107
Table 3.5. Representation of cognitive processes in PIAAC and PISA 122
Table 3.6. Representation of content areas in PIAAC and PISA 122
Table 3.7. The representation classifications in PIAAC 122
Table 3.8. Representation of context in PIAAC and PISA 123
Table 4.1. Descriptions of the three APS proficiency levels proposed 186
Table 4.2. Proposed distribution of the information environments and problem contexts 188
Table 4.3. Proposed distribution of the three main cognitive and metacognitive processes 189
Table 4.4. PIAAC Cycle 2 APS and literacy assessments 194
Table 4.5. PIAAC Cycle 2 APS and digital competence 195
Table 4.6. PIAAC Cycle 2 APS and PIAAC Cycle 1 PS-TRE 197

Annex Table 1.A.1. Literacy (Reading) 32
Annex Table 1.A.2. Managing numerical and mathematical information 34
Annex Table 1.A.3. Problem solving 36
Annex Table 4.A.1. Description of difficulty drivers 201

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Executive summary

This publication contains the frameworks for the assessment of literacy, numeracy and adaptive problem solving in the second cycle of the OECD's Programme for the International Assessment of Adult Competencies (PIAAC Cycle 2).

The assessment frameworks represent key documents for understanding what is measured by PIAAC and for interpreting its results.

The introductory chapter provides an overview of the PIAAC assessment and its relationship to previous international assessments of adult skills. It also describes the purpose of the assessment frameworks and the evolution of the concepts of literacy, numeracy and problem solving since the first international assessment of adult literacy was conducted in the mid-1990s as well as the relationship of PIAAC to the OECD's assessment of 15-year-old school students, PISA.

The individual frameworks are presented in separate chapters: literacy (Chapter 2), numeracy (Chapter 3) and adaptive problem solving (Chapter 4). They define the particular skills assessed, describe their salient features, outline a recommended approach to the assessment of the skill and identify other matters relevant to test development. The similarities and differences with the frameworks of previous assessments are outlined with a focus on the social, theoretical and measurement considerations that have contributed to the development of the frameworks over time.

1 The assessment frameworks for Cycle 2 of PIAAC: An introduction and overview

This chapter introduces the assessment frameworks that define and describe the skills assessed in Cycle 2 of PIAAC. It provides some background to the PIAAC assessment, outlines the purposes of the assessment frameworks and explains how the understanding and conception of the skills measured in PIAAC has evolved over time.

Introduction

This volume contains the frameworks for the assessment of literacy, numeracy and adaptive problem solving in the second cycle of the OECD's Programme for the International Assessment of Adult Competencies (PIAAC Cycle 2). This introductory chapter provides some context and background to the study as well as to the frameworks guiding the assessment. In particular, it describes:

- the main features of the PIAAC assessment and how it relates to previous international assessments of adult literacy, numeracy and problem solving
- the purposes of the assessment frameworks
- the way in which the constructs assessed in PIAAC and its predecessors have been conceived.

The PIAAC assessment

PIAAC is an international assessment of the information-processing skills of adults. It assesses three broad skills: reading and understanding written texts (literacy), understanding and using mathematical and numerical information (numeracy) and solving problems. A comprehensive background questionnaire is also administered in conjunction with the assessment.

PIAAC is the third in a series of international adult assessments conducted since the mid-1990s. It builds on the experience of the International Adult Literacy Survey (IALS) and the Adult Literacy and Life Skills Survey (ALL).[1] IALS collected data in three waves between 1994 and 1998 in 22 countries and regions. ALL collected data in two waves over the period 2002-2008 in 11 countries and regions.

The study is designed as a repeated cross-sectional study that provides comparable estimates of proficiency in literacy and numeracy over time. The first cycle of the assessment took place over the period 2008-2019 with three data collection rounds: the first in 2011-12, the second in 2014-15 and the third in 2017-18.[2] A total of 39 countries/regions took part in the first cycle of the study and 33 are currently preparing to collect data in the second cycle (see Table 1.1). Preparations for Cycle 2 of the assessment began in 2018. Data collection was originally planned for 2021-22, ten years after data collection in the first round of Cycle 1, but due to the Covid-19 crisis of 2020 which delayed the Field Trial, it has been rescheduled to 2022-23.

Data are collected in PIAAC using a combination of personal interview and a self-completed assessment. Data collection takes place in the respondent's own home[3] under the supervision of trained interviewers. The background questionnaire is administered in Computer Aided Personal Interview (CAPI) mode by the interviewer. Following completion of the background questionnaire, the respondent completes the assessment under the supervision of the interviewer. In the first cycle of the study, the assessment could be completed on a laptop computer or in paper-and-pencil format. The computer-based assessment (CBA) format constituted the default format with the paper-based assessment (PBA) option being made available to those respondents who had little or no familiarity with computers, had poor information communications technology (ICT) skills, or who did not wish to take the assessment on computer. In the second cycle of the study, the assessment will be delivered on a tablet device. The assessment interface has been designed in such a way as to ensure that most, if not all, respondents will be able to take the assessment on the tablet even if they have limited experience with such devices. It will still be possible for participating countries to provide a paper-based option to respondents who cannot or are unwilling to take the assessment on the tablet.

Table 1.1. Countries and regions participating in PIAAC

PIAAC Cycle 1			PIAAC Cycle 2
Round 1	Round 2	Round 3	
Main study 2011-12	Main study 2014-15	Main study 2017-18	Main study 2022-23
Australia	Chile	Ecuador	Australia
Austria	Greece	Hungary	Austria
Canada	Jakarta (Indonesia)[2]	Kazakhstan	Canada
Cyprus[1]	Israel	Mexico	Chile
Czech Republic	Lithuania	Peru	Croatia
Denmark	New Zealand	United States	Czech Republic
England (UK)	Singapore		Denmark
Estonia	Slovenia		England (UK)
Finland	Turkey		Estonia
Flanders (Belgium)			Finland
France			Flanders (Belgium)
Germany			France
Ireland			Germany
Italy			Hungary
Japan			Ireland
Korea			Israel
Netherlands			Italy
Northern Ireland (UK)			Japan
Norway			Korea
Poland			Latvia
Russian Federation			Lithuania
Slovak Republic			Netherlands
Spain			New Zealand
Sweden			Norway
United States[3]			Poland
			Portugal
			Russian Federation
			Singapore
			Spain
			Sweden
			Switzerland
			United States

1. Note by Turkey:
The information in this document with reference to "Cyprus" relates to the southern part of the Island. There is no single authority representing both Turkish and Greek Cypriot people on the Island. Turkey recognises the Turkish Republic of Northern Cyprus (TRNC). Until a lasting and equitable solution is found within the context of the United Nations, Turkey shall preserve its position concerning the "Cyprus issue".
Note by all the European Union Member States of the OECD and the European Union:
The Republic of Cyprus is recognised by all members of the United Nations with the exception of Turkey. The information in this document relates to the area under the effective control of the Government of the Republic of Cyprus.
2. Indonesia's data was subsequently withdrawn.
3. The United States also collected data as part of a PIAAC National Supplement in 2013-14. This included representative samples of a) unemployed adults (aged 16-65); b) young adults (aged 16-34) and c) older adults (aged 66-74). See Krenzke et al. (2019[1]) for details.

The basic specifications for the design of PIAAC (common across the two cycles of the study) are summarised in Table 1.2. More details regarding Cycle 1 of the study can be found in PIAAC (2014[2]).

Table 1.2. Key features of the PIAAC study design

Target population	Non-institutionalised adults aged 16-65 years normally resident in the national territory of the participating country.
Sample frame	The sample frame should cover the target population. Exclusions of up to 5% of the target population permitted.
Sample design	Probability-based sample with each individual in the target population having a known probability of selection.
Sample size	Minimum sample size of 5 000 completed cases per reporting language.
Data collection method	Computer-aided personal interview and self-completed assessment under the supervision of the interviewer.
Mode of assessment	Computer (Cycle 1) and tablet (Cycle 2) delivered assessment with a paper-based alternative for respondents with insufficient experience of the use of digital devices.
Quality assurance and quality control	Central review of key elements of the study such as sampling, translation and adaptation of instruments. Monitoring of data collection. Data adjudication based on indicators of data quality.

Instrumentation

As noted above, respondents complete both a background questionnaire and a skills assessment.

The background questionnaire in PIAAC Cycle 2 will consist of 11 modules collecting information on demographic characteristics, social and language background, education, labour-force participation, employment, the task composition of the respondent's job, literacy and numeracy practices and personality traits.[4]

The direct assessment involves the following components:

- a locator test
- an assessment of reading and numeracy components
- assessments of literacy, numeracy and adaptive problem solving.

The *locator test* consists of eight literacy and eight numeracy items of low difficulty. It is designed to provide an initial estimate of the proficiency of the respondent. This is used to direct the respondent to the testing pathway appropriate to his/her proficiency (see below).

The *reading and numeracy components assessment* consists of set of items assessing:

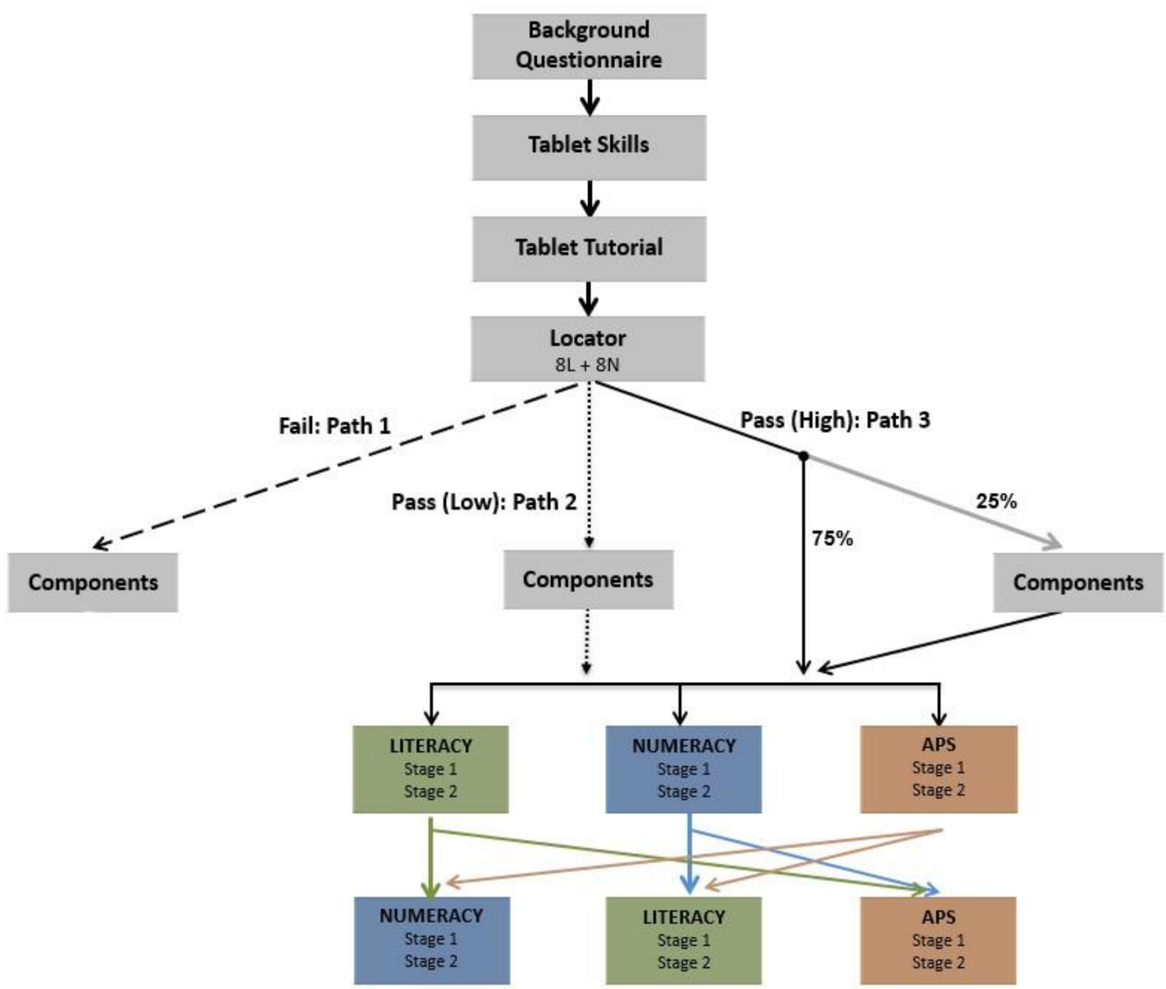
- the ability to understand the meaning of simple sentences and to read and understand short passages fluently (reading)
- understanding basic notions of quantity and magnitude (numeracy).

The *assessments of literacy, numeracy and adaptive problem solving* each consist of around 80 items. Any individual respondent is administered test items covering only two of the three domains and in each of these domains he or she is presented a subset of the test items. In all three domains, the assessments use an adaptive design. The goal is to maximise the efficiency and precision of the assessment by presenting respondents with test items that are neither too easy nor too difficult for them.

In each domain, the assessment consists of a set of units in which each unit is made up of one or more stimuli (e.g. a description of a problem situation, a text, a table – see Figure 1.2 below) and a set of questions or tasks. These units are combined into groups called 'testlets' with different average levels of difficulty. The testlets are presented to respondents in two stages. Information from the background questionnaire, the component measures and the locator are used to assign a testlet that is most appropriate for the respondent at Stage 1. The respondent's performance on the Stage 1 testlet is automatically scored. The test application then assigns a second testlet to the respondent based on his/her performance on the first. While all respondents have a small probability of being allocated any testlet, they have a greater probability of being allocated a testlet closer to their estimated proficiency. For example, at each stage in the assessment, a respondent of high estimated ability has a greater chance of being allocated a testlet of high average difficulty than does a respondent with lower estimated proficiency.

The design for the main study in PIAAC Cycle 2 is presented in Figure 1.1 below. The background questionnaire is administered in CAPI mode by the interviewer and is estimated to take 20-45 minutes to complete depending on the situation of the respondent (with an average of around 30 minutes). The direct assessment is completed by the respondent on a tablet device supplied by the interviewer. The average time for completion of the assessment is estimated to be 60 minutes. However, as PIAAC is not a timed assessment, actual completion times are expected to vary widely.

Figure 1.1. Assessment design: PIAAC Cycle 2



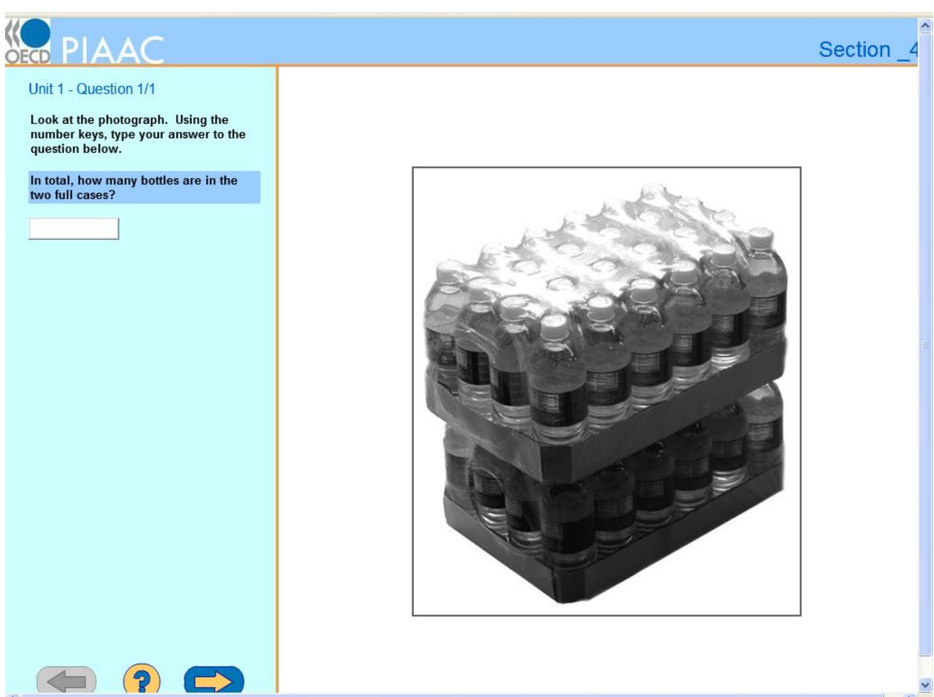
Respondents undertake the assessment in the following sequence:

- The interviewer administers the background questionnaire. The background questionnaire is answered by all respondents and includes a set of questions dealing with the familiarity of the respondent with electronic devices.

- After agreeing to continue with the survey, the respondent is handed the tablet device on which he/she completes the assessment. The interviewer demonstrates the basic skills required to complete the direct assessment tasks, e.g., tapping, using drag and drop, and highlighting.
- Respondents then complete a tutorial in which they perform each of the skills independently.
- The respondent then completes the locator test.
- Depending on their responses to relevant background items and their performance on locator tasks, respondents are directed to one of three paths:
    - Respondents who ‘fail’ the locator follow path 1 and receive the reading and numeracy components only.
    - Respondents who ‘pass’ the locator, but perform relatively poorly, follow path 2 and receive the components plus the two-stage adaptive modules of literacy, numeracy, or adaptive problem solving (APS).
    - Respondents who perform well on the locator test follow path 3. A quarter of these respondents are randomly assigned to the reading and numeracy components assessments before moving on to the two-stage adaptive modules of literacy, numeracy, or adaptive problem solving (APS), while the other 75% of respondents proceed directly to the two-stage cognitive modules.

The assessment tasks in PIAAC consist of 1) a set of instructions and a question or task statement that defines what the respondent must do to complete the task, 2) stimulus material (e.g. texts, graphic representations, simulated websites) with which the respondent must interact to complete the task and 3) a means of registering a response. All items in the assessment have the same format. The instructions to the respondent and the task question/statement together with forward and back arrows and access to help are on the left-hand side of the screen with the stimulus materials(s) on the right. Responses are recorded on the left-hand side as in the sample item below or through interaction with the stimulus material. Figure 1.2 provides an example of a PIAAC computer-based test item.

Figure 1.2. Sample PIAAC test item (Numeracy)



The response modes used in PIAAC Cycle 1 were numeric entry, clicking on multiple choice check boxes, radio buttons and pull-down menus (left-hand side of the screen), and highlighting or clicking on elements in the stimulus material – text, graphic element, links (in simulated we environments) and check boxes (right-hand side) [see OECD (2019[3]), Section 5.2.1)]. In PIAAC Cycle 2, similar response modes will be used with the interaction with the test application interface being via the use of a stylus or tapping with fingers. A simulated calculator will be used for numeric entry. No constructed responses are used in PIAAC.

Assessment frameworks

In large-scale international assessments, the constructs measured are usually described by an assessment framework.[5] The framework has a dual purpose: 1) to guide the development of the items (tasks) used to assess the skill in question and 2) to guide the interpretation of the results of the assessment. To this end, the framework provides a definition and detailed description of the features of the construct assessed. In addition, it outlines the recommended approach to the assessment of the skill in question and identifies (e.g. the recommended coverage of the various aspects or dimensions of the construct) and discusses other matters relevant to test development such as the factors that affect the difficulty of items.

Table 1.3. Main features of the assessment frameworks for PIAAC Cycle 2

	Literacy	Numeracy	Adaptive Problem Solving
Definition	Literacy is accessing, understanding, evaluating and reflecting on written texts in order to achieve one's goals, to develop one's knowledge and potential and to participate in society.	Numeracy is accessing, using and reasoning critically with mathematical content, information and ideas represented in multiple ways in order to engage in and manage the mathematical demands of a range of situations in adult life.	Adaptive problem solving involves the capacity to achieve one's goals in a dynamic situation, in which a method for solution is not immediately available. It requires engaging in cognitive and metacognitive processes to define the problem, search for information, and apply a solution in a variety of information environments and contexts.
Cognitive processes	• Accessing text • Understanding • Evaluating	• Access and assess situations mathematically • Act on and use mathematics • Evaluate, critically reflect, make judgements	• Definition • Searching • Application
Content	Texts characterised by their: • Type (description, narration, exposition, argumentation, instruction, transaction) • Format (continuous, non-continuous, mixed) • Organisation (the amount of information and the density of content representation and access devices) • Source (single vs. multiple texts)	Mathematical content information and ideas • Quantity and number • Space and shape • Change and relationships • Data and chance Mathematical representations • Text or symbols • Images of physical objects • Structured information • Dynamic applications	Task dimensions • Problem configuration • Dynamics of the situation • Features of the environment • Information environment
Contexts	• Work and occupation • Personal • Social and civic	• Personal • Work • Societal/community	• Personal • Work • Social/community

In PIAAC, the skills assessed are described in terms of 1) a broad definition, and 2) the dimensions of:

- Cognitive processes: the mental processes that form part of the skill in question.
- Content: the artefacts, knowledge, representations, situations that constitute the 'object(s)' to which these cognitive processes are applied.
- Contexts: the settings in which the skill is used.

The main components of the assessment frameworks for PIAAC Cycle 2 are summarised in Table 1.3.

Some of the key implications for the assessment of these skills arising from the frameworks are briefly discussed below.

Coverage of the constructs

In order for the assessment to represent the construct adequately, the set of tasks that constitute the assessment must include tasks designed to cover the range of cognitive processes, type of content and contexts identified by the framework. To this end, each of the framework documents proposes a desirable distribution of tasks across the different dimensions of the framework.

Factors affecting the difficulty of assessment tasks

The PIAAC assessment is intended to measure the entire range of proficiency in the skills of interest that exists in the adult population – from very low to very high. The adult population in participating countries includes individuals who have completed no more than primary education as well as adults who have completed post-doctoral studies. In addition, in countries with relatively high levels of immigration, a substantial proportion of the population may have limited proficiency in the language or languages in which the assessment is delivered.[6]

The frameworks identify the factors that affect task difficulty and can be manipulated to ensure that tasks covering the full spectrum from very easy to very difficult are included in the assessments. In broad terms, these can be categorised as encompassing features of:

- the task statement (e.g. the instructions provided to test-takers, the explicitness of the presentation and definition of the task to be completed)
- the stimulus material (e.g. its complexity, length, organisation)
- the interaction of task and stimulus (e.g. the presence of distracting/irrelevant material, the number of operations/steps required to be undertaken to successfully complete the task).

Authenticity of tasks

The skills assessed in PIAAC are primarily conceived as skills that enable adults to engage and function effectively in social and economic life and perform the range of tasks required in their various social roles. In line with this focus, assessment tasks are intended to represent the types of reading, mathematical and problem solving demands and situations that the generality of adults face in their everyday lives. In the words of the numeracy framework document: 'PIAAC is interested in the ability of individuals to cope with tasks that are embedded in the real world, rather than assessing decontextualised mathematical tasks'. Stimulus materials (e.g. the texts that respondents must read, the presentations and representations of numerical and mathematical information and problem situations to which they must respond) represent the kinds of texts, mathematical information and problems that adults encounter in 'real-world' situations. Regarding the stimulus material used in literacy tasks, for example:

> *Many of them are directly drawn from authentic materials with little, if any adaptation. This means that no effort is made to make these texts easier to read or to improve their organisation or presentation. Using naturalistic*

> *texts, sometimes even clearly suboptimal ones (for instance, poorly organised or using complex language), ensures a high level of face validity. However, no artificial difficulty or flaw is introduced at the time of test design. (see literacy framework)*

Content appropriate to the entire adult population

As PIAAC is an assessment of the entire adults aged 16-65 years, the assessment tasks do not assume highly technical or occupation-specific knowledge. In addition, they do not assume knowledge or skills relevant in formal educational settings such as the use of formal mathematical notation and symbolisation. This reflects the fact that there are countries in which a significant proportion of adults (especially older adults) have very low educational attainment and, more importantly, the reality that most adults left the formal education system long ago. In the case of adults aged 55-65 years, for example, most will have completed their education between 40-50 years ago.

Assessment at low skill levels: Reading and numeracy components

One of the challenges in the assessment of the information-processing skills of adults is to gain information regarding the skills of adults with low proficiency. Low skills are manifested through the inability of a test-taker to successfully complete most tasks in the assessment. In other words, for this group, a lot is known about what they *cannot do* and little about what they *can do*.

To provide more information regarding the skills of low-skilled readers, an assessment of reading component skills was introduced in PIAAC Cycle 1 (Sabatini and Bruce, 2009[4]). This covered three skills: print vocabulary, sentence processing and passage fluency. Print vocabulary assessed basic vocabulary knowledge, sentence processing evaluated the ability to understand the semantic logic of simple sentences, and passage fluency assessed the capacity to understand passages of text. Reading components will continue to be assessed in PIAAC Cycle 2 with some modifications. Only two skills (sentence processing and passage fluency) will be assessed.

An assessment of numeracy components has been developed and will be administered as part of PIAAC Cycle 2. This involves two types of tasks designed to measure number sense: 1) identifying how many objects are displayed in photographs of real-life items, and 2) selecting the biggest number from a set of four choices.

No components measures have been developed in the domain of APS. The experience with previous assessments of problem solving has been that a reasonable level of proficiency in literacy and numeracy is a precondition for the successful completion of problem solving items. This is expected to be true also for the assessment of APS. As can be seen from the presentation of the drivers of task difficulty in APS (APS Framework, Table 4.A1.1), even simple problems have a level of complexity and difficulty far in excess of the type of tasks forming the literacy and numeracy components measures.

The evolution of assessment frameworks in international adult assessments

As noted above, PIAAC Cycle 2 is the latest in a series of related international assessments of adults. Table 1.4 presents the domains assessed in each successive study from IALS to PIAAC Cycle 2. The domains in which results are psychometrically linked and can be compared over time are indicated by shading.

The assessment frameworks in each of broad domains assessed in adult skills assessments have evolved considerably since IALS was conducted in the mid- to late-1990s. This is most obvious in the domain of problem solving where different (albeit related) constructs were measured in ALL and PIAAC Cycles 1 and 2 and that of managing numerical and mathematical information where the construct of numeracy was introduced in ALL in place of that of quantitative literacy. However, even within the domains of reading and of numeracy, there has been considerable change in the conceptualisation of the constructs between assessments. These changes are briefly described below and summarised in Tables 1.A.1-1.A.3 in Annex 1.A.

Table 1.4. Domains assessed in IALS, ALL and PIAAC

	Domains assessed						
	Reading		Managing numerical and mathematical information		Problem solving		
IALS	Prose Literacy	Document Literacy	Quantitative Literacy				
ALL	Prose Literacy	Document Literacy		Numeracy	Analytic Problem Solving		
PIAAC Cycle 1	Literacy + Literacy Components			Numeracy		Problem Solving in Technology-Rich Environment	
PIAAC Cycle 2	Literacy + Literacy Components			Numeracy + Numeracy Components			Adaptive Problem Solving

Understanding the evolution of the assessment frameworks and, therefore, of the constructs measured is important for the interpretation of the distributions of skills observed both within and between assessments. The link between the most recent and the older assessments becomes more attenuated over time as the constructs continue to evolve. While the different international adult assessments have been designed to be linked psychometrically in the domains of literacy (IALS and its successors) and numeracy (ALL and its successors), the constructs measured have undergone considerable revision and extension even if a common core remains. Literacy as it will be measured in PIAAC Cycle 2 in 2022-23 is not exactly the same as literacy as measured in PIAAC Cycle 1, ALL and IALS, and the same is true for numeracy. In particular, although IALS and ALL recognised the growing importance of electronic texts, those two earlier assessments were delivered only on paper. Starting with PIAAC Cycle 1, the assessment moved to computer delivery which provided a means to include various types of electronic texts and materials.

The evolution of the assessment frameworks in large-scale assessments (including adult assessments) is the outcome of competing demands: on the one hand, the desire for continuity in measures (to provide reliable measures of change over time) and, on the other, the need for measures to be relevant to contemporary realities and understandings of the phenomena measured. Three main factors push in the direction of change: developments in the understanding of the skills measured, technological and social developments that affect the nature and practice of these skills in everyday life, work and study and technological and methodological advances in the science and practice of measurement.[7]

The assessment of problem solving provides a particular illustration of the impact of the forces that lead to change in large-scale assessment. Of the domains assessed in PIAAC and its predecessors, it is the one in which the impact of the introduction of computer-based testing has been greatest as it opened up possibilities for its assessment that did not exist in a world of paper-based tests. In addition, the demand for measures of problem solving that speak to current understandings of the phenomenon has been evident in the changes in the points of view from which the assessment of problem solving have been approached over time.

As in any area of scientific endeavour, the understanding of the skills assessed in large-scale assessments changes over time. This is a consequence of theoretical developments as well as reflection on the outcomes of empirical research including the results of large-scale assessments themselves.

Comprehensive discussions of the theoretical and conceptual considerations that led to the development of the assessment of APS and to the substantial revision of the numeracy assessment framework in Cycle 2 of PIAAC, can be found in Greiff et al. (2017[5]) for APS and Tout et al. (2017[6]) and Tout (2020[7]) for numeracy as well as in framework documents included in this publication.

The nature of skills such as literacy, numeracy and problem solving has changed in many ways since the early 1990s. Information and communications technologies have altered what it is to read, engage with numerical and mathematical information and solve problems by changing the ways in which information is accessed, communicated and analysed and transformed. For example, print-based texts and representations constituted the source of much of the information accessed by adults in the mid-1990s. At the start of the third decade of the 21st century, electronic texts and representations accessed through digital devices (e.g. computers, tablets, and smartphones) and applications (e.g. web browsers, hypertext, pdf and html files) have become primary sources of information. This has involved the appearance of new types of texts and representations; new forms of navigation within and between texts and representations (scrolling, clicking on icons or radio buttons, hyperlinks); and new tools for the processing and communication of information and increased interlinkages between texts, documents and representations (hypertext, strings of related texts). In addition, on-line service delivery has increased the information-processing demands on adults through the reduction (or removal) of the role of intermediaries in providing access to information and assistance with decision making in many domains (e.g. health, finances and travel).

ICTs have also transformed assessment. The introduction of computer-based assessment (CBA) has had a major impact on the design, delivery and processing of assessments and on the quality, amount and complexity of the resulting data. It has made possible the assessment of proficiency in the digital dimensions of information-processing skills (e.g. the reading of electronic texts, interaction with digital tools presenting and transforming mathematical information, the use of ICT applications to access and transform information to solve problems). It has also enabled the development of more complex assessment tasks. For example, digital assessment platforms make it possible to design tasks that are iterative in nature, and in which not all information is given as part of the initial conditions, as well as tasks involving complex displays of information, modelling and exploration of variation in a range of parameters. This is particularly important in the assessment of problem solving. The introduction of CBA has also permitted the implementation of more complex and efficient test designs (e.g. adaptive testing) as well as features such as automatic scoring. It has also allowed the development of more efficient and timely quality assurance and control procedures and considerably increased the possibilities of identifying data fabrication and fraud. The availability of log-files in which interactions between test-takers and the testing application are captured and stored has provided a new and rich source of data for analysts and test developers interested in understanding test-taking behaviour.[8]

The introduction of CBA as the default assessment mode in PIAAC Cycle 1 constituted one of the major factors influencing the evolution of the assessment frameworks of adult skills assessments between IALS and PIAAC. This made it possible for PIAAC to 1) reflect the changes in the practices of reading, managing mathematical and numerical information and problem solving brought about by the diffusion of digital tools and media in the way it assessed these skills and 2) use much more efficient test designs for adults.

Box 1.1. Assessment frameworks for previous assessments of adult literacy

Presentations of the assessment frameworks for IALS, ALL and PIAAC Cycle 1 can be found in the following documents:

IALS

Murray, S., I. Kirsch and L. Jenkins (eds) (1998[8]), *Adult Literacy in OECD Countries: Technical Report on the First International Adult Literacy Survey*, National Center for Education Statistics, Office of Educational Research and Improvement, Washington, DC.

OECD/Statistics Canada (2000[9]), *Literacy in the Information Age: Final Report of the International Adult Literacy Survey*, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264181762-en.

ALL

Murray, S., Y. Clermont and M. Binkley (eds) (2005[10]), *Measuring Adult Literacy and Life Skills: New Frameworks for Assessment*, Statistics Canada, Ottawa, Catalogue No. 89-552-MIE, No. 13.

PIAAC Cycle 1

OECD (2012[11]), *Literacy, Numeracy and Problem Solving in Technology-Rich Environments: Framework for the OECD Survey of Adult Skills*, OECD Publishing, Paris, http://dx.doi.org/10.1787/9789264128859-en.

PIAAC Expert Group in Problem Solving in Technology-Rich Environments (2009[12]), "PIAAC Problem Solving in Technology-Rich Environments: A Conceptual Framework", *OECD Education Working Papers*, No. 36, OECD Publishing, Paris, http://dx.doi.org/10.1787/220262483674.

PIAAC Literacy Expert Group (2009[13]), "PIAAC Literacy: A Conceptual Framework", *OECD Education Working Papers*, No. 34, OECD Publishing, Paris, http://dx.doi.org/10.1787/220348414075.

PIAAC Numeracy Expert Group (2009[14]), "PIAAC Numeracy: A Conceptual Framework", *OECD Education Working Papers*, No. 35, OECD Publishing, Paris, http://dx.doi.org/10.1787/220337421165.

Sabatini and Bruce (2009[4]), "PIAAC Reading Component: A Conceptual Framework", *OECD Education Working Papers*, No. 33, OECD Publishing, Paris, http://dx.doi.org/10.1787/220367414132.

Developments in literacy

The evolution of the constructs of literacy from IALS to PIAAC Cycle 2 has occurred in four main areas: 1) a reduction of the number of separate domains of literacy assessed, 2) the expansion of the range of text types covered in the assessment, 3) an increasing emphasis placed on evaluation and evaluating metacognition as cognitive strategies required for effective reading and 4) the disentangling of the description and specification of cognitive strategies from questions of task difficulty.[9]

In IALS, three separate domains of literacy were assessed and represented by separate scales: prose, document and quantitative literacy (Murray, Kirsch and Jenkins, 1998[8]). Prose literacy covered the reading of continuous texts or texts organised in paragraphs. Document literacy covered the reading of written information presented in matrix formats (e.g. tables and lists). Quantitative literacy represented the knowledge and skills required to apply arithmetic operations to numbers embedded in printed materials. ALL continued to assess prose and document literacy as separate domains (Murray, Clermont and Binkley, 2005[10]). However, the assessment of quantitative literacy was dropped in ALL and replaced by the assessment of numeracy (see below). The construct of 'literacy' as a single domain was introduced in PIAAC Cycle 1.

'Literacy' as defined in PIAAC Cycle 1 represented a global construct that no longer differentiated between the reading of prose and document texts. The other major (and probably the most significant) development was the expansion of the range of texts covered by the assessment to include digital or electronic texts.[10] In PIAAC Cycle 2, the classification of texts has been revised to include the dimensions of organisation (density of content, representations and access devices) and source (single or multiple authors/publishers) to better represent the universe of texts accessible in digital environments, including the interactive texts typical of the Web 2.0.

The conceptualisation of the cognitive strategies brought into play by competent readers has also evolved between assessments. In IALS/ALL, the cognitive strategies were conceived in terms of the 'matching' of information in the question (the given information) to the information in the stimulus text to respond correctly to a question or directive. These 'matching' strategies included the identification of pieces of information in the text (locating/cycling), connecting different parts of the text (integrating), and developing some understanding of the text as a whole (generating). In PIAAC Cycle 1, 'evaluation and reflection' (the making of judgements regarding aspects of the text such as truthfulness, relevance and quality) was added as a cognitive strategy required of competent readers. The dimension of evaluation has been further emphasised in Cycle 2 where it is conceived in terms of the evaluation of the accuracy, soundness, and task relevance of a text in relation to both its source and content.

There has also been a gradual separation of the identification and description of cognitive processes involved in literacy from the description of the factors that make assessment tasks more or less difficult. In IALS/ALL, matching strategies were treated as one of the three main factors determining task difficulty, the second being the type of information requested by the question and the third, the plausibility of distractors (the presence of other information in the stimulus text that could distract the test-taker's attention from the information needed to answer the question) (Murray, Clermont and Binkley, 2005, pp. 101-103[10]). The Cycle 2 framework treats cognitive strategies and the factors affecting task difficulty independently. Task difficulty is conceived as being driven by the features of the stimulus text(s), the formulation of the question/task description and the interaction of the text and question/task description (see literacy framework, Table 2.5).

The assessment of reading components was another new element of the assessment of literacy introduced in PIAAC Cycle 1 (Sabatini and Bruce, 2009[4]) to provide more detailed information about adults with poor literacy skills. Reading components were defined as the basic set of decoding skills essential for extracting meaning from written texts: knowledge of vocabulary (word recognition), the ability to process meaning at the level of the sentence, and fluency in reading passages of text. In PIAAC Cycle 2, the assessment of reading components will be continued but cover only the sentence meaning and passage fluency dimensions. Performance on the reading components tasks will also be integrated as part of the literacy proficiency scale in Cycle 2,[11] adding precision to its lower end.

Developments in numeracy

The measurement of 'numeracy' was introduced in ALL. This replaced the assessment of 'quantitative literacy' conducted in IALS. The rationale for the development of an assessment of numeracy was that the assessments of quantitative and document literacy represented 'only a subset of the much wider range of tasks and responses that are typical of many every day and work tasks' (Murray, Clermont and Binkley, 2005, p. 148[10]) relating to the engagement with mathematical information. In particular, key aspects of mathematical information such as measurements and shapes as well as information in formats that did require comprehension of text were not covered. The construct of 'numeracy' was developed to more comprehensively cover the mathematical knowledge and skills relevant in work and the everyday life of adults.

> *[Numeracy's] key concepts relate in a broad way to situation management and to a range of effective responses (not only to application of arithmetical skills). It refers to a wide range of skills and knowledge (not only to computational operations) and to a wide range of situations that present actors with mathematical information of different types (not only those involving numbers embedded in printed materials). (Murray, Clermont and Binkley, 2005, p. 151[10])*

In contrast with the domain of literacy, only minor changes to the specification of the numeracy domain were made in PIAAC Cycle 1 compared to ALL. These concerned presentation more than content. One of the major drivers for the revision of the numeracy assessment framework for PIAAC Cycle 2 was the view that the assessment of numeracy in the 21st century had to be expanded to cover the engagement with mathematical information in digital environments as well as to increase use of the possibilities offered by CBA.[12] The revised framework reflects the importance of digital information, representations, devices and applications as realities that adults have to deal with in responding to the numerical demands of everyday life. To this end, the content dimension of the numeracy framework has been significantly updated to include representations of mathematical information in the form of 'structured information' (infographics, etc.) and also 'dynamic applications' (e.g. online interactive websites and applications alongside more standard software applications and tools). The dimension of cognitive processes has also been revised to emphasise the ability to recognise and identify how and when to use mathematics; to be able to understand, use and apply mathematical concepts and procedures; and the capacity to use strategic, reasoning and reflective skills when using and applying mathematics.

In PIAAC Cycle 2, the assessment of numeracy will be accompanied by an assessment of 'numeracy components'. As for literacy, the numeracy components assessment focuses on some of the skills essential for achieving automaticity and fluency in managing mathematical and numerical information. The focus is on 'number sense' defined as 'the sense of quantities and the sense of how numbers represent quantities' (see numeracy framework). The items to be used will be of two types: items relating to quantities (using the stem 'How many…?') and items relating to relative magnitudes ('The biggest?').

Developments in problem solving

Problem solving represents the domain in which the changes in the conceptualisation of the skill in question have been greatest.[13] This is one of the reasons why the assessments of problem solving have not been linked across assessments. An assessment of problem solving was first undertaken in ALL, based on the construct of 'analytical problem solving' (Murray, Clermont and Binkley, 2005[10]) and assessed in paper-based format. This was replaced with the assessment of 'problem solving in technology-rich environments' (PS-TRE) in PIAAC Cycle 1 which has been replaced, in its turn, by adaptive problem solving (APS) in PIAAC Cycle 2.

Analytical problem solving in ALL focused on the generic aspects of the process of problem solving understood as 'goal-directed thinking and action in situations for which no routine solution procedure is available' (Murray, Clermont and Binkley, 2005, p. 197[10]), in particular the steps of:

- identifying a problem
- searching for relevant information and integrating it into a coherent problem representation
- evaluating the problem situation with respect to given goals and criteria
- devising a plan for the solution – i.e. an ordered sequence of appropriate actions
- monitoring its execution.

The assessment of problem solving in ALL was a paper-based assessment involving static problems in which all necessary information was provided up front. The limitations of this approach were explicitly acknowledged. In particular, computer simulated tasks were seen as the only way to address the dynamic aspects of task regulation (continuous processing of incoming information, coping with processes that cannot be influenced directly, coping with feedback and critical incidents).

In Cycle 1 of PIAAC, the assessment of problem solving moved to CBA mode in the form of the assessment of PS-TRE. PS-TRE represented a hybrid construct, at the intersection of the capacity to use information and communication technologies (ICTs) on the one hand, and of the ability to solve problems on the other. This was reflected in the restriction of the domain of problems covered to that of 'information problems' – problems that involved interaction with digital devices and applications (PIAAC Expert Group in Problem Solving in Technology-Rich Environments, 2009, pp. 8-9[12]):

- The problem is primarily a consequence of the availability of new technologies.
- The solution to the problem requires the use of computer-based artefacts (applications, representational formats, computational procedures).
- The problems are related to technology-rich environments themselves (e.g. how to operate a computer, how to fix a settings problem, how to use an Internet browser).

The focus on the assessment of problems in digital environments constituted both the strength and the weakness of PS-TRE. By design, only test-takers who had some (basic) ICT skills could display proficiency in this domain. Non-response for reasons of lack of familiarity with ICT devices or poor computer skills was construct relevant and could be interpreted as lack of proficiency. The downside was that a sizeable proportion (between 8 to 57%) of respondents in all participating countries did not take the assessment at all as they either lacked familiarity with computers or did not wish to undertake PIAAC on a laptop[14] (OECD, 2019[15]). This created difficulties in comparisons of results between participating countries[15] and meant that there was a considerable gap in the knowledge regarding the problem solving skills *per se* of the adult population.

APS, as conceptualised for PIAAC Cycle 2, represents the return to a concept of general problem solving that is relevant to a range of information environments and contexts and is not limited to digitally embedded problems even though digital aspects as a mode of problem solving play an important role in APS. What differentiates it from analytical problem solving as assessed in ALL is its focus on the dynamic and adaptive aspects of problem solving – the capacity to react to unforeseen changes and new information that emerge during the process – and on metacognition – the capacity to reflect on the process of problem solving as it takes place (monitoring progress, adjusting goals and strategies in the light of new information and changes in the problem situation).

Relationship of the PIAAC and PISA assessments

In addition to PIAAC, the OECD manages the Programme of International Student Assessment (PISA), an assessment of 15-year-old school students that has been administered every three years since 2000. In each assessment cycle, PISA assesses skills in three core domains (reading literacy, mathematical literacy and scientific literacy) as well as an additional domain unique to each cycle. Assessments of problem solving were administered as the additional domain in 2003, 2012 and 2015.

While similar skills are assessed in PIAAC and PISA in the domains of literacy/reading literacy, numeracy/mathematical literacy and problem solving, the two studies have followed separate development paths and have not been designed to be linked psychometrically. The measurement scales in related domains (e.g. literacy/reading literacy) are independent and the assessments have no items in common.[16] This reflects a degree of path dependency (PIAAC is designed to be linked to IALS and ALL) as well as the fact that the two assessments have different target populations.

At the same time, PIAAC and PISA share much at a conceptual level. They belong to the same measurement tradition, share a similar approach to the conceptualisation and definition of the constructs that they measure and a similar assessment methodology. In addition, there have been many experts who have worked on both studies. Reviewing the relationship between the assessment of numeracy in PIAAC and the assessment of mathematical literacy in PISA, Gal and Tout (2014, p. 52[16]) conclude that:

> *Both assessments of numeracy in PIAAC and mathematical literacy in PISA appear to have substantial conceptual similarities and quite a few practical commonalities in the nature of their test items and their design principles, as well as the range of content areas and skills they cover. The two surveys are highly consistent in their descriptions and structures for contexts and real world content classifications, along with how they describe the types and breadth of responses and actions expected of the respondents.*

Much the same comments could be made regarding the literacy/reading literacy and problem solving frameworks in both studies [see OECD, 2019 (pp. 91-93[17])].

Over time, there has been considerable mutual influence between adult and student assessments, particularly regarding the conceptualisation and definition of skills in reading and managing mathematical and numeric information. The IALS literacy frameworks were extremely influential on the development of the first PISA reading framework (OECD, 1999[18]) at the end of the 1990s. The adoption in PISA of an approach to the assessment of reading, mathematics and science that focused on the use of these skills in settings outside school owes much to the IALS approach to the assessment of literacy with its emphasis on the role of reading for social functioning. The PISA 2000 reading framework took over the classification of text types developed in IALS, particularly the prose/document distinction. In many ways, PISA could be viewed as an IALS for school students.[17] The PISA frameworks have in their turn influenced PIAAC, particularly in the domain of reading/literacy. The single reading scale adopted by PISA prefigured the single PIAAC literacy scale, for example, and the classifications of texts and cognitive processes adopted in PIAAC Cycle 1 reflects that used in PISA.

Reflecting the conceptual links between the two studies, one of the considerations in the development of the assessment frameworks for PIAAC Cycle 2 was to maximise the conceptual and terminological consistency between the PIAAC and PISA frameworks where relevant and appropriate. At the same time frameworks continue to reflect the fact that the PIAAC represents an assessment of adults.

The framework documents

The framework documents included in this volume were each prepared by a dedicated expert group[18] over the 2018-19 with the process being managed and coordinated by the PIAAC international contractor led by Education Testing Service (ETS). Members were selected to include experts from different backgrounds and countries. In all groups, some members had also served as members of the groups responsible for the Cycle 1 frameworks, thus ensuring continuity between the cycles and others had also worked on the PISA project in various capacities. While each expert group worked independently, there was close communication between the groups, particularly between the Chairs. In addition, there was overlap in membership with the Chair of the reading group also serving a member of the problem solving group.

In both adaptive problem solving and numeracy, the work of the expert groups built on earlier exploratory work commissioned by the PIAAC Board of Participating Countries (BPC), the steering committee for the PIAAC project. An initial conceptual framework for the assessment of adaptive problem solving was prepared in 2017 (Greiff et al., 2017[5]) as was a review of the PIAAC numeracy framework (Tout et al., 2017[6]).

The framework documents represent a work in progress. They will be updated following the completion of the main study data collection. At this point, the expert groups will review and revise the descriptors for the proficiency levels used to describe the measurement scales in the case of literacy and numeracy and develop the described scale in the case of APS.

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Annex 1.A. Summary of the evolution of assessment frameworks – from IALS to PIAAC Cycle 2

Annex Table 1.A.1. Literacy (Reading)

	IALS/ALL		PIAAC Cycle 1	PIAAC Cycle 2
Construct	Prose Literacy	Document Literacy	Literacy	Literacy
Definition	Literacy is using printed and written information to function in society, to achieve one's goals, and to develop one's knowledge and potential. Prose literacy is the knowledge and skills needed to understand and use information from texts, including editorials, news stories, brochures and instruction manuals.	Literacy is using printed and written information to function in society, to achieve one's goals, and to develop one's knowledge and potential. Document literacy is the knowledge and skills required to locate and use information contained in various formats, including job applications, payroll forms, transportation schedules, maps, tables and charts.	Literacy is the ability to understand, evaluate, use and engage with *written texts* to participate in society, to achieve one's goals, and to develop one's knowledge and potential. Literacy encompasses a range of skills from the decoding of written words and sentences to the comprehension interpretation, and evaluation of complex texts.	Literacy is accessing, understanding, evaluating and reflecting on written texts in order to achieve one's goals, to develop one's knowledge and potential and to participate in society.
Cognitive processes	• Locating • Cycling • Integrating • Generating		• Access and identify • Integrate and interpret (relating parts of text to one another) • Evaluate and reflect	• Accessing text • Understanding • Evaluating
Content	Continuous texts: • Description • Narration • Exposition • Argumentation • Instruction • Document or record	Non-continuous texts: • Matrix documents • Graphic documents • Locative documents • Entry documents • Combination documents	Texts characterised by their medium (*print-based* or *digital*) and by format: • *Continuous* or *prose texts* which involve narration, argumentation or descriptions for example • *Non-continuous* or *document texts*, for example, tables, lists and graphs • *Mixed texts* which involve combinations of prose and document elements • *Multiple texts* which consist of the juxtaposition or linking of independently generated elements	Texts characterised by their: • Type (description, narration, exposition, argumentation, instruction, transaction) • Format (continuous, non-continuous, mixed) • Organisation (the amount of information and the density of content representation and access devices) • Source (single vs. multiple texts)

	IALS/ALL	PIAAC Cycle 1	PIAAC Cycle 2
Contexts	• Home and family • Health and safety • Community and citizenship • Consumer economics • Work • Leisure and recreation	• Personal • Work • Community • Education	• Work and occupation • Personal • Social and civic
Factors affecting task difficulty	• Type of match • Type of information requested • Plausibility of distractors	• Transparency of the information • Degree of complexity in making inferences • Semantic complexity and syntactic complexity • Amount of information needed • Prominence of the information • Text features (such as text cohesion signals)	• Text factors (length, type of text, familiarity of content, presence of content signalling devices) • Task factors (length of stem, explicitness of guidance) • Text-by-task factors (type of match, presence of distracting or irrelevant information)
Assessment mode	Paper-based	Computer-based (laptop device) + paper-based option	Computer-based (tablet device) + paper-based option in a limited number of countries

Sources: For IALS: Murray, Kirsch and Jenkins (1998[8]). For ALL: Murray, Clermont and Binkley (2005[10]). For PIAAC Cycle 1: (OECD, 2019[17]). For PIAAC Cycle 2: the frameworks included in this volume.

Annex Table 1.A.2. Managing numerical and mathematical information

	IALS	ALL	PIAAC Cycle 1	PIAAC Cycle 2
Construct	Quantitative Literacy	Numeracy	Numeracy	Numeracy
Definition	Quantitative literacy is the knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials, such as balancing a chequebook, figuring out a tip, completing an order form or determining the amount of interest on a loan from an advertisement.	Numeracy is the knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations. Numerate behaviour is observed when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, factors and processes.	Numeracy is the ability to access, use, interpret and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life. To this end, numeracy involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways.	Numeracy is accessing, using and reasoning critically with mathematical content, information and ideas represented in multiple ways in order to engage in and manage the mathematical demands of a range of situations in adult life.
Content	Non-continuous texts: • Matrix documents • Graphic documents • Locative documents • Entry documents • Combination documents	Mathematical information: • *Dimension and shape* • *Pattern, functions and relationships* • *Data and chance* • *Change* Representations of mathematical information: • *Objects* • *Pictures* • *Symbolic notation* • *Formulae* • *Visual displays* • *Texts*	Mathematical content, information and ideas: • *Quantity and number* • *Dimension and shape* • *Pattern, relationships, change* • *Data and chance* Representations of mathematical content: • *Objects and pictures* • *Numbers and symbols* • *Diagrammes, maps, graphs, tables* • *Texts* • *Technology-based displays*	Mathematical content information and ideas: • *Quantity and number* • *Space and shape* • *Change and relationships* • *Data and chance* Mathematical representations: • *Text or symbols* • *Images of physical objects* • *Structured information* • *Dynamic applications*
Cognitive processes	• Locating • Cycling • Integrating • Generating	• Identify or locate • Act upon or react • Interpret • Communicate	• Identify, locate or access • Act upon and use (order, count, estimate, compute, measure, model) • Interpret, evaluate and analyse • Communicate	• Access and assess situations mathematically • Act on and use mathematics • Evaluate, critically reflect, make judgements
Contexts	• Home and family • Health and safety • Community and citizenship • Consumer economics • Work • Leisure and recreation	• Everyday life • Work-related • Society and community • Further learning	• Everyday life • Work-related • Society and community • Further learning	• Personal • Work • Societal/community

	IALS	ALL	PIAAC Cycle 1	PIAAC Cycle 2
Factors affecting task difficulty	• Type of match • Type of information requested • Plausibility of distractors • Type of calculation • Operation specificity	• Type of match/problem • Plausibility of distractors • Complexity of mathematical information • Type of operation • Expected number of operations	• Type of match/problem • Plausibility of distractors • Complexity of mathematical information • Type of operation • Expected number of operations	• Type of match/problem • Plausibility of distractors • Complexity of mathematical information • Type of operation • Expected number of operations
Assessment mode	Paper-based	Paper-based	Computer-based (laptop device) + paper-based option	Computer-based (tablet device) + paper-based option in a limited number of countries

Sources: For IALS: Murray, Kirsch and Jenkins (1998[8]). For ALL: Murray, Clermont and Binkley (2005[10]). For PIAAC Cycle 1: (OECD, 2019[17]). For PIAAC Cycle 2: the frameworks included in this volume.

Annex Table 1.A.3. Problem solving

	ALL	PIAAC Cycle 1	PIAAC Cycle 2
Construct	Analytical Problem Solving	Problem Solving in Technology-Rich Environments	Adaptive Problem Solving
Definition	Problem solving involves goal-directed thinking and action in situations for which no routine solution procedure is available. The problem solver has a more or less well defined goal, but does not immediately know how to reach it. The incongruence of goals and admissible operators constitutes a problem. The understanding of the problem situation and its step-by-step transformation based on planning and reasoning, constitute the process of problem solving.	Problem solving in technology-rich environments involves the ability to use digital technology, communication tools and networks to acquire and evaluate information, communicate with others and perform practical tasks. The assessment focuses on the abilities to solve problems by setting up appropriate goals and plans, and accessing and making use of information through computers and computer networks.	Adaptive problem solving involves the capacity to achieve one's goals in a dynamic situation, in which a method for solution is not immediately available. It requires engaging in cognitive and metacognitive processes to define the problem, search for information, and apply a solution in a variety of information environments and contexts.
Cognitive processes	• Defining the goal • Analysing the given situation and construct a mental representation • Devising a strategy and plan the steps to be taken • Executing the plan, including control and – if necessary – modification of the strategy • Evaluating the result	• Setting goals and monitoring progress • Planning • Acquiring and evaluating information • Using information	• Definition • Searching • Application
Content	Problems	Technology: • *Hardware devices* • *Software applications* • *Commands and functions* • *Representations* (e.g. text, graphics, video) Nature of problems: • *Intrinsic complexity* which includes the number of steps required for solution, the number of alternatives, complexity of computation and/or transformation, number of constrains • *Explicitness of the problem statement*, for example largely unspecified or described in detail	Aspects of problems: • Problem configuration • Dynamics of the situation • Features of the environment • Information environment
Contexts	Not specified	• Personal • Work and occupation • Civic	• Personal • Work • Social/community

	ALL	PIAAC Cycle 1	PIAAC Cycle 2
Factors affecting task difficulty	Not specified	• Minimal number of steps required to solve the problem • Number of options or alternatives at various stages in the problem space • Diversity of operators required, complexity of computation/transformation • Likelihood of impasses or unexpected outcomes • Number of constraints to be satisfied • Amount of transformation required to communicate a solution • Ill defined (implicit, unspecified) vs. well defined (explicit, described in detail)	• Number of elements, relations, and operations • Salience and accessibility of operators • Interactions between problem elements • Number of parallel tasks and goals • Number of features that change and their relevance • Salience of change (if something changes) • Frequency of change • Degree of impasse • Wealth of information • Proportion of irrelevant information • (Lack of) Structure of the environment • Number of sources of information
Assessment mode	Paper-based	Computer-based (laptop device)	Computer-based (tablet device)

Sources: For ALL: Murray, Clermont and Binkley (2005[10]). For PIAAC Cycle 1: (OECD, 2019[17]). For PIAAC Cycle 2: the frameworks included in this volume.

Notes

[1] Results from IALS can be found in OECD/Statistics Canada (2000[9]) and results from ALL in OECD/Statistics Canada (2005[25]; 2011[26]).

[2] Results have been published in OECD (2013[21]; 2016[22]; 2019[15]). A comprehensive bibliography of publications based on PIAAC over the period 2008 to 2019 is provided in Maehler, Jakowatz and Konradt (2020[29]).

[3] The PIAAC Technical Standards and Guidelines [(PIAAC, 2014[2]), Guideline 10.4.1] provide that the interview should be completed in the respondent's home. However, if the respondent prefers, it may be conducted at a neutral location such as a library, community centre or office. On average, across all countries, around 91% of interviews took place in the respondent's home [see Keslair, 2018 (pp. 11-13[19])]. In a small number of countries, around a third of interviews took place in a location other than the respondent's residence.

[4] The background questionnaire used in Cycle 1 of PIAAC can be accessed at: http://www.oecd.org/skills/piaac/BQ_MASTER.HTM. The background questionnaire for Cycle 2 will be largely similar, although it will be improved and updated in a number of dimensions.

[5] See, for example, the frameworks for PISA (OECD, 2019[23]), TIMSS (Mullis and Martin, 2013[27]) and PIRLS (Mullis and Martin, 2015[28]).

[6] The assessment is usually delivered in the national language or languages only. In a small number of participating countries, the assessment is also made available in widely spoken minority languages [see Table 4.11 in OECD (2019[17]).

[7] Tout (2020[7]) offers a comprehensive overview of the changes in the conceptualisation of 'numeracy' between IALS and PIAAC Cycle 2. A good discussion of the factors that influence the evolution of assessment frameworks in reading in PISA which is also relevant to PIAAC can be found in OECD (2019, pp. 22-27[23]).

[8] See OECD (2019[24]) for an exploration of the log-file data derived from PIAAC.

[9] One aspect of the assessment of literacy has remained constant since IALS in adult assessments is that it has been undertaken as an assessment of *reading* (of the understanding of and engagement with written texts) and has not included the dimension of *writing* or the production of text. This represents a pragmatic choice rather than a theoretical position. It is acknowledged that writing represents an important dimension of a broad concept of literacy. However, the challenges of directly assessing proficiency are sufficiently large to make it impractical in large-scale cross-national assessments such as PIAAC.

[10] As well as text formats common in digital environments (e.g. multiple texts or texts constituted by series of juxtaposed texts).

[11] Performance in the reading components assessment was reported separately from performance in literacy in PIAAC Cycle 1.

[12] In the words of the numeracy framework, Cycle 1 numeracy test items were 'based predominantly around static images and associated responses' and were 'more like paper-based assessments transferred onto a computer' (numeracy framework).

[13] This is also true of PISA where three separate constructs have been assessed: analytical problem solving (2003), creative problem solving (2012) and collaborative problem solving (2015).

[14] Paper-based versions of the assessments of literacy and numeracy were available for respondents.

[15] As a variable proportion of the 16-65 year-old population took the assessment on computer, comparison of mean scores between countries was not possible. Presentation of country differences focusses on the proportion of the population performing at different proficiency levels.

[16] The exception is the reading assessment in PISA 2000 in which fifteen prose literacy items from IALS were included. The intention was to see whether the results of the two studies could be reported on a common scale. Chapter 8 of (OECD, 2002[20]) discusses the findings of an analysis of the performance of students on the IALS items.

[17] The description of PIAAC as a 'PISA for adults' [see, for example, Wallin (2018[30])] ignores the fact that adult assessments (in the form of IALS) predated PISA and fails to acknowledge the strong influence of IALS on PISA. It is important to note that PISA also owes a considerable debt to the International Evaluation Association (IEA) studies TIMSS and PIRLS which demonstrated the feasibility and utility of large-scale international assessments of school students.

[18] The members of the expert groups are listed in the acknowlegments.

2 PIAAC Cycle 2 assessment framework: Literacy

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Literacy skills play an essential part in adults' personal, social and professional life. In addition, the spread of digital technologies further emphasises the importance of reading literacy. As a set of cognitive abilities, literacy involves: accessing texts, or passages within texts, that match readers' tasks and needs; understanding the literal contents of text(s) and drawing adequate inferences both within and across texts; and evaluating texts and their sources for accuracy, soundness, and relevance, as well as reflecting on authors' purposes and strategies. The PIAAC assessment of literacy draws from a broad range of contexts and text types, from personal narratives to descriptions and arguments. It is designed as a set of scenarios involving one or several texts and a set of questions using various response formats. The main factors expected to drive item difficulty and to define proficiency levels are identified in this framework document.

Introduction

The term literacy (from the Latin "litera": letter, written sign) refers to one's ability to comprehend and use written sign systems. Literacy may be defined both as a set of generalised abilities [e.g., decoding words and comprehending sentences; (Perfetti, 1985[1])] and a set of cultural practices and values that vary across human groups and communities (Street and Street, 1984[2]). Thus, the literate individual is both a person who is able to make use of a broad diversity of written materials in the service of wide range of activities, and a person who is knowledgeable of the cultural standards of their communities of practice (Rouet and Britt, 2017[3]).

Since the invention of written sign systems some five thousand years ago, written communication has played an increasing role in societies throughout the world. The percentage of humans who can read and write has increased steadily over the past centuries, even though an estimated 750 million adults still cannot read and write fluently, with the highest rates of illiteracy matching the lowest levels of economic development (UNESCO, 2017[4]). In countries where people are given a chance to become literate, teenagers' and adults' actual levels of mastery vary to a remarkable extent. Furthermore, individual levels of literacy are usually associated with better living conditions, jobs, and health (Morrisroe, 2014[5]; OECD, 2013[6]).

One reason why literacy has become so important is that, in the modern world, written communication pervades most aspects of people's lives, whether personal, social, or professional. A study found that typical American adults read on an average of nine occasions per day, slightly more on working days than on weekends and holidays, and mostly in relation with practical tasks (White, Chen and Forsyth, 2010[7]). Depending on the context and purpose, reading may take a wide diversity of forms. Adults sometimes read extended pieces of continuous texts for the sake of enjoyment or just to comprehend an author's main points, but they more often scan pages to search for information that matches specific needs or questions. To serve these purposes, adults read a wide variety of texts ranging from e-mails to leaflets to timetables and instruction manuals. While doing so, they use a broad diversity of strategies and tactics, which all belong to the construct of literacy (Alexander and The Disciplined Reading and Learning Research Laboratory, 2012[8]; Britt, Rouet and Durik, 2018[9]; Goldman, 2004[10]).

The spread of computers and Internet access over the past two decades has further exacerbated the importance of literacy skills in contemporary societies (Leu et al., 2017[11]). There is little that an illiterate person can do with a smartphone, a tablet or a laptop. Written signs are ubiquitous in most computer applications, including the most widely used video sharing platforms. Digital reading is increasingly important for people to access jobs, services and goods, and to participate in communities.

For these reasons, acquiring valid and reliable estimates of what adults can do with printed texts has become a prominent target for public institutions. Several rounds of studies have been conducted at an international level over the past decades.

The second PIAAC study in the context of past international literacy studies

Since the early 1990s, three large-scale cross-country assessments of literacy and basic skills of the adult population have taken place. The first was the International Adult Literacy Survey (IALS) (Murray, Kirsch and Jenkins, 1998[12]), which was conducted in 22 countries and regions over the period 1994-1998. The second, known as the Adult Literacy and Life Skills Survey (ALL) (OECD/Statistics Canada, 2005[13]; 2011[14]), was undertaken over 2002-2008 in 11 countries. A successor to IALS and ALL – the Programme for the International Assessment of Adult Competencies (PIAAC Cycle 1) (OECD, 2013[6]) was administered in 39 countries and regions over the period 2011-2019 (National Center for Education Statistics (NCES), n.d.[15]).

IALS, ALL and PIAAC share a common conceptual framework and approach to the assessment of literacy skills, covering the conceptualisation of literacy, the approach to measurement, data quality and reporting of results (Kirsch and Lennon, 2017[16]).

Developments between IALS and PIAAC

One of the major areas in which there has been a change between the three assessments concerns the skill domains assessed. IALS included three separate domains of literacy: prose literacy, document literacy and quantitative literacy. The major change between IALS and ALL was that a new numeracy scale replaced the quantitative scale, while the prose and document scales were kept.

The measurement framework for literacy in PIAAC Cycle 1 was heavily based on those used in IALS and ALL, but in PIAAC literacy was assessed on a single scale rather than on two separate scales (prose and document literacy in ALL). PIAAC Cycle 1 also expanded the kinds of texts covered by including electronic texts in addition to the continuous (prose), non-continuous (document) and combined texts of the IALS and ALL frameworks. In addition, the assessment of literacy was extended to include a measure of reading component skills. This was designed for people with low levels of literacy competence and focused on assessment of the foundational skills needed to gain basic meaning from texts. The skills tested were print vocabulary, sentence processing and passage fluency.

PIAAC Cycle 1 also differed from IALS and ALL in that it mainly was an integrated computer-based assessment. The majority of respondents were assessed using a laptop computer. A pen-and-paper version of the literacy (and numeracy) assessment was available for respondents who had insufficient familiarity with computers or preferred the paper-and-pencil version for other reasons (26%).

Information technology and the changing nature of literacy

During the past 10 years, the use of internet has grown rapidly all over the world. According to a recent estimate (ITU, 2017[17]), more than half (53.6%) of the world's households has internet access – a dramatic increase from just less than 20% of the households having internet access in 2005, and just over 30% in 2010. The number of individuals using the internet has naturally grown as the internet access has become more common. It is estimated that there are 3.5 billion internet users today, representing almost half (48%) of the world's population (ITU, 2017[17]).

The rapid growth of the use of internet means that in today's world, reading often takes place in digital environments: people search and read timetables, maps and calendars online, they look for products and product reviews and purchase them on the internet, look up information in Wikipedia, read newspapers and blogs online, and participate in social media. The medium for accessing information is rapidly moving from print to screens to handheld devices, such as smartphones. As digital media affords different types of activities than traditional print media, reading in digital environments poses different cognitive demands and challenges to the reader than reading in print (Mangen and van der Weel, 2016[18]). While digital environments allow features that can support comprehension, recent evidence suggests that reading comprehension of informational texts may suffer when text material is presented in digital form in comparison to print (Delgado et al., 2018[19]).

One notable difference between print and digital media is that printed text is static and linear in nature, whereas digital texts often are hypertexts, which can include embedded hyperlinks to other sources, including multimedia. The ability to navigate within the interrelated network of documents, and the ability to locate relevant information among the potentially distracting information, are thus crucial aspects of skillful digital reading (Salmerón et al., 2018[20]).

The current framework aims at describing reading literacy in the present day context, in which digital reading is a central aspect of active participation in society. Three core sets of abilities are required for skilful reading in the complex information environments readers interact with: 1) ability to navigate within and between networked documents, 2) ability to comprehend and integrate multiple and sometimes disparate sources of information, and 3) ability to critically evaluate the information presented (Britt and Gabrys, 2001[21]; Rouet and Potocki, 2018[22]; Salmerón et al., 2018[20]).

Evolution of the PIAAC Cycle 2 Literacy domain in comparison with previous frameworks

As a consequence of the increasing uses of digital communication, there is a need to expand the construct of literacy to account for the advanced skills that enable people to interact with complex repositories of information. These include an ability to identify relevant items within sets of texts, and to scan the selected texts in order to locate information of interest. During their search for relevant information, readers use a range of criteria to discard irrelevant or inadequate information while identifying the most helpful resources. In addition, proficient readers need to comprehend information not just from one text, but also across multiple texts potentially containing fixed or animated graphs, still pictures and video segments in addition to written information. As evidenced in research studies, integrating information from multiple documents requires specific mental processes that come on top of the more traditional comprehension processes (Rouet, Britt and Potocki, 2019[23]). Finally, being literate increasingly requires readers to distance themselves from the information they are processing, questioning the accuracy, completeness, actuality of the information, as well as the competence, perspective and potential biases of the authors and publishers. These validation processes (Britt, Richter and Rouet, 2014[24]; Singer, 2013[25]) rest on specific types of knowledge and heuristics that any assessment of literacy should give due consideration.

As the domain expands to represent more sophisticated strategies, care must also be taken to describe the skills of those who only have a limited ability to comprehend and use written texts. Studies like PIAAC have found that in many countries a substantial proportion of adults still experience difficulties with the foundational processes that support any kind of literate activities: identify written words or symbols, make sense of simple sentences, draw basic inferences. There have been calls to increase the precision of the assessment at the lower end of the proficiency scale. The PIAAC framework acknowledges the role of these foundational skills and aims to provide satisfactory coverage of their distribution in the population.

Finally, an assessment of literacy must also consider people's active engagement in literate activities both at work and in their daily life. Exposure to written texts has been found to be a factor of children's acquisition of literacy skills (Stanovich and West, 1989[26]). Likewise, adults who encounter frequent opportunities to use texts are likely to develop better skills and to maintain them over time. Therefore, information about individual exposure to and engagement with texts may provide helpful information to understand the links between skill use and proficiency.

Definition of literacy

PIAAC Cycle 2 uses a parsimonious definition of literacy that aims to highlight a set of core cognitive processes involved in most, if not all literate activities. At the same time, the definition acknowledges that literate activities "do not happen in a vacuum" (Snow and the RAND reading study Group, 2002[27]). Instead, they are done in the service of one's goals, one's development and participation in society. These diverse purposes and contexts contribute to shaping the way individuals make use of written texts, hence their inclusion in the definition.

> "Literacy is accessing, understanding, evaluating and reflecting on written texts in order to achieve one’s goals, to develop one’s knowledge and potential and to participate in society."

We elaborate on each part of the definition below, emphasising some important theoretical advances in the domain, as well as evidence from the first PIAAC cycle and former research studies.

"Literacy…"

Although the etymology of the word literacy directly points to written language, in past decades the term has been used to refer to an increasingly broad array of domains and interests, for instance in "health literacy", "financial literacy" or "computer literacy". In some definitions, the activities denoted by these phrases have only remotely and incidentally to do with written language. In the present framework, the word is taken in its broadest but also most literal sense, to describe the proficient use of written language artefacts such as texts and documents, regardless of the type of activity or interest considered. This characterisation of literacy highlights both the universality of written language (i.e., its potential to serve an infinite number of purposes in an infinite number of domains) and the very high specificity of the core ability underlying all literate activities, that is, the ability to read written language. As demonstrated in neuroscience research, learning to read is a very special experience with consequences on the organisation of some areas of the brain (Dehaene, 2009[28]).

"is accessing…"

Proficient readers are not just able to comprehend the texts they are faced with. They can also reach out to texts that are relevant to their purposes, and search passages of interest within those texts (McCrudden and Schraw, 2007[29]; Rouet and Britt, 2011[30]). Searching text is cognitively distinct from reading for comprehension (Guthrie and Kirsch, 1987[31]). When searching, the proficient reader makes use of text organisers (such as tables of contents and headers) in order to inform relevance decisions; the proficient reader can also adjust the pace and depth of processing, alternating phases of quick skimming with phases of sustained, deep reading for comprehension. Finally, proficient readers are parsimonious: they may decide to quit a passage upon realising that it does not contain helpful information. In the PIAAC literacy framework, these processes are subsumed under the term "accessing".

"understanding…"

Most definitions of literacy acknowledge that the primary goal of reading is for the reader to make sense of the contents of the text. This can be as basic as comprehending the meaning of the words, to as complex as comprehending the dispute between two authors making opposite claims on a social-scientific issue. Whatever the context, any literate activity (including accessing a piece of text or a passage within a text) requires some level of understanding. Theories of text comprehension (Kintsch, 1998[32]) usually distinguish the literal understanding of the message from a deeper level of understanding in which the reader integrates their prior knowledge with the text contents through the production of various types of inferences (i.e., a situation model). Prior knowledge of the domain has a strong (usually positive) impact on the deeper level of understanding.

"evaluating and reflecting…"

Readers continually make judgements about a text they are approaching. They evaluate whether the text is appropriate for the task at hand and whether it will provide the information they need. Readers also make judgements about the accuracy and reliability of both the content and the source of the message (Bråten, Strømsø and Britt, 2009[33]; Richter, 2015[34]). They attempt to detect and explain any biases and gaps in the coherence or persuasiveness of the text. And, for some texts, they must make judgements about the quality of the text, both as a craft object and as a tool for acquiring information.

"on written text…"

In the context of PIAAC Cycle 2, the phrase "written text" designates pieces of discourse primarily based on written language. Written texts may include non-verbal elements such as charts or illustrations. However, pictures, video and other visual media are not considered written texts per se.

A text typically includes two broad components: a source and a content. The source of the text is a set of parameters that identify the origin and dissemination of the text. The most typical source parameters are a description of the author (for instance, "Alfred Nobel, a Swedish chemist and businessman"), the publication medium and date of the text. But source information sometimes includes more specific details about the text, for instance "second edition", or "confidential". Although all texts have a source, source information is not always provided together with the content. In addition, emerging practices of online publishing and social media have tended to make it more challenging for the reader to identify the source of the text.

As in the first cycle of PIAAC (and in related studies such as PISA), the assessment of literacy will include a wide variety of text types, such as narrative, descriptive or argumentative. Texts in various formats, such as continuous, non-continuous or mixed will be included. Just as in the real world, some of these texts may be presented in a static way, meaning that the reader has only a limited opportunity to navigate through them,[1] whereas others, especially in digital environments, contain interactive navigation tools such as interactive tables of contents, hyperlinks and other devices. The PIAAC definition of written texts encompasses both static and interactive materials.

"in order to achieve one's goals,"

Just as written languages were created to meet the needs of emergent civilisations, at an individual level, literacy is primarily a means for one to achieve their goals. Goals relate to personal activities but also to the workplace and to interaction with others. Literacy is increasingly important in meeting those needs, whether simply finding one's way through a building, or negotiating complex bureaucracies, whose rules are commonly available only in written texts (and increasingly only in digital forms). Literacy is also important in meeting adult needs for sociability, for entertainment and leisure, for developing one's community and for work.

"to develop one's knowledge and potential and to participate in society."

Developing one's knowledge and potential highlights one of the most powerful consequences of being literate. Written texts may enable people to learn about topics of interest, but also to become skilled at doing things and to understand the rules of engagement with others.

Written communication is primarily and ultimately a consequence of humans being a sophisticated social species. Texts are communication artefacts, they serve the purpose of transmitting information but also feelings and values to others. As such, literacy contributes to building, nurturing and preserving social cohesion.

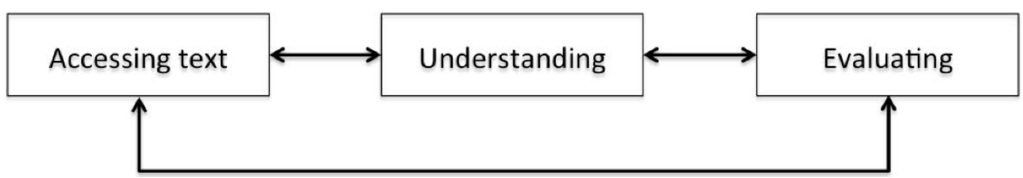
Core dimensions of the literacy domain

The PIAAC literacy assessment aims to provide a complete and accurate description of what adults can do with texts in a broad range of contexts and tasks. To that aim, the literacy domain is organised along a set of dimensions that ensure a broad coverage and a precise description of what people can do at each level of proficiency. In this section we describe the most important dimensions, which will be used to help define the proficiency levels for literacy.

Cognitive task demands

Naturalistic reading is a complex and versatile process. Proficient readers can read systematically and intensely extended passages of texts, but they can also quickly scan a page in search for a single keyword. How readers approach texts is primarily determined by their reading goals, which themselves are informed by the reader's understanding of the context and the task demands (Britt, Rouet and Durik, 2018[9]). PIAAC identifies three groups of processes that support most reading activities: accessing text, understanding, and evaluating (Figure 2.1).

Figure 2.1. Three core cognitive processes supporting literacy



Note: These processes may unfold in any order and even in parallel.

The three processes correspond to those included in related assessments such as PIAAC Cycle 1 and PISA 2018. Table 2.1 shows the correspondence between the processes in these frameworks.

Table 2.1. Correspondence between the processes in PIAAC Cycle 2, PIAAC Cycle 1 and PISA 2018

PIAAC Cycle 2 (processes)	**PIAAC Cycle 1 (aspects of tasks)**	**PISA 2018 (processes)**
Accessing text	Access and identify information in the text	Locate information
Understanding	Integrate and interpret	Understand
Evaluating	Evaluate and reflect	Evaluate and reflect

Accessing text

Accessing text encompasses a number of literacy processes whereby readers examine the text(s) available, select the most relevant text, scan contents in search for specific pieces of information and locate these pieces through various types of cues. In addition, accessing conveys the sense of navigating across various texts or passages within texts as a function of task demands and the reader's progress towards their goal.

Ability to access information within and across texts is a core component of skilful reading in print and perhaps even more in digital environments (Salmerón et al., 2018[20]). Successful navigation means that the reader is capable of searching and locating relevant information within the texts, and this is influenced by the type of the question posed to the reader, as well as the nature of the materials. When searching, the proficient reader also calibrates their depth of processing of the information, merely scanning task-irrelevant contents while pausing and engaging in deeper processing of passages they deem relevant to the task.

The task or the question the reader has in mind has a big impact on how readers navigate within and between text documents (McCrudden and Schraw, 2007[29]). Identifying what information is relevant is only possible if the reader has formed an appropriate task model that provides specific criteria and guides the strategies utilised in searching and locating relevant information (Britt, Rouet and Durik, 2018[9]). Theories of purposeful reading suggest that when reading with specific objectives in mind, the incoming text information is constantly processed in the light of the task model (Britt, Rouet and Durik, 2018[9]). When task-relevant information is detected, attention is zoomed in to meet the task demands (Kaakinen and Hyönä, 2014[35]). The complexity of the task model depends on the question posed to the reader: simple questions may only require the search for a match between the question item and information within the text, whereas forming an appropriate task model for a more complex question may require background knowledge and inferencing. Lack of related prior knowledge may thus make it harder to search and locate relevant information (Kaakinen, Hyönä and Keenan, 2003[36]), as the reader's task model might not specify what is relevant, and reader has to scrutinise all information in order to decide whether it is relevant or not.

The nature of the text materials obviously influences how easy or hard it is to access information from a text or set of texts. The PIAAC literacy framework distinguishes two types of search processes: identifying a relevant text from a set, and locating information within a single text.

<u>Identifying a relevant text in a set</u>. If the available material consists of multiple texts (for instance, several documents on the same topic), readers have to first search and select the text that is expected to contain the most helpful information, disregarding the other items. Then readers need to search and locate relevant information within that text (Britt, Rouet and Durik, 2018[9]). Searching a relevant text in a set often involves using lists such as a table of contents (Dreher and Guthrie, 1990[37]) or the page showing the results of a query in a search engine. In selecting an item in this type of list, readers often use very simple heuristics such as the ranking of the items [priority given to the first items in the list, see (Fu and Pirolli, 2007[38]; Pan et al., 2007[39]; Wirth et al., 2007[40]) for evidence from search engine tasks] or the presence of highlighted information (Rouet et al., 2011[41]). However, in some tasks these simple heuristics may lead to suboptimal selections. For instance, in the Rouet et al. (2011[41]) study, 5th and 7th grade students were more likely to select irrelevant items when the items contained capitalised keywords. Moreover, if the materials contain a lot of distracting (irrelevant) information, the reader has to work harder to reject that information, which poses extra demands on their reasoning and working memory skills (Kaakinen and Hyönä, 2008[42]), and may cause them to forget the question (Rouet and Coutelet, 2008[43]).

<u>Locating information within a text</u>. When readers need to locate a relevant passage within a single text, signalling devices, such as headings and highlighting, can be used to facilitate the visual scanning and the identification of the relevant passage (Lemarié et al., 2008[44]). Knowing the function of text signals and using them while scanning a text are characteristics of proficient readers (Garner et al., 1986[45]; Potocki et al., 2017[46]).

Readers' search and locate processes pervade the whole reading cycle, from readers' initial decision of which text or passage they want to focus on, to their post-reading assessment of whether the passage contributes to reaching their goal (see below, "Evaluate and reflect").

Understanding

A large number of reading activities involve the parsing and integration of one or several extended passage(s) of text in order to form a complete representation of what the text is about. Cognitive theories of text comprehension usually distinguish two levels of representation (Kintsch, 1998[32]): a representation of the literal content of the text (literal comprehension), and a representation integrating the literal content with the reader's prior knowledge through mapping and inference processes [inferential comprehension or "situation model"; (McNamara and Magliano, 2009[47]; Zwaan and Singer, 2003[48])]. In addition, theories of multiple text comprehension (Perfetti, Rouet and Britt, 1999[49]; Britt and Rouet, 2012[50]) consider that text comprehension sometimes includes a representation of source features together with the respective contents.

Literal comprehension requires readers to comprehend the meaning of written words (e.g., "the kitten") and semantic propositions (i.e., small groups of words usually containing a substantive and a verb, adverb or an adjective, such as "the kitten is sleeping"). Propositions are then organised into hierarchies corresponding to one or a few sentences (Kintsch and van Dijk, 1978[51]). Literal comprehension tasks involve a direct or paraphrase type of match between the question and target information within a passage (for instance "what is the kitten doing?"). The reader may need to hierarchise or condense information at a local level in order to answer literal comprehension questions. Tasks requiring integration across entire text passages, such as identifying the main idea, summarising, or giving a title, are not considered literal, but rather inferential comprehension.

Inferential comprehension is the outcome of readers' integration of text information with their prior knowledge. The outcome is often labelled a "situation model" or "integrated text representation". Integrated text representations may be based on sentences but also on paragraphs or even on extended passages of text. As readers proceed through several sentences and paragraphs, they need to generate various types of inferences ranging from simple connecting inferences (such as the resolution of anaphora) to more complex coherence relationships (e.g. spatial, temporal, causal or claim-argument links). Sometimes the inference connects several portions of the text; in other cases, the inference is needed to connect the question and a text segment. Finally, the production of inferences is also needed in tasks requesting the reader to identify an implicit main idea, in order to produce a summary or a title for a given passage.

Multiple text inferential comprehension. When readers are faced with more than one text, integration and inference generation may be based on pieces of information located in different texts (Perfetti, Rouet and Britt, 1999[49]). Integration of information across texts poses a specific problem when the texts provide inconsistent or conflicting information. In those cases, readers must engage in evaluation processes in order to acknowledge and handle the conflict (Bråten, Strømsø and Britt, 2009[33]; Stadtler and Bromme, 2014[52]).

Evaluating

Competent readers can critically assess the quality of information in a text, even when the task does not explicitly require such an evaluation. The importance of evaluation as part of literacy has increased with the amount and heterogeneity of written information readers are faced with. Adult readers need to be able to evaluate to protect themselves from misinformation and propaganda and to make sense of conflicting information, such as political or scientific controversies. Evaluation can be based on attending to and assessing the accuracy, soundness, and task relevance of a text. The focus of these evaluations can be on the content or on the source of a text. Source evaluation plays a critical role when evaluating information from multiple texts, which sometimes provide discrepant or conflicting information (Bråten et al., 2011[53]; Leu et al., 2015[54]; Rouet and Britt, 2014[55]; Stadtler and Bromme, 2014[52]; Stadtler et al., 2013[56]). Handling conflict can require readers to assign discrepant claims to their respective sources and assess the credibility of the sources or believability of the claims (accuracy), to assess the relevance of the support or evidence provided for the discrepant claims (relevance), to evaluate the completeness of the provided perspectives and information from those possible (sufficiency), and to coordinate these outcomes to inform one’s weight to make a decision about the conflict.

Evaluating accuracy. The information conveyed in written texts can be more or less accurate, ranging from agreed upon facts to intentionally false information. Even websites conveying science information often contain inaccurate or misleading information (Allen et al., 1999[57]). The evaluation of the accuracy of claims and statements can be based on the content or on the source of the text. Content evaluation includes validation against one’s beliefs and knowledge (is the assertion true? Is it plausible? What information is presented to support the claim?) (Richter, Schroeder and Wöhrmann, 2009[58]). Readers can also assess accuracy indirectly, by identifying and assessing the source of the information (sourcing) (Britt and Aglinskas, 2002[59]; Wineburg, 1991[60]). For instance, the reader may ask whether the author is competent, well-informed and benevolent. When reading from web sources, readers may also check whether the information offered was submitted to any kind of editorial control prior to its publication (i.e., academic institutions, professional journalism vs. personal blogs or sites).

When dealing with conflicting information, readers have to be able to assign conflicting claims to different sources and use the credibility of the sources to assess the quality of information (Bråten, Strømsø and Britt, 2009[33]; Stadtler and Bromme, 2014[52]). Readers of multiple texts can also evaluate accuracy by comparing information across different sources (i.e., corroboration) (Britt and Aglinskas, 2002[59]; Wineburg, 1991[60]).

Evaluating soundness. The modern reader has to deal with texts that vary on a continuum of internal quality or soundness (Magliano et al., 2017[61]). In this framework, soundness encompasses two characteristics of discourse, namely completeness and internal consistency (Blair and Johnson, 1987[62]). Readers have to identify the completeness of the set of facts or evidence that is presented and to identify what is not accounted for or considered. Readers also have to identify perspectives presented in a text and assess whether all the important perspectives are represented. They may also have to account for any biases they find in the text. Evaluating bias may be based on language (does the text use neutral, factual language or rather colourful, evaluative language), or on the source of the text (i.e., interpreting, explaining or resolving different author biases that may impact sufficiency).

When evaluating internal consistency, readers must identify the structure of a text (e.g., persuade, inform) and evaluate the quality of the information in achieving that goal (e.g., warranted or sound claim-reason connections or reasonable cause-effect relationships). Does the author provide the type of information that is expected given the structural organisation of the text and what is the quality of that information for achieving the goal of the text? The evaluation of internal consistency can be especially challenging for argumentative texts (those attempt to convince the readers to accept a proposition, or claim by presenting supporting reasons; (Galotti, 1989[63]) because consistency cannot be determined by formal logic (Toulmin, 1958[64]).

When facing multiple texts that contradict each other, readers need to become aware of the conflict, understand where the conflict comes from (e.g., texts reporting discrepant facts or proposing discrepant interpretations) and to find ways to deal with the conflict (Britt and Rouet, 2012[50]; Stadtler and Bromme, 2014[52]).

Evaluating task relevance. As discussed in the section on "Accessing text" above, evaluating task relevance takes place throughout the reading process, from the reader's attempt to locate a text or passage of interest, to their post-reading assessment of whether the text or passage they have read was helpful (i.e., post-reading task relevance assessment); (Rieh, 2002[65]). When evaluating task relevance after reading a passage, readers must reconsider the task or question using an activated schema to understand what is being asked for and how to achieve that goal state (Britt, Rouet and Durik, 2018[9]; Rouet, Britt and Durik, 2017[66]). They must then assess whether a text they have just read contributes to reaching the goal state.

Research considers that there are two main routes in assessing task relevance. One consists in evaluating the content of the text, the other consists in evaluating the source (i.e., the person or the organisation responsible for authoring and disseminating the text). Both content and source evaluation can focus on accuracy, soundness or task relevance (Table 2.2). For instance, a layperson may realise that the text comes from a specialised medium (e.g., an academic journal or institution) and that the level of language and details is not suited to their prior knowledge and goals. Importantly, task relevance evaluation requires task readers to interpret the task or question using activated schema to understand what is being asked for and how to achieve that goal state (Britt, Rouet and Durik, 2018[9]).

The PIAAC literacy assessment will include tasks involving multiple, possibly discrepant texts and a series of items assessing each of the evaluate processes.

Table 2.2. Summary of different types of evaluation processes

	Accuracy	Soundness	Task relevance
Content evaluation	Plausibility Quality of evidence	Completeness of facts or perspectives; bias in explanation or interpretation Internal consistency	Contribution to reading goals
Source evaluation	Author competence, bias Editorial control	Author's explicit or covert interests	Appropriateness of text type with respect to one's goals and abilities

Reflecting on the author's intent, purpose, and effectiveness. When evaluating texts, readers need to be aware of the author's intent or purpose for writing. Author purposes include to entertain, to inform, to explain or to describe, or to persuade. Author purposes generally have to be inferred from the structure and form of the text, although they are sometimes stated explicitly, for instance in a preface, an overview, or in a separate text, for instance a publisher leaflet or an interview with a journalist. Readers can also infer authors' purposes by acquiring information about the author's opinion, beliefs, attitude, assumption, or bias.

In addition to identifying the author's purpose and viewpoint, the reader can evaluate how the author conveyed their points and whether it was effective. The structure of the text as well as tone, word choice and writing style can provide cues to author purpose and perspective. In the context of the PIAAC literacy study, "Reflect" represents tasks in which the reader is explicitly asked about authors' intentions, purposes or effectiveness.

Because handling conflict across texts includes all aspects of evaluating and reflecting, it is important to include units involving multiple, discrepant texts to assess the extent to which adults can meet the challenges involved in contemporary reading situations.

Texts

Texts are vehicles that convey the ideas, beliefs and intentions of their authors. They are communication artefacts anchored in space and time (Wineburg, 1994[67]). Every text involves a source (where the text comes from: author, date and so forth) and some content (what is said in the text). Source and content information are both important for comprehending and making use of texts (Perfetti, Rouet and Britt, 1999[49]). Moreover, with the advent of digital technology, laypersons have access to a growing diversity of textual materials. In addition to traditional genres such as a novel, a newspaper article or a cooking recipe, new genres have appeared such as blogs, forums, or instant messaging systems (e.g. Twitter). Furthermore, text genres tend to be presented in combination, such as when readers react to an online article or offer their versions of a cooking recipe. The profusion of text genres represents new opportunities, but also new challenges for contemporary readers. In addition, readers are increasingly faced with multiple texts that they may have to read in parallel in order to achieve their purpose. For instance, a person who seeks advice about a health issue may look up a web forum and read several messages posted by different people. The person may then turn to the website of a hospital to seek further information, and so on and so forth. Therefore, modern text comprehension involves an ability to make sense of multiple and sometimes heterogeneous sets of texts.

In this context, ensuring the coverage of the literacy domain is a challenge, as there is no universal categorisation of text types, genres and formats. The PIAAC literacy framework rests on a distinction between single and multiple texts (as defined by a distinct source). In addition, the framework relies on distinctions made in previous assessments, such as text types (e.g., narration, description), text format (i.e., continuous vs. non-continuous texts) and the presence of organising devices enabling readers to navigate within and across texts.

Text types

Text types describe the diversity of texts as prototypical representations of the world and communication acts. The most frequently encountered text types are description, narration, exposition, argumentation, instruction and transaction. Naturalistic texts are usually difficult to categorise, as they tend to cut across these prototypical categories. For example, a newspaper article might start with a specific story (narration), then engage in some definitions and context (explanation), and a critical analysis (argumentation). Nevertheless, it is useful to categorise texts according to the text type, based on the predominant characteristics of the text, in order to ensure that the instrument samples across a range of texts that represent different types of reading. The classification of texts used in the PIAAC literacy assessment is borrowed from that used in the previous PIAAC and PISA assessments.

<u>Description</u> is the type of text where the information refers to properties of objects in space. Descriptive texts are mostly meant to answer "what" or "how" type of questions. Descriptions can take several forms. Impressionistic descriptions present information from a subjective point of view reflecting the viewer's impressions of elements, relations, qualities and directions in space. Technical descriptions present information from a more objective and perspective-independent viewpoint. Frequently, technical descriptions use non-continuous text formats such as diagrams and illustrations. Typical examples of descriptions are a depiction of a particular place in a travelogue or diary, a catalogue, a geographical map, an online flight schedule or a description of a feature, function or process in a technical manual.

<u>Narration</u> is the type of text where the information refers to properties of characters and objects in time. Narration typically answers questions relating to "what", "when", "how" or "in what sequence". Why characters in stories behave as they do is another important question that narration typically answers. Narration can take different forms. Narratives present change from the point of view of subjective selection and emphasis, recording actions and events from the point of view of subjective impressions in time. Reports present change from the point of view of an objective situational frame, recording actions and events which can be verified by others. News stories intend to enable the readers to form their own independent opinion of facts and events based on the reporter’s account. Typical examples narrations are a novel, a biography, a play, a comic strip and a newspaper report of an event.

<u>Exposition</u> is the type of text meant to communicate concepts, phenomena and other mental constructs involving a set of interacting elements. The text provides an explanation of how the different elements interrelate in a meaningful whole and often answers questions about "how" and "why" (referring to enabling conditions and causal relationships). Expositions can take various forms. Expository essays provide a simple explanation of concepts, mental constructs or conceptions from a subjective point of view. Definitions explain how terms or names are interrelated with mental concepts. In showing these interrelations, the definition explains the meaning of words. Explications are a form of analytic exposition used to explain how a concept can be linked with words or terms. Minutes are a record of the results of meetings or presentations. Typical examples of expositions are a scholarly essay about the metabolism of sugar, a diagram showing a model of memory, and a graph of population trends.

<u>Argumentation</u> is the type of text that presents factual or interpretive claims about a situation, together with supporting reasons and warrants. Argumentative texts often answer "why" (as in, for instance, "why did this happen?" or "why should we do this?"), but also "what if" questions. An important subcategory of argumentative texts is persuasive and opinionative texts, referring to opinions and points of view. A "comment" relates the concepts of events, objects and ideas to a private system of thoughts, values and beliefs. "Scientific argumentation" relates concepts of events, objects and ideas to systems of thought and knowledge so that the resulting propositions can be verified as valid or non-valid. Examples of text objects

in the text type category argumentation are a poster advertisement, the posts in an online forum and a web-based review of a book or film.

Instruction (sometimes called injunction) is the type of text that provides directions on what to do. Instructions present directions for certain behaviours in order to complete a task. Rules, regulations and statutes specify requirements for certain behaviours based on impersonal authority, such as practical validity or public authority. Examples of textual instruction are a cooking recipe, a series of diagrams showing a procedure for giving first aid and guidelines for operating digital software.

Transaction represents a written text that supports interpersonal communication, such as requesting that something is done, organising a meeting or making a social engagement with a friend. Before the spread of electronic communication, this kind of text was a significant component of some kinds of letters and, as an oral exchange, the principal purpose of many phone calls. Transactional texts are often personal in nature, rather than public, and this may help to explain why they do not appear to be represented in some of the corpora used to develop many text typologies. With the extreme ease of personal communication using e-mail, text messages, blogs and social networking websites, this kind of text has become much more significant as a reading text type in recent years. Transactional texts often build on common and possibly private understandings between communicators – though clearly, this feature is difficult to explore in a large-scale assessment. Examples of text objects in the text type transaction are everyday e-mail and text message exchanges between colleagues or friends that request and confirm arrangements.

Text format: Continuous, non-continuous and mixed texts

The building blocks of texts are written words, which can be organised according to the rules of syntax, coherence and cohesion, but also according to spatial dimensions such as in lists, tables and charts. In the PIAAC literacy framework, continuous texts are defined as sequences of sentences and paragraphs. These may fit into even larger structures such as sections, chapters and books. Non-continuous texts are defined as words, sentences or passages organised in a list or matrix format (Kirsch and Mosenthal, 1990[68]).

In both print and digital environments, written texts are often associated with non-verbal representations, such as graphics and pictures. The PIAAC assessment does not focus on these representations *per se*, but some tasks may involve the use of text in combination with graphics or pictures.

The PIAAC literacy framework also considers mixed texts, which involve both continuous and non-continuous components. In well-constructed mixed texts, the components (for example, a prose explanation including a graph or table) are mutually supportive through coherence and cohesion links at the local and global level. Mixed text is a common format in magazines, reference books and reports, where authors employ a variety of presentations to communicate information. In digital texts, authored web pages are typically mixed texts, with combinations of lists, paragraphs of prose and often graphics. Message-based texts, such as online forms, e-mail messages and forums, also combine texts that are continuous and non-continuous in format.

Text organisation: Layout, content representation and access devices

Naturalistic texts vary from a few lines to several hundreds of pages. Depending on the length and purpose, texts may include a range of devices aimed at representing content and facilitate access to passages of interest.

Organisation is primarily signalled by the sequence of sentences and texts, along with the use of different font sizes, font types such as italic and boldface or borders and patterns. Various types of discourse markers also provide information about how ideas are organised in the text. For example, sequence markers (first, second, third, etc.), signal the relation of each of the units introduced to each other and indicate how the units relate to the larger surrounding text. Causal connectors (therefore, for this reason, since, etc.) signify cause-effect relationships between parts of a text.

Larger texts often come with titles and headers, paragraphs and sections. These markers also provide clues to text boundaries (with space and a new header showing section completion, for example). Yet longer texts are organised into chapters, they include a table of contents and one or several indexes. Readers' awareness and use of these devices is critical to their effectiveness when reading texts for specific purposes (Goldman and Rakestraw Jr., 2000[69]).

Digital texts also come with a number of tools that let the user access and display specific passages. Some of these tools are identical to those found in printed texts (e.g., headers), whereas others are more specific to the electronic medium. Examples include windows, scroll bars, tabs, but also embedded hyperlinks. There is growing evidence that the processes involved in reading printed and digital texts differ, partly because of differences in presentation formats and navigation tools (Delgado et al., 2018[19]; Naumann, 2015[70]; OECD, 2011[71]). Therefore, it is important to assess readers' ability to deal with texts featuring a diversity of content representation and navigation tools.

The PIAAC literacy assessment will implement texts that vary on a continuum of length (i.e., single vs. multiple pages), but also diversity and density of content representation and access devices.

Source: Single vs. multiple texts

As mentioned in the introduction to this section, a text is defined by its source and its content. The PIAAC literacy framework defines single texts as texts that originate in a single source, i.e., an author, a publication medium, and a date of publication [other dimensions of the complex construct of a "source" will not be discussed here; see (Britt et al., 1999[72]), for a more detailed analysis of the construct of a source)]. Multiple texts are defined by having different authors, or being published through different channels or at different times.

It is important to note that in this framework the distinction between single and multiple texts is in principle independent from the amount of information contained in the text(s). A single text can be as short as a single sentence and as long as a whole book or website, as long as it has a single author (or group of authors), publication medium and date. Conversely, multiple texts can take the form of a series of brief passages, for instance in a web forum where different people post messages at different times. A single text can also contain embedded sources, that is, references to various authors or texts (Rouet and Britt, 2014[55]; Strømsø et al., 2013[73]).

Items in a set of multiple texts may have different relationships to each other: some texts may corroborate, complete, support or provide evidence for other texts, whereas others may disagree, contradict or conflict with others. Readers' cognitive representation of a set of texts together with their respective sources and the network of intertext relationships has been termed a "documents model" (Perfetti, Rouet and Britt, 1999[49]).

Table 2.3 summarises the dimensions of texts that are considered in the PIAAC literacy framework.

Table 2.3. Main dimensions of texts considered in the PIAAC literacy framework

Dimension	Levels
Text type	Description, narration, exposition, argumentation, instruction, transaction
Text format	Continuous, non-continuous, mixed
Text organisation	Continuous dimension involving the amount of information (number of pages) and the density of content representation and access devices
Source	Single vs. multiple texts

Social contexts

Reading pervades all domains of an individual's life. Reading activities are normally situated in a social situation and may serve a range of purposes from personal to professional and civic. Both the motivation to read and the interpretation of the content may be influenced by the context. As a result, the PIAAC literacy framework defines three main types of contexts that will be represented in the assessment:

a) Work and occupation. Written texts play an important role in a wide range of occupations. Uses of text in an occupational context includes finding employment, finance, and being on the job (i.e., regulations, organisation, safety instructions). However, the materials used in the PIAAC literacy assessment do not include specialised job-specific texts, which obviously would pose the problem of prerequisite background knowledge.

b) Personal use. Reading is also important for personal purposes. Many adults engage in reading when dealing with interpersonal relationships, personal finance, housing, and insurance. They also increasingly make use of written materials in addressing health and safety issues (e.g., disease prevention and treatment, safety and accident prevention, first aid, and staying healthy). Adults also use texts in relation to their consuming habits: credit and banking, savings, and advertising, making purchases, and maintaining personal possessions. Finally, texts are important in organising leisure and recreation time, including travel, restaurants, and material read for leisure and recreation itself (games etc.).

c) Social and civic contexts. Finally, literacy is essential in adults' participation in social and civic life. Community and citizenship includes materials dealing with community resources, public services and staying informed. Education and training includes materials that deal with opportunities for further learning.

Assessing literacy

General organisation of literacy tasks

The construct of literacy encompasses what readers can do with texts and also what they comprehend and remember from the texts. This warrants the design of testing situations in which test-takers may be asked to complete tasks either with the text available or after they have read the text, based on their memory for text information. Research suggests that answering comprehension questions with or without text availability tap in part on distinct mental processes, and that assessment tasks without the text available might be more sensitive to the quality of the reading processes and less dependent from reader motivation and test-taking strategies (Ozuru et al., 2007[74]; Schroeder, 2011[75]). However, the PIAAC literacy assessment focuses on what adults can do with texts, and therefore it is based on scenarios involving questions and one or several texts that remain available throughout the task. This is arguably the most common scenario in adults' daily uses of text (White, Chen and Forsyth, 2010[7]).

The PIAAC assessment of literacy is based on test units in which participants are asked to make use of one or several texts in order to answer a set of questions. A short introduction usually provides some context and motivation for the unit. Each question elicits one of the core processes defined in the framework (see section on cognitive task demands). Questions are presented one by one in a blocked format in order to decrease the influence of test-taking strategies and to reduce variance in test completion time.

The texts used as stimuli reflect texts that test-takers may encounter in real life. Many of them are directly drawn from authentic materials with little, if any adaptation. This means that no effort is made to make these texts easier to read or to improve their organisation or presentation. Using naturalistic texts, sometimes even clearly suboptimal ones (for instance, poorly organised or using complex language), ensures a high level of face validity. However, no artificial difficulty or flaw is introduced at the time of test design.

Response formats

Questions can be designed using a wide range of response formats, such as constructed (open) responses, true-false judgements, multiple choice, or responses based on filling a blank or highlighting a text passage, to cite just some of the most common types. Computerised test delivery also affords additional response modes, such as "drag and drop". The form in which responses are collected – the response format – varies according to what is considered appropriate given the kind of evidence that is being collected, and also according to the pragmatic constraints of a large-scale assessment.

Response formats can involve demands on specific cognitive processes. For example, multiple-choice comprehension questions are typically dependent on decoding skills, because readers have to decode distractors or items, when compared to open constructed response items (Cain and Oakhill, 2006[76]; Ozuru et al., 2007[74]). Conversely, constructed responses tap on written production as much as on comprehension skills. Several studies suggest that the response format has a significant effect on the performance of different groups (Grisay and Monseur, 2007[77]; Schwabe, McElvany and Trendtel, 2015[78]). Finally, participants in different countries may be more or less familiar with different response formats. Consequently, the use of a diversity of response formats is recommended to ensure precision and to reduce potential biases. However, consistent with the general guidelines for PIAAC Cycle 2, the assessment of literacy will not include any constructed response. Besides removing the need for human scoring, this reduces the confounding of comprehension and written production skills.

Adaptive testing design

The deployment of computer-based assessment in PIAAC creates the opportunity to implement adaptive testing. Adaptive testing enables higher levels of measurement precision using fewer items per individual participant. This is accomplished by targeting more items that are aligned to the ability range of participants at different points in the ability distribution.

Adaptive testing has the potential to increase the resolution and sensitivity of the assessment, most particularly at the lower end of the performance distribution. For example, participants who perform low on items that assess their ease and efficiency of reading (e.g. reading fluency) will likely struggle on highly complex multiple text items. Thus, there would be benefit in providing additional lower-level texts for those participants to better assess specific aspects of their comprehension.

Recommended distribution of items

The Literacy Expert Group recommends the following distribution of items based on a typology of cognitive task demands, text size and contexts.

Recommended distribution by cognitive task demands and number of sources

The rationale for the recommended distribution per cognitive task demands is as follows: a substantial number of items (45%) should involve text understanding, both literal and inferential, as this is considered a core process present in most if not all reading activities. Due to its increased importance in digital environments, the category "access" (which involves identifying texts in a set and locating information within texts) should also be broadly represented (35%). Finally, about 20% of the tasks should involve one type of evaluation or reflection about the text.

As regards text size, most tasks (60%) will involve texts presented on a single page, with the view that some of these need to be simple enough so as to describe basic levels of literacy. Some of these short texts may involve multiple sources (such as, e.g., a series of short messages on a web forum page). However, acknowledging that readers most often face texts distributed across multiple pages (either from one or from several sources), the test will also include multi-page units. It is expected that tasks focusing on the process of "understanding" will be proportionally more represented in single page units, whereas "access" and "evaluate" tasks should be more frequent in multi-page units.

Table 2.4 presents the recommended distribution of items as a function of text size (i.e., single vs. multiple pages) and cognitive task demands.

Table 2.4. Recommended distribution of items as a function of text size and cognitive task demands

Cognitive task demands	Single page	Multiple pages	Total
Access	20%	15%	35%
Understand	30%	15%	45%
Evaluate	10%	10%	20%
Total	60%	40%	100%

It is further recommended that a majority of the test units (goal: 60%) include single source texts.

Recommended distribution by context

A broad range of tasks drawn from realistic contexts is meant to help ensure that no group of respondents will be either advantaged or disadvantaged based on their familiarity with, or interest in, a particular context. The recommended percentage of tasks for work, personal, community and education types of contexts is 15, 40, 30, and 15%, respectively.

Distribution across other relevant dimensions

No specific recommendation is made regarding a distribution of tasks across dimensions of text types or response formats, beyond the general recommendation to ensure a broad diversity and a representation of as many types as possible.

The role of fluent reading, engagement and metacognition

Reading fluency can be defined as an individual’s ability to read words, sentences and connected text efficiently (Kuhn and Stahl, 2003[79]), i.e. both quickly and accurately. Fluent readers master the basic reading processes of recognising written words, assigning meaning to these words, and establishing a coherent sentence meaning by way of syntactic parsing and semantic integration. They do so without using a large amount of working memory and attentional resources (LaBerge and Samuels, 1974[80]; Perfetti, 1985[1]). Therefore, fluent readers have more cognitive resources available to invest in higher-level comprehension processes such as inferences and reading strategies (Walczyk et al., 2004[81]). The differential allocation of mental resources to low- vs. higher-level processes in struggling vs. fluent readers accounts for the strong link between fluent reading and text-level comprehension outcomes found in many studies and in all age groups ranging from primary school to adult readers (García and Cain, 2014[82]; Klauda and Guthrie, 2008[83]; Richter et al., 2013[84]).

To better assess reading fluency, the PIAAC Cycle 2 assessment will again include a measure of reading component skills. The components assessment tasks are designed to inform our understanding of the basic reading skills that underlay proficient literacy performance levels. These tasks help describe what low literate adults can do and therefore form a basis for learning, instruction, and policy with respect to helping low literate adults achieve higher literacy levels (Sabatini and Bruce, 2009[85]). In response to the OECD's requirement that the results of the components assessment be generalisable to the overall population, the components tasks will be administered to a representative subsample of all individuals who take the full literacy assessment.

The reading components assessment will include two sets of tasks, both of which were administered in the first cycle of PIAAC. The first set focuses on the ability to process meaning at the sentence level. Respondents will be shown a series of sentences, which increase in complexity, and be asked to identify if the sentence does or does not make sense in terms of properties of the real world or the internal logic of the sentence. The second set of tasks focuses on passage comprehension. For these tasks, respondents are asked to read passages where, at certain points, they must select a word from two provided alternatives so that the text makes sense [see sample tasks in (OECD, 2019[86])].

Because PIAAC Cycle 2 will be administered on tablets, it will be possible to precisely record both accuracy and response times for the component tasks. The accuracy data in the sentence verification and passage comprehension tasks will serve as indicators of the mastery of basic reading comprehension processes. They will be included in the scaling of the items in the PIAAC literacy assessment, increasing measurement precision in the lower range of the scale. The response times will serve as an indicator of fluency in basic reading processes, allowing researchers to explore its potential contribution to the mastery of the more complex literacy tasks in the PIAAC literacy assessment.

The concept of reading engagement refers to the degree of importance of reading to an individual and to the extent that reading plays a role in their daily life. Empirical studies with children and adults have shown that differences in engagement are systematically related to differences in performance on assessments. In particular, studies with different age groups provide evidence for an upward causal spiral: more proficient readers will read more and the exposure to printed texts will promote their reading development and lead to higher proficiency (Guthrie and Wigfield, 2000[87]; Mol and Bus, 2011[88]). The construct of engagement encompasses objective aspects such as the amount and diversity of reading one experiences in daily life, and also subjective aspects such as one's interest in reading, perception of control over reading, and reading efficacy. The PIAAC literacy assessments will capture core aspects of the objective aspects of reading engagement as part of the background questionnaire.

Metacognition, or one's awareness, monitoring and control of their own cognitive processes, is also considered an important aspect of reading literacy (Baker, 1989[89]). However due to methodological and practical constraints the PIAAC literacy study will not include any specific assessment of metacognition in reading. Metacognition will be indirectly assessed through its contribution to the more complex reading tasks which require strategic decisions and self-regulation to different degrees.

Factors driving task difficulty

The difficulty of literacy tasks is expected to depend on three series of factors, namely a) characteristics of the text(s); b) characteristics of the question; and c) the specific interaction between a question and a text (or set of texts).

In addition, some of these factors affect the difficulty of the task regardless of the specific cognitive demands involved, whereas other factors are specific to a certain type of task demand. Table 2.5 lists the main text, task, and text-by-task factors driving difficulty in general, and then more specifically for each type of cognitive task demand.

Table 2.5. Text, task, and text-by-task factors driving difficulty as a function of cognitive task demands

	Text factors	Task factors	Text-by-task factors
Factors affecting all tasks	Longer, multiple texts are generally more difficult because they increase processing load and require readers to sustain their attention over a longer time span. Longer texts are also more likely to contain distracting (task-irrelevant) information. Text dealing with unfamiliar contents, using unfamiliar words and/or a complex syntax or organisation are also more difficult regardless of the task. Content representation and signalling devices such as tables of contents, headers, boldface, underlining, and bullet points generally decrease the text difficulty.	Tasks involving a longer stem and/or unfamiliar words are more likely to be forgotten en route, thus requiring the reader to re-read the question. Readers with low-levels of self-monitoring may fail to realise that they need to refresh their memory. The lack of explicit guidance regarding which portion(s) of the materials should be inspected increases the difficulty of the question, compared to questions that include instructions as to where to look the answer.	Tasks involving a direct match between the question and the text are easier than tasks that require the reader to infer the link between the question and the relevant portion of the text. Texts that contain a large number of distracting information (for instance, passages sharing keywords with the question though irrelevant content wise) are more difficult than those in which a single passage is related to the question.
Difficulty drivers for "Accessing" tasks	Texts distributed across multiple pages require multiple stages of selection: selecting the right text and then the right portion of that text. Multi-page texts that are organised in non-linear ways, with several levels of links, are more difficult to search through than texts organised linearly or in the form of more shallow hierarchies.	Questions requiring the reader to gather multiple pieces of information across texts are more difficult than questions involving a single piece of information.	Texts containing content organisers (e.g., headers) that match the topic of the question are easier to access than those in which the location of information remains implicit.
Difficulty drivers for "Understanding" tasks	In addition to the general factors listed above, texts involving an implicit and/or unfamiliar structure are more difficult to understand. In sets of multiple texts, the presence of inconsistencies add the burden of identifying and resolving them.	Questions that require a large amount of information are more difficult that those that can be answered based on a single piece of information. Simple, connecting inferences are considered easier to perform than elaborative inferences, which require using one's prior knowledge.	Comprehension questions that require the test-taker to draw an inference based on text information are more difficult than questions whose answers are explicit in the text. Questions that require the test-taker to relate several pieces of information located in distant portions of the text(s) are more difficult than those for which the relevant information is grouped within a single section.
Difficulty drivers for "Evaluating" tasks	Unfamiliar, incomplete or less salient source indications make accuracy assessment more difficult. Unusual argument structures and incomplete arguments are more difficult to evaluate.	For familiar contents, factual inaccuracies are easier to detect than flaws in an argument structure (connection of claims and supporting reasons).	Texts involving low-quality sources issuing topically-matching information make it more difficult for the reader to evaluate the relevance of the information.

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Note

[1] Navigation in a static piece of continuous text is always possible by simply shifting one's focus of attention from one passage of the text to another, by skimming through passages, and by browsing through pages and sections in the case of long texts.

3 PIAAC Cycle 2 assessment framework: Numeracy

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This chapter presents the framework for conceptualising and assessing adult numeracy and developing a reporting scale for the direct assessment of numeracy as part of the OECD's Programme for the International Assessment of Adult Competencies (PIAAC Cycle 2). Numeracy as described here refers to adults' skills in accessing, using and reasoning critically with mathematical content, information and ideas represented in multiple ways in order to engage in and manage the mathematical demands of a range of situations in adult life. The framework describes the conceptual and theoretical foundations behind the adult numeracy construct and the principles applied for assessing numeracy in PIAAC and the distribution of the numeracy assessment items by a range of task characteristics.

Introduction

This chapter presents the framework for conceptualising numeracy as part of the OECD's Programme for the International Assessment of Adult Competencies (PIAAC Cycle 2). It builds on conceptual and assessment frameworks and cumulative wisdom developed in connection with prior surveys of adult skills, primarily the first cycle of the PIAAC, the Adult Literacy and Lifeskills project (ALL) and the International Adult Literacy Survey (IALS), but also surveys of school-age students e.g., the Programme for International Student Assessment (PISA).

Structure of the chapter

This chapter has six separate sections, followed by references:

- The assessment of numeracy in PIAAC
- Conceptual and theoretical foundations
- Numeracy assessment construct in PIAAC Cycle 2
- Operationalisation of the PIAAC numeracy assessment
- Relationship between PIAAC and PISA
- Numeracy components.

The first section provides a summary of the 2017 review of the PIAAC numeracy framework and assessment, gives an indication of some other conceptual issues considered, and includes a brief rationale for assessing numeracy in PIAAC. The second section addresses the conceptual construct for numeracy. The third section addresses the assessment construct, and describes the different dimensions of numeracy being assessed, including contexts, expected responses, content areas of mathematical information and ideas, and representations, as a way of operationalising the numeracy construct for scale development. It also discusses enabling processes, both cognitive and non-cognitive or dispositional, which underlie numerate behaviour. The fourth section discusses the operationalisation of the construct of numeracy in a large-scale assessment such as PIAAC and how this is affected by many factors that determine and shape the extent to which the theoretical construct can be fully addressed by the actual collection of items used in the direct assessment. It describes what can, and what cannot, be assessed in PIAAC Cycle 2. Subsequent sections comment on differences and commonalities between PIAAC's numeracy assessment and the related construct of mathematical literacy assessed in the Programme for International Student Assessment (PISA); and another section is dedicated to describing the new numeracy components assessment.

Why have an assessment framework and construct for PIAAC?

An assessment framework and construct is required for any valid assessment. The assessment framework provides a definition of the domain and the features of the construct. Such a framework usually includes:

- the background, purpose and rationale for, and description of, the assessment programme based on a theoretical and conceptual framework
- the target groups for the assessment
- a definition of the domain
- the description of any variables that are part of the description and describe its depth and breadth (e.g., contexts, processes, content)
- a blueprint for the test development against the above descriptions and variables, which might also include item types, representations, the length of the assessment, the number of items and the spread against the different variables.

Together, these aspects and content create the conceptual and assessment framework that will guide the assessment, as is the case with this document and its role for the assessment of numeracy components in PIAAC. It defines the construct of numeracy that steers the development of the test items and eventually the interpretation of results. The assessment construct provides a formal definition of the domain and the features of the construct in terms of any key parameters or dimensions of content, cognitive strategies and range of applications that need to be covered by the content of the assessment.

It is important to note that the PIAAC numeracy assessment describes the full range of numeracy capability in the adult population. This covers at one extreme, adults who have university level training and, at the other, adults who have very limited levels of education (e.g. who left school at or before the age of 15). At the same time, it covers both young adults still in education and adults who completed their formal education 30-50 years prior to undertaking the assessment.

The assessment of numeracy in the second cycle of PIAAC will link to the assessment used in the first cycle of PIAAC and also the earlier ALL study through the use of linking items. As a result, this revised conceptual framework for assessing numeracy in PIAAC will need to maintain key conceptual and pragmatic links to the numeracy framework used for the ALL study and PIAAC Cycle 1.

At the same time, it is important that the framework identifies a construct of numeracy that is relevant to the realities of the third decade of the 21st century as well as reflecting contemporary understandings of adult numeracy and that it incorporates relevant developments in testing practice and makes the best use of the available testing technologies.

The assessment of numeracy in PIAAC

This part of the report provides a summary of the 2017 review of the PIAAC Cycle 1 numeracy framework and assessment, gives an indication of some other conceptual issues to consider—identified by the new PIAAC Cycle 2 Numeracy Expert Group (NEG)—and finishes with a brief rationale for assessing numeracy in PIAAC.

Review report

The conceptual and assessment framework for the second cycle of PIAAC numeracy was expected to be updated and revised based on a review of the numeracy assessment framework used in the first cycle of PIAAC. This review was commissioned by the OECD Secretariat and published at the beginning of 2017 (Tout et al., 2017[1]). The aim of this review project was to prepare a paper reviewing the framework that guided the assessment of numeracy in the first cycle of PIAAC.

The review aimed to evaluate the extent to which the framework developed in 2009 reflected current understandings of adult numeracy and continued to be an appropriate basis for the assessment of the capacity of adults to successfully undertake the range of numeracy tasks that they will face in their everyday and working lives in the third decade of the 21st century. In particular, the review addressed the following:

- theoretical developments in the understanding and conceptualisation of adult numeracy that are relevant for the assessment of numeracy in PIAAC
- how to ensure that the assessment reflects the importance of digital information, representations, devices and applications as realities that adults have to manage in dealing with the numerical demands of everyday life
- developments in the assessment of numeracy (particularly of adults) that could be relevant for PIAAC (e.g., item types and formats, use of animation, and modelling)

- how the relationship between the PIAAC numeracy framework and the PISA mathematical literacy framework and assessment should be conceived, developed (if appropriate) and presented
- the utility and feasibility of the development and implementation of an assessment of numeracy components equivalent to the PIAAC reading components assessment.

The review recommended a range of areas for potential improvements and enhancements, including the definition and elaborations of adult numeracy used in the framework, and of the assessment content. Many of the suggestions arose out of the concern that the existing Cycle 1 framework and assessment did not reflect some of the realities of the skills and knowledge adults needed to succeed in work, life, and citizenship in the 21st century. Some of the key elements arising from the review paper included:

- addressing 21st century skills including critical thinking and reflection, reasoning and understanding of degree of accuracy
- taking on board technology/ICT advancements while keeping a balance with more traditional modes and means of communication and undertaking numeracy tasks
- making better use of technology for assessment in relation to both authenticity and making items accessible
- addressing a number of issues regarding adults' numeracy performance and understandings, including a person's disposition to use mathematics and to see mathematics in a numeracy situation
- developing an assessment of numeracy components, which would have parallel aims to the existing reading components assessment, and provide insights into the skills and knowledge of the significant number of adults with low levels of numeracy.

This report and the recommendation have been instrumental in the writing of this framework and assessment construct. The review and this document build on conceptual and assessment frameworks and cumulative wisdom developed in connection with prior surveys of adult skills, primarily the first cycle of PIAAC – see PIAAC Numeracy Expert Group (2009[2]) and Gal and Tout (2014[3]). The PIAAC Cycle 1 framework and assessment drew heavily on the numeracy assessment framework of the Adult Literacy and Life Skills Survey (ALL) (Murray, Clermont and Binkley, 2005[4]). It also built on the work in the International Adult Literacy Survey (IALS), and surveys of school-age students, especially the Programme for International Student Assessment (PISA).

Some new issues

In the process of considering the recommendations and content of the 2017 review paper by the new PIAAC Cycle 2 Numeracy Expert Group (NEG) at its first meeting in March 2018, a number of additional issues were identified which needed some further exploration. As a result, there was a literature review undertaken by the NEG exploring the following five conceptual issues: Big Ideas in mathematics, Number sense, Embeddedness, Authenticity, and Numerate behaviour and practices. While this work has been incorporated and embedded throughout this revised numeracy framework, a brief summary is included below.

Big Ideas in mathematics

Big Ideas in mathematics, is a term that is used to talk about powerful mathematical ideas (Jones, Langrall and Thornton, 2002[5]) central to the learning of mathematics, linking numerous mathematical understandings into a coherent whole [e.g., see (Charles, 2005[6]; Hurst, 2014[7]; Hurst and Hurrell, 2014[8]; Kuntze et al., 2009[9]; Kuntze et al., 2011[10]; Steen, 1990[11])]. Initially, the term "Big Ideas" referred to how mathematical information can be classified in different ways compared with the traditional school mathematics curriculum content areas. They often include the following content domains (which are elaborated in the PIAAC framework): quantity and number, space and shape, change and relationships, and data and chance. "Big Ideas" are also used as focal points to add some structure to sometimes "overcrowded curricula" (Siemon, 2017[12]; Siemon, Bleckly and Neal, 2012[13]).

Number sense

In PIAAC Cycle 2, particularly in relationship to the challenge to develop a separate, new, numeracy components assessment, number sense is seen as relating to a person's general understanding of different types of number and operations, and it involves a critical understanding in order to make decisions and solve problems using numbers in flexible ways in *Personal*, *Work*, and *Societal/community* contexts (Ontario Ministry of Education, 2006[14]; Peters, 2012[15]; Wagner and Davis, 2010[16]; Yang, Reys and Reys, 2009[17]). McIntosh, Reys and Reys (1992[18]) define number sense as: "It reflects an inclination and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information" (p. 3). In addition, "using numbers is more than reasoning about number and more than skilled calculations. It is about making sense of the situation to which we apply numbers and calculations" (Thompson, 1995, p. 220[19]). Numbers, and quantitative expressions, may be presented in a range of different representational systems, including: text or symbols, images of physical objects, structured information and dynamic information. An understanding of *number sense* has been identified as a key element and is addressed throughout the framework, and has helped underpin the development of the new numeracy components assessment for PIAAC Cycle 2. This is elaborated further in the sixth section: *Numeracy components*.

Embeddedness, authenticity, numerate behaviour and practices

These four issues are all interrelated, and the NEG has attempted to address them more explicitly throughout the new framework, its elaboration, and to some extent in the content of the assessment itself, including in the background questionnaire (BQ) questions that relate to numeracy and mathematics skill use. These issues of embeddedness, authenticity, numerate behaviour and numerate practices, all relate to an understanding of the vital and underpinning connection to the real-world context in which mathematics is utilised by adults in their daily lives as individuals, citizens, family members or as workers.

The embeddedness of mathematics refers to the deep connections the mathematics has to the context in which it is utilised. This means that the way mathematics is used to operate on a task is fundamentally shaped by the context in which it is employed, which includes socio-cultural influences that afford or constrain action in civic, personal or workplace environments. In this view there is a clear separation between school mathematical knowledge, how it is taught, learned and practised, and the use of this knowledge outside of schooling. The issue of authenticity is a significant issue in the development of the test questions in PIAAC as it relates to how alike a task is in an international assessment like PIAAC to the actual real-life situation that the assessment task has been adapted from. PIAAC items are developed on the basis of finding situations and tasks that are based on authentic stimuli and then composing sets of questions that someone would want answered about, or based on, the information in the stimulus. Numerate behaviours and practices are distinct, but complementary, issues. Numerate behaviours relate to the cognitive responses by an individual to situations where mathematics is embedded in a real-world problem where a response or action is expected. Numeracy practices relate to the different uses of mathematics within a context, defined not just by the problem itself, but also by the physical and social context in which it is situated. These issues are further elaborated and discussed throughout the framework paper.

Rationale for assessing numeracy in PIAAC

As was argued in the PIAAC Cycle 1 numeracy framework (PIAAC Numeracy Expert Group, 2009[2]), this framework and description of numeracy is founded on the assumption that a direct assessment of numeracy in PIAAC is an essential and worthwhile undertaking for four separate but related reasons (PIAAC Numeracy Expert Group, 2009, pp. 8-9[2]):

- Numeracy is essential for adults and for the societies in which they live.
- Public policy in most countries includes separate investments in literacy and numeracy.
- The policy and programme responses are different for numeracy than for literacy.
- Numeracy skill levels are not measured well by literacy measures.

Basic computational and mathematical knowledge has always been considered as part of the fundamental skills that adults need to possess to function well and be able to accomplish various goals in their everyday, work, and social life. As will be demonstrated later in this framework, societies now present increasing amounts and a wider range of information of a quantitative or mathematical nature to citizens from all walks of life, in diverse contexts such as regarding health risk factors, environmental impacts, or financial planning and insurance purchasing, to name just a few. As workplaces are becoming more technology rich and concerned with involving all workers in improving efficiency and quality, the importance of numeracy and mathematical skills is growing. Numeracy-related skills have been shown to be a key factor in labour market participation, sometimes even more so than literacy skills. Adults with lower level skills in numeracy and literacy are more likely to be unemployed or require social assistance. Further, sound numeracy skills are deemed essential for post-secondary education in many areas, including but not limited to hard sciences, engineering and technology (Benn, 1997[20]; Bynner and Parsons, 2005[21]; Coben et al., 2003[22]; Coben, O'Donoghue and FitzSimons, 2000[23]; Condelli et al., 2006[24]; Coulombe, Tramblay and Marchand, 2004[25]; Forman and Steen, 1999[26]; Gal, 2000[27]; Gal et al., 2005[28]; Ginsburg, Manly and Schmitt, 2006[29]); (Hoyles et al., 2002[30]; Johnston, 1994[31]; Jonas, 2018[32]; Jones, 1995[33]; Murnane, Willett and Levy, 1995[34]; National Research and Development Centre (NRDC), 2006[35]; OECD/Statistics Canada, 2005[36]; Tout and Gal, 2015[37]; Tout and Schmitt, 2002[38]; Willis, 1990[39])

Public policy in most countries includes separate investments in literacy and numeracy. The separate acquisition of skills in these two fundamental areas is emphasised throughout both primary and secondary school systems, and in adult education or nonformal learning schemes. Countries expect that investment in literacy and numeracy will increase citizens' ability to act independently towards their own progress and income security, thereby reducing future social expenditures as well as contributing to citizens' participation in economic and social life in an information-laden society.

The policy and programme responses are different for numeracy than for literacy. Efforts to improve literacy and numeracy levels of specific population groups are not necessarily implemented via the same mechanisms—they often require different experts, resources, and learning systems because of differences in the underlying knowledge components and learning trajectories. It is vital that nations have information about their workers' and citizens' numeracy, independently of other competency areas, in order to evaluate the human capital available for advancement, to plan school-based and lifelong learning opportunities, and to better understand the factors that affect citizens' acquisition and usage of numeracy (Johnston and Maguire, 2005[40]).

It is not possible to represent the numeracy levels in a population solely via people's performance on literacy measures that examine how well people read, process, and comprehend various types of texts and documents, or communicate about such texts. As found in PIAAC and other research that compares adult's skills and performance in literacy with numeracy , there are substantial differences in the performances, outcomes and implications/consequences of lower or higher numeracy skilled adults compared to literacy skills [e.g., see (Bynner and Parsons, 2005[21]; Jonas, 2018[32]; OECD, 2017[41]; Tout and Gal, 2015[37])]. As is explained in more detail later, numeracy involves, among other things, the handling of not only arithmetical processes, but also the understanding of proportions and probabilistic ideas, the understanding of numerical, geometric, graphical and algebraic types and representations of mathematical information, and the critical interpretation of statistical or mathematical messages. Most of these elements and processes bear little relation to what is subsumed by literacy measures (Coben, O'Donoghue and FitzSimons, 2000[23]; Gal et al., 2005[28]).

It follows that a direct assessment of numeracy in PIAAC can provide policy makers and other stakeholders with a unique and sound basis for evaluating the distribution of the actual numeracy competence in the adult population.

Conceptual and theoretical foundations

The conceptualisation of *numeracy* in an international context is a challenging undertaking. Like literacy, the term numeracy has multiple meanings across countries and languages. In some countries the term numeracy relates to basic skills which school children are expected to acquire as a prerequisite to learning formal mathematics at higher grades. In other countries the term numeracy encompasses a broad range of skills, knowledge and dispositions that adults should possess but it does not necessarily relate to formal schooling. This is discussed further in the next section. Some countries do not even have a word such as numeracy; therefore, as part of educational or policy-oriented discourse in such countries, experts or translators either had to invent a special new word for it (e.g., *Numératie* in Canada, *Numeralitet* in Denmark), or use other phrases such as "mathematical literacy", "functional mathematics", or terms equivalent to "computational ability". Such diversity in terminology, or the lack of an accepted term with which policy makers feel comfortable, can complicate the communication with and among policy makers and educators interested in PIAAC.

The range of meanings attached to the term numeracy and the lack of an equivalent term across languages may create miscommunications or gaps in expectations regarding what will be measured by a numeracy scale in PIAAC. This can affect the perceived policy relevance of a numeracy scale. Thus, attention has to be given to making sure that discussions regarding numeracy assessment in PIAAC are based on a clear description and consensus about the scope of the term and recognition of its centrality in a wide range of adult life circumstances.

It must also be remembered that what will be measured by a numeracy assessment is *jointly* determined by two interrelated factors:

- the conceptual construct describing numeracy and its elements
- the assessment construct describing how the general conceptualisation of numeracy is operationalised and manifested in the nature and range of tasks used in the assessment and the mode of administration and scoring.

Developing perspectives on adult numeracy

Formulation of what numeracy encompasses has evolved since the term was first introduced in the 1959 Crowther Report in England and Wales [e.g., see (Karaali, Villafane-Hernandez and Taylor, 2016[42])], when it was defined as something "more than mere ability to manipulate the rule of three" (Crowther, 1959, p. 270[43]). Another significant milestone in the conceptualisation and description of numeracy was in the Cockcroft report of 1982, where it was defined as:

> *[n]umeracy is…an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life…[and] an ability to have some appreciation and understanding of information, which is presented in mathematical terms, for instance graphs, charts or tables or by reference to percentage increase or decrease. (Cockcroft, 1982, p. 11[44])*

The use and meaning of the term *numeracy* has gained momentum in the years since the Crowther and Cockcroft Reports. Some relevant papers and research include: (Baker and Street, 1994[45]; Benn, 1997[20]; Coben, O'Donoghue and FitzSimons, 2000[23]; Coben et al., 2003[22]; Condelli et al., 2006[24]; Forman and

Steen, 1999[26]; Gal, 2000[27]; Gal et al., 2005[28]; Ginsburg, Manly and Schmitt, 2006[29]; Hoyles et al., 2002[30]); (Johnston, 1994[31]; Lindenskov and Wedege, 2001[46]; Maguire and O'Donoghue, 2003[47]; National Research and Development Centre (NRDC), 2006[35]; Tout and Gal, 2015[37]; Tout and Schmitt, 2002[38]; Willis, 1990[39]). In the United Kingdom in 2000, Coben, O'Donoghue and FitzSimons published a work titled *Perspectives on Adult Learning Mathematics*, in which they provided a review of research related to adults' learning of mathematics. At the same time, a similar volume, *Adult Numeracy Development: Theory, Research, Practice* was published in the United States (Gal, 2000[27]). A conceptualisation of adult numeracy for the Adult Literacy and Life Skills (ALL) survey, the precursor to PIAAC, was developed around the same time (1998-2000) by an international team (Gal et al., 2005[28]). This was the first time the construct of numeracy had to be defined in an international comparative assessment context and not purely in an educational context. The ALL international team defined numeracy alongside a more elaborate definition of numerate behaviour. Coben, in 2003, led a team who wrote *Adult Numeracy: Review of Research and Related Literature* and noted that numeracy was increasingly defined as "mathematics in work and mathematics in everyday adult life" (Coben et al., 2003, p. 38[22]).

Maguire and O'Donoghue (2003[47]) reviewed and organised conceptions of numeracy from several countries (Australia, Canada, Denmark, Ireland, the Netherlands, United Kingdom and United States) along a continuum of increasing levels of complexity or sophistication: *formative*, *mathematical* and *integrative*. *Formative* conceptions view numeracy as related to basic arithmetic skills. *Mathematical* conceptions consider numeracy in a contextualised way, as a broader set of mathematical knowledge and skills (beyond basic computations) of relevance in everyday life. Finally, *integrative* conceptions consider numeracy as a multifaceted, sophisticated construct incorporating not only mathematics but also communicative, cultural, social, emotional, and personal elements which interact and pertain to how different people function in their social contexts.

At this time, formative conceptions were often associated with how numeracy was viewed in connection with goals of primary schooling, and reflected in how numeracy was defined when classifying literacy/numeracy levels worldwide e.g., UNESCO (1997[48]). Most extant conceptions which adult education, workplace training, and national and international assessments have adopted fall at different points across the mathematical and integrative phases described by Maguire and O'Donoghue. The range of conceptions and definitions of adult numeracy from late last century to more recent times illustrate that conceptions evolve over time and that variability can be noticed even within the same national system.

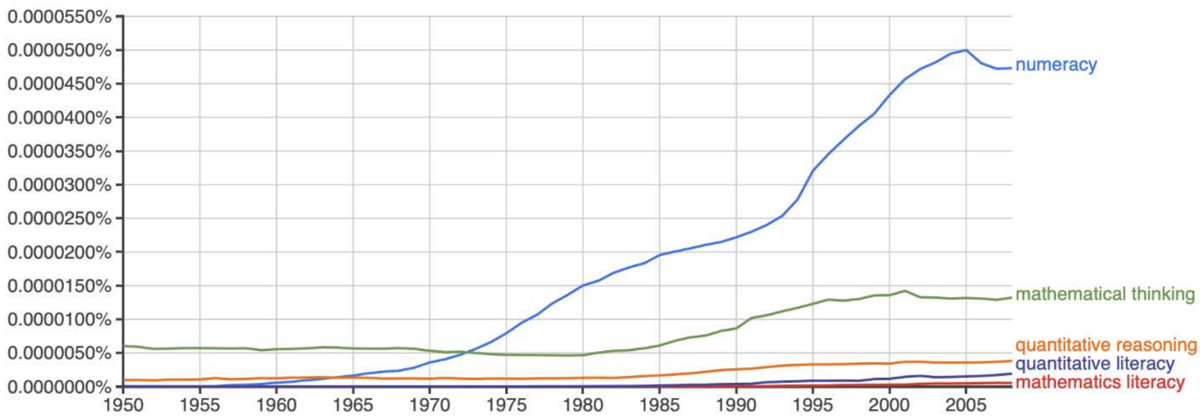
Lindenskov and Wedege (2001[46]) offer an interesting case study of defining numeracy. Based on their work in adult and mathematics education in Denmark, they imported numeracy from English-speaking countries and introduced a new term, *Numeralitet*, with a conceptual framework that was later adopted by the Danish Ministry of Education. According to this perspective, it is essential to distinguish between what numeracy is, or ought to be, from the individual's and from society's points of view. Lindenskov and Wedege advocated a societal view, whereby numeracy is seen as a competence that involves a dynamic interaction between functional mathematical skills and conceptions and operations on the one hand, and a series of activities and various types of data and media on the other. They argued that this skill- and activity-based view should be coupled with the understanding that in principle all people need to have this competence, and that numeracy is a competence determined by society and technology and that it changes in time and space along with social change and technological development.

The definition quoted from the United Kingdom's Cockcroft Committee (1982[44]) earlier has been quite influential in that its conception of numeracy implied it is an ability to cope with various functional tasks in real-world contexts as well as interpretive tasks, but also pointed to the centrality of underlying supporting non-cognitive components. These key ideas are reflected, albeit with different terminologies and foci, in other views of numeracy, including in the definitions used in both ALL and PIAAC Cycle 1. Another important commonality is the presence of mathematical elements or ideas in real situations, and the notion that these can be used or addressed by a person in a goal-oriented way, dependent on the needs of the individual within the given context, i.e., home, community, workplace, societal action, etc.

Common use of the term numeracy in the 2000s

Looking at publishing sources, the evidence is growing that the use and concept of the term numeracy has been displacing other terms such as mathematical literacy and quantitative literacy, and has become more popular, even though in many languages it does not have a direct translation. This is illustrated in Figure 3.1 below, which is based on work by Karaali, Villafane-Hernandez and Taylor (2016[42]), but updated.

Figure 3.1. Use of the term numeracy versus other terms in the books published between 1950 and 2008, included in Google Books



Source: Data from Google Books Ngram Viewer. Retrieved on 15th December 2018.

Competence versus skill

The two terms *Competency* and *Skill* are both used in current PIAAC documents and reports. Competencies can mean different things in different situations, and in different cultures. For example, a reductive notion of competence is used as a synonym for skill in some adult education settings within Australia. Competencies can however also mean the combination of skills and aspects of higher order thinking such as strategic planning. The latter is reflected in the OECD's Definition and Selection of Competencies project (DeSeCo) (OECD, 2005[49]), which was developed to provide a framework informing the identification of key competencies in international surveys measuring the competence level of young people and adults [see Rychen (2004, p. 321[50])].

In *The Survey of Adult Skills – Reader's Companion, Second Edition* (OECD, 2016[51]), the OECD discusses this issue of terminology and the use of these two terms (*skill* and *competency*) and concludes that there is much overlap in their understanding and use, and while acknowledging that there can be differences in the use and meaning of the two terms, concludes:

> *In the context of the Survey of Adult Skills (PIAAC), however, no attempt is made to differentiate competency and skill, and the terms are used interchangeably ... Both terms refer to the ability or capacity of an agent to act appropriately in a given situation. Both involve the application of knowledge (explicit and/or tacit), the use of tools, cognitive and practical strategies and routines, and both imply beliefs, dispositions and values (e.g. attitudes). In addition, neither competency nor skill is conceived as being related to any particular context of performance, nor is a skill regarded as one of the atomic units that combine to form competency. (OECD, 2016, p. 17[51]).*

This numeracy framework report similarly does not make or attach an explicit meaning or use to either term.

Numerate behaviour and practices

Establishing and extending numeracy capability requires the adoption, development or appropriation of both numerate behaviours and practices. These two constructs are distinct but complementary. Numerate behaviours are cognitive responses by an individual to particular situations where mathematics might provide advantage in addressing a real-world problem. On the other hand, numeracy practices relate to the use of mathematics within a context defined not just by the problem but also the physical and social context in which it is situated. The notion of situatedness is tied to ways of thinking, modes of reasoning and means of knowledge generation within communities that are defined by distinct social or cultural types of activity. From this perspective, numeracy is viewed as a social practice. As Yasukawa et al. (2018[52]) explain:

> *A NSP [numeracy as a social practice] perspective focuses on what people do with numeracy through social interactions in particular contexts, rather than on people's performance of mathematical skills in isolation of context…Moreover, a focus on practice entails viewing numeracy activity as culturally, historically and politically situated. (p. 13)*

Thus, employing numeracy behaviours to address real-world problems in different contexts requires the accommodation of unique ongoing activities, social relationships and community-based modes of thinking and reasoning (Lave, 1988[53]). This means that the use of mathematics within a practice requires that mathematical capability is nuanced by holistic strategies shaped by the specific contexts in which they are deployed (Geiger, Goos and Forgasz, 2015[54]).

The notion of numeracy as a social practice, however, has implications for the question of transfer—the use of numerate behaviours developed in one context in a new or different situation. This issue of transfer, according to Hoyles, Noss and colleagues (Hoyles, Noss and Pozzi, 2001[55]; Noss, Hoyles and Pozzi, 2002[56]), can be achieved through the abstraction of underlying invariants that are relevant across situations—a process they term situated abstraction.

> *Mathematical conceptualization may be finely tuned to its constructive genesis-how it is learned, how it is discussed and communicated—and to its use in a cultural practice, yet simultaneously can retain mathematical invariants abstracted within that community of practice. (Noss, Hoyles and Pozzi, 2002, p. 205[56])*

While the nature of the assessment content in PIAAC limits its primary focus to cognitive aspect of numeracy, that is, the numeracy behaviours and skills that underpin the questions, their contexts and the specific items, the notion of practices has influenced the development of the skills use questions that form part of the background questionnaire. PIAAC's background questionnaire (BQ) includes collecting a wide range of information which can help to explain differences in performance among adults, further informing our understanding of factors that affect skill acquisition and retention or motivation for further learning. The skills use questions are structured around two themes, *work practices* and *everyday practices*, where questions elicit responses about the frequency of use a different numeracy practices. The Numeracy Expert Group (NEG) worked with the OECD and the PIAAC BQ Expert Group to revise and improve the consistency and research validity and usefulness of the BQ questions on numeracy practices at work and in everyday life for the second cycle of the PIAAC. The NEG's work and recommendations helped to homogenise the set of questions, keep a sense of consistency between work/professional practices and personal uses, while also trying to preserve a continuity between the two cycles. The NEG used the descriptions of the four different content areas of the PIAAC numeracy framework to help guide them in this, alongside reviewing the existing research about numeracy practices at work and in everyday life from PIAAC Cycle 1.

Questions related to work practices and use include: the calculation of prices; counting stock; reviewing inventories; planning delivery routes; preparing budgets; undertaking measurements; interpreting charts or performing data analysis. Everyday practice questions are related to examples such as: calculations related to purchase and discounts; decisions regarding financial matters such as budgets, insurance, loans, or savings; measurements needed when you cook, garden, make clothes. Thus, while PIAAC's capacity to assess numeracy activity from a social practice perspective is necessarily limited, the role of practices in documenting and researching numeracy capability and performance is recognised as vital.

Theoretical developments and foundations

The theoretical conceptualisation of numeracy for PIAAC Cycle 2, discussed and presented below, is built on the previous review of literature and research findings reported in the two previous numeracy frameworks for ALL survey (Gal et al., 2005[28]) and PIAAC Cycle 1 (PIAAC Numeracy Expert Group, 2009[2]). As well it incorporates the review of the numeracy assessment framework used in the first cycle of PIAAC commissioned by the OECD Secretariat (Tout et al., 2017[1]), along with further research done by the Numeracy Expert Group for PIAAC Cycle 2. This is then used as the basis for the elaboration of how the assessment of numeracy in PIAAC Cycle 2 will be implemented and what the key dimensions are that will be assessed. This is described in *the third section: Numeracy assessment construct in PIAAC Cycle 2.*

This conceptualisation operates on two levels. It relates to numeracy as a construct describing a skill or competence, and to numerate behaviour and practices, which is the way a person's numeracy is manifested in the face of situations or contexts which have mathematical elements or carry information of a quantitative nature. In this way, inferences about a person's numeracy are possible through analysis of performance on assessment tasks designed to elicit numerate behaviour. In congruence with the above view of a competence, numeracy will be described as comprised both of cognitive elements (i.e., various knowledge bases and skills) as well as non-cognitive or semi-cognitive elements (i.e., attitudes, beliefs, habits of mind, and other dispositions) which together shape a person's numerate performance, behaviour and practices. This conceptualisation includes ways of knowing, the means of generating new knowledge, and using different modes of reasoning.

The following sections summarise some of the theoretical and conceptual foundations in the previous PIAAC framework document and adds more recent research and understanding about adult numeracy, mainly from the review paper. It starts with the same structure as the previous framework and first addresses the contexts and demands for numeracy, but then adds to this with a new section that adds in further research insights from the PIAAC Cycle 1 framework review and more.

But it should be noted that most recent reviews indicate that there continues to be a shortage of any empirical or theoretical developments in research on adult numeracy [see e.g., (Carpentieri, Litster and Frumkin, 2009[57]; Condelli et al., 2006[24]; Geiger, Goos and Forgasz, 2015[54]; Windisch, 2015[58])]. However, the 2017 PIAAC numeracy review team's research (Tout et al., 2017[1]) included reading and reviewing recent reports about the teaching, learning and descriptions of adult numeracy practices; e.g., (Chisman, 2011[59]; Griffiths and Stone, 2013[60]; National Institute of Adult Continuing Education (NIACE), 2011[61]). The review team found that a number of issues should be considered and addressed in the review and rewriting of the PIAAC numeracy framework for this second cycle of PIAAC. These have been incorporated into the discussions and outcomes below.

Contexts and demands for numeracy

What is encompassed by numeracy (and numerate behaviour and practices) can initially be addressed by identifying the nature of the contexts that contain quantitative and mathematical[1] elements that adults face and which pose demands with which they have to cope. This in turn provides the basis for describing the knowledge elements and supporting processes which enable adults to cope with real-world numeracy tasks (Ginsburg, Manly and Schmitt, 2006[29]), and can later help to form a road map which can guide the design and selection of tasks for inclusion in the numeracy assessment in PIAAC.

The literature pertaining to the uses of numeracy in the real world can be divided into three strands:

- the role of numeracy in adults' lives
- the mathematical demands of workplace settings
- educational perspectives on the mathematical needs of school graduates and citizens.

These three areas are certainly intertwined but also offer complementary ideas; hence, each is reviewed separately below.

Implications of 21st century skills and demands on numeracy needs

Research shows that 21st century skills requirements have changed compared with the previous century, and new ways of working, reasoning and thinking are required, and that increasingly the new skills interact with technology [e.g., see (Binkley et al., 2011[62]; Expert Group on Future Skills Needs (Ireland), 2007[63]; Foundation for Young Australians, 2017[64]; Griffin, McGaw and Care, 2012[65]; Partnership for 21st Century Skills, 2016[66]; Pellegrino and Hilton, 2012[67])]. In the literature, this is often referred to as '21st century skills' or '21st century competences' (Voogt and Roblin, 2012[68]), 'global competences' (OECD, 2019[69]) or 'the 4th industrial revolution' (Schwab, 2016[70]). Common is the acknowledgement that across education, government, and business, the skills and knowledge needed to succeed in work, life and citizenship have significantly changed in the twenty-first century. As has been argued and documented by many sources, and summarised in PIAAC planning documents, adults are presented with ever-increasing amounts of information of a quantitative or mathematical nature through Internet-based or technology-based resources. New means of communication and types of services have changed the way individuals interact with governments, institutions, services and each other, and social and economic transformations have, in turn, changed the demand for skills as well. More so than in prior decades, a wider range of quantitative and mathematical information is more readily available, but this information has to be located, selected or filtered, interpreted, at times questioned and doubted, and analysed for its relevance to the responses needed.

The implications of such 21st century skills and ICT demands on the numeracy needs for adults in their daily lives, as citizen and as workers are discussed in the following sub-sections.

The role of numeracy in adults' lives

Analysis of the purposes served by adults' numeracy skills has often focused on workplace numeracy practices or on the outcomes of school education, which are both discussed in the following two sub-sections. In the 21st century, young people and adults need to be able to cope with the aspects of the world as they encounter them, which includes the digital and technological aspects of information and society—and society already has all kinds of technological aspects that interact with numerical and mathematical information. Therefore, the focus must also be on both life as an individual, and as part of society and citizenship, and that includes with the digital aspects of information and society—the reality is that technology is now ubiquitous with all aspects of many societies. Services, interactions and communications outside the workplace have all changed in the 21st century, often driven by technological advances. This includes online processes such as banking, purchases, bookings, reviewing information (health, housing, etc.) and making decisions based on that information. It includes functioning in the bureaucratic world (applying for permits, social security applications and processes, applying for jobs, managing insurances, etc.), use of different media (e.g., the Internet, online news, Facebook, podcasts, videos, etc.), use of different aspects of communication (e-mail, SMS, apps, social media, etc.), and of a range of software and technology at home and in the community. Technology has meant greater market penetration and influence. The influences of social and mass media has implications for informed and participatory citizenship, and hence for citizens to be critical consumers of all forms of media.

Further, it has been argued that in a society in which the media constantly present information in numerical or graphical form, the ability to interpret and critically reflect on quantitative and statistical messages is vital for all adults [e.g., see (Benn, 1997[20]; Paulos, 1988[71]; Paulos, 1995[72]; Steen, 1990[11]; Utts, 2003[73]; Willis, 1990[39])]. It is seen as essential for all adults to possess the ability to critically reflect on quantitative information encountered in various media sources and documents (Frankenstein, 1989[74]), and to understand how to be a careful or critical consumer of statistical arguments of various kinds (Gal, 2002[75]; Utts, 2003[73]; Watson and Callingham, 2003[76]). This view of needing to be critical as part of being numerate was often espoused by adult education experts, focusing on the role of adults as reflective communicators and critical consumers of information in society who are involved in the exchange and interpretation of messages encountered in media or in political and community contexts (Frankenstein, 1989[74]). For example, Johnston (1994[31]) argued:

> *To be numerate is more than being able to manipulate numbers, or even being able to 'succeed' in school or university mathematics. Numeracy is a critical awareness which builds bridges between mathematics and the real-world, with all its diversity. (p. 34)*

Efforts to formally describe numeracy use in society more generally have been undertaken in several countries [e.g., see (McLean et al., 2012[77]; Quality and Qualifications Ireland (QQI), 2016[78]; Tertiary Education Commission, 2008[79]; U.S. Department of Education, 2013[80])]. In Australia, for example, two frameworks (Kindler et al., 1996[81]; Victorian Curriculum and Assessment Authority (VCAA), 2008[82]) proposed four broad categories regarding the uses of numeracy. The four categories are *Numeracy for practical purposes*; *Numeracy for interpreting society*; *Numeracy for personal organisation*; and *Numeracy for knowledge*. Numeracy for practical purposes addresses aspects of the physical world that involve designing, making, and measuring. Numeracy for interpreting society relates to interpreting and reflecting on numerical and graphical information in public documents and texts. Numeracy for personal organisation focuses on the numeracy requirements for personal organisational matters involving money, time and travel. Numeracy for knowledge describes the mathematical skills needed for further study in mathematics, or other subjects with mathematical underpinnings or assumptions.

Another scheme was developed by Steen (1990[11]), who outlined five dimensions of numeracy:

- practical, focused on mathematical and statistical knowledge and skills that can be put to immediate use to cope with tasks in daily life
- professional, focused on the mathematical skills required in specific jobs
- civic, focused on benefits to society
- recreational, related to the role of mathematical ideas and processes in games, puzzles, sports, lotteries, and other leisure activities
- cultural, concerned with mathematics as a universal part of human culture (and related to appreciation of mathematical aspects such as in cultural or artistic artefacts).

Overall, the purposes regarding numeracy use appear to be consistent and suggest that adults need to be able to apply their numeracy (and literacy) skills to tasks with a social or personal purpose in both informal and more formal contexts. Such perspectives supplement Bishop's (1988[83]) proposal that there are six modes of mathematical actions that are common in all cultures and pertain both to children and to adults: counting, locating, measuring, designing, playing and explaining.

Numeracy in the workplace

Mathematical and statistical skills that are important in adults' work have in part been described in large-scale efforts to define "core skills" or "key competencies" that workers should have, usually in response to the need to maintain economic competitiveness and improve employability of adults and school graduates.

In addition, several projects looked specifically at the mathematical skills of workers in a range of occupational groups or workplace clusters.

Basic computational knowledge has always been considered as part of the fundamental skills that adults need to possess, but recent research and skills framework developments claim that workers need to possess a much broader range of mathematical skills. Examples exist in many countries and the following selective description is indicative of the nature of such efforts. In the United States [see (Carnevale, Gainer and Meltzer, 1990[84]; Secretary's Commission on Achieving Necessary Skills (SCANS), 1991[85])], reviews differentiated between mastery of basic arithmetical skills and a much broader and flexible understanding of mathematical skills. The higher level skills included "choosing appropriately from a variety of mathematical techniques; uses quantitative data to construct logical explanations for real-world situations; expresses mathematical ideas and concepts orally and in writing; and understands the role of chance in the occurrence and prediction of events" (Secretary's Commission on Achieving Necessary Skills (SCANS), 1991, p. 83[85]). Forman and Steen (1999[26]) similarly argued that quantitative skills desired by employers are much broader than mere facility with the mechanics of addition, subtraction, multiplication, and division and familiarity with basic number facts. They also include some knowledge of statistics, probability, mental computation strategies, some grasp of proportional reasoning or modelling relationships, and broad problem solving and communication skills about quantitative issues.

Work in the 21st century

In relation to work in the 21st century, research is showing that there is a significant and increasingly important and underpinning role that science, technology, engineering, and mathematics (STEM) skills play [e.g., (Foundation for Young Australians, 2017[64]; PwC, 2015[86])]. In their recent 2017 review, the National Council of Teachers of Mathematics (NCTM) (2017[87]) argued that mathematics is at the heart of most innovations in the information economy. They saw mathematical and statistical literacy as needed more than ever to filter, understand, and act on the enormous amount of data and information that we encounter every day.

As well, there is a significant amount of research that has looked into specific numeracy and mathematics practices in workplaces, including in relation to 21st century skills [e.g., see (Australian Association of Mathematics Teachers (AAMT) and Australian Industry Group (AiGroup), 2014[88]; Bessot and Ridgway, 2000[89]; Buckingham, 1997[90]; Coben et al., 2010[91]; FitzSimons, 2005[92]; Geiger, Goos and Forgasz, 2015[54]; Hoyles et al., 2002[30]; Hoyles et al., 2010[93]; Kent et al., 2011[94]; Marr and Hagston, 2007[95]); (Straesser, 2015[96]; Wake, 2015[97]; Weeks et al., 2013[98]; Zevenbergen, 2004[99])]. One of the key outcomes of the research is that because of the impact of technology and digital tools and processes, the mathematics or numeracy tasks that people undertake at work involve more than basic calculation skills or 'by hand' skills and straightforward procedural competence, consistent with the above research. These practices involve more sophisticated mathematical problem solving skills and understandings, new ways of reasoning and thinking, and entail the ability to recognise and engage with the mathematics that is fully embedded within complex and "messy" workplace settings. Many 21st century workplace mathematics and numeracy practice are integrated with technology, particularly information and communications technologies (ICT), and have profoundly altered what are considered to be the key knowledge and skills that individuals need as economies and society continue to evolve.

The skills required in the 21st century include a range of mathematical capabilities such as understanding and interpreting graphical information, interpreting measures in terms of what the data are saying about a manufacturing process, making use of spreadsheets, interpreting visual, computer-generated 3D representations or virtual images, and more. Hoyles et al. (2010[93]) argue that this requirement for mathematical capabilities will be driven by the need to improve production processes and productivity; that is, there will be greater demand for what they call techno-mathematical literacies:

> *We therefore decided to introduce the term Techno-mathematical Literacies, developing from the idea of mathematical literacy that was used in our previous research … This literacy involves a language that is not mathematical but 'techno-mathematical', where the mathematics is expressed through technological artefacts. (Hoyles et al., 2010, p. 14[93])*

In relation to technology at work, a 2014 Australian study about the use of mathematics in the workplace found similar connections and entanglements between mathematics and technology:

> *Many people in the workplace are engaged with technology, particularly in using spreadsheets and graphical outputs. There is an inter-dependency of mathematical skills and the use of technology in the workplace in ways that are not commonly reflected in current teaching practice. The perception is that technology is transforming workplace practices and the use of technology has changed the mathematical skills required – while not reducing the need for mathematics. (Australian Association of Mathematics Teachers (AAMT) and Australian Industry Group (AiGroup), 2014, p. 2[88])*

The same report (Australian Association of Mathematics Teachers (AAMT) and Australian Industry Group (AiGroup), 2014[88]) found that workers needed a blend of the following skills:

- ability to recognise and identify how and when mathematics is used in the workplace
- an understanding of mathematical concepts, procedures and skills
- an understanding of the kinds of practical tasks they need to perform
- the strategic processes they should be able to use in using and applying mathematics.

Overall, these studies complement the earlier research and studies, and suggest that employees need to possess a range of specific numeracy-related skills or knowledge, such as in the following (but not the only) areas of mathematics:

- fast and accurate computations but also estimation, and knowing when each skill is required and why
- ability to deal with proportions and percents
- understanding measurement concepts and procedures
- working with, or creating, simple formulas
- a sense for the use of models and modelling in foreseeing future needs
- understanding basic statistical concepts and interpreting data and displays.

However, it is not simply the demands of 21st century workplaces and practices that are driving the use of digital technologies in the workplace; workers themselves also use technology to support their *thinking*. That is, it is not just about the use of digital technologies and tools to replace traditional physical or cognitive skills. In particular, digital tools increasingly mediate young workers' ways of reasoning, acting, and working (Jorgensen Zevenbergen, 2010[100]; Zevenbergen, 2004[99]). At the same time, these new ways of thinking and acting are reshaping the structuring practices and deployment of skills in workplaces. Zevenbergen argues that this allows young workers to solve problems in often more inventive ways than their more experienced co-workers do.

In addition, on a broader and less technical level, these studies argue that workers need to be able to make decisions in the face of uncertainty in real situations, prioritise actions and make choices regarding the approach to handling different tasks, depending on changing external demands. As well, there is a need for workers to be able to communicate with other workers or clients or understand written documentation (e.g., through text or with tables, charts, and graphs) about issues such as quantities, schedules, variation over time, results of quantitative projections, or analysis of different courses of action in this regard. Such findings echo the earlier distinctions made by the SCANS analysis between the need to attend both to basic arithmetical skills and more elaborate and complex mathematical skills in the workplace, including ways of reasoning and thinking, making connections within and between different aspects of mathematics, and also highlight some areas where specific literacy and communication skills are intertwined with numeracy skills.

School mathematics versus everyday or workplace mathematics

Important research literature has also accumulated over the last decades regarding the ways in which people use mathematical skills or cope with mathematical tasks in both formal (i.e., school-based) and informal (i.e., everyday, workplace) contexts (e.g., (Carraher, Carraher and Schliemann, 1985[101]; Nunes, 1992[102]; Nunes, Schliemann and Carraher, 1993[103]; Presmeg, 2007[104]; Resnick, 1987[105]; Rogoff and Lave, 1984[106]; Saxe and Gearhart, 1988[107]). While too complex to discuss in detail here −see Greeno (2003[108]), for one of several reviews of this literature, among other things these studies highlight the situatedness of mathematical knowledge used in functional contexts and the need for actors in different contexts to develop situation-specific mathematical procedures and know-how.

Research suggests that, for adults as well as for children, mathematical knowledge develops both in and out of school [e.g., (Lave, 1988[53]; Saxe, 1992[109]; Saxe et al., 1996[110]; Schliemann and Acioly, 1989[111])]. Saxe and his colleagues have written about the importance of cultural practice in the development of mathematical thinking and how such practices profoundly influence an individual’s cognitive constructions and mathematical ideas, depending, e.g., on the artefacts or tools they use, the nature of the measurement systems in their culture, the counting or calculating devices (abacus, calculator) they use, the distribution of work among family members, or general patterns and types of social activity.

Further, numerous researchers [e.g., (FitzSimons and Coben, 2009[112]; Kent et al., 2007[113]; Marr and Hagston, 2007[95]; Straesser, 2003[114]; Wedege, 2004[115]; Wedege, 2010[116]; Williams and Wake, 2006[117])] have argued, based on ethnographic analyses of workers’ activities in diverse industries, that important portions of the mathematical activities at work are made “invisible” to occasional observers as well as to the workers themselves, or are disguised as non-mathematical. This means that mathematics can be fundamental to activities that are not obviously mathematical. This is most clearly apparent in the use of technology in the workplace where digital tools used to complete tasks often obscure underpinning mathematical activity. As Kent et al. (2007[113]) argues, within techno-mathematical situations in workplaces “there is a shift from fluency in doing explicit pen-and-paper mathematical procedures to a fluency with using and interpreting output from IT systems and software, and the mathematical models deployed within them” (p. 2-3). Building on this point, Wedege (2010[116]) defines two forms of invisible mathematics as (a) subjectively invisible mathematics where people do not recognise the mathematics that they do as mathematics and (b) objectively invisible mathematics in which mathematics is hidden in technology.

Various factors have been posited as causing this phenomenon, such as the encapsulation of many mathematical activities into routines or automated procedures; the use of tools and instruments or information technology (e.g., spreadsheets); the normative use of job-specific linguistic terms that are different than traditional school terms; or the division of labour among different workers.

Based on such and related findings, many reports have argued that mathematical skills as used in the workplace are often different and broader in scope than what is traditionally taught in school mathematics, but also take on different forms depending on the specific work context [e.g., (Australian Association of Mathematics Teachers (AAMT) and Australian Industry Group (AiGroup), 2014[88]; Marr and Hagston, 2007[95])]. The above Australian study (Australian Association of Mathematics Teachers (AAMT) and Australian Industry Group (AiGroup), 2014[88]) about the use of mathematics in the workplace summed up much of this research:

> *Although the skills observed appear to be fundamental, it is their use and application in work contexts that is not straightforward. (p. 1)*

This report went on to describe more fully the differences between school mathematics and workplace mathematics use:

> *Mathematics is applied in both routine and complex tasks requiring sophisticated use of fundamental mathematical skills and 'judgement' or 'problem-solving' procedures. Workplace mathematics is performed differently to school mathematics. Mathematical demands may be present implicitly in the workplace tasks, often through tasks that are not obviously mathematical. (Australian Association of Mathematics Teachers (AAMT) and Australian Industry Group (AiGroup), 2014, p. 2[88])*

This is consistent with earlier research by Steen in the United States:

> *"Mathematics in the workplace makes sophisticated use of elementary mathematics rather than, as in the classroom, elementary use of sophisticated mathematics. Work-related mathematics is rich in data, interspersed with conjecture, dependent on technology, and tied to useful applications. Work contexts often require multi-step solutions to open-ended problems, a high degree of accuracy, and proper regard for required tolerances. None of these features are found in typical classroom exercises." (Steen, 2004, p. 55[118])*

It needs to be emphasised that sense of number still underpins much of the mathematical thinking required—including fluency and flexibility in mental calculations and estimations.

The updated conceptualisation of numeracy for PIAAC Cycle 2 was derived with reference to the types of numeracy and mathematical demands as depicted in this sub-section. However, a working assumption has been made that it is not feasible to employ assessment items that are too workplace-specific (e.g., couched in the context of a single workplace or occupation) because mathematics or statistics as used in this context may not be visible or familiar to most other adults (Hoyles et al., 2002[30]).

School-based perspectives on numeracy and informed civic participation

A growing dialogue about the goals and impact of mathematics education in schools has intensified in recent years. This is in part due to economic pressures and industry expectations on the one hand, but also due to the realisation that mathematical knowledge and skills serve multiple and separate gateway functions on the other hand. Specifically, mathematical competencies affect chances of entry into key occupational tracks (mainly in science, technology, and economics) and may affect employability and labour-force participation, underlie some important aspects of civic participation, and may impact on the possibilities of certain population groups for social equality and mobility. While the dialogue about these issues admittedly overlaps to some extent the points raised earlier in discussing the role of numeracy in society, it is worth elaborating upon because it brings forward some additional points and broadens the understanding of contexts where demands on adults' numeracy skills exist.

Various arguments have been forwarded over the last few decades to support a broadening of the conceptions regarding the mathematical skills and knowledge that school graduates should possess, and the ways in which learned knowledge serves adults. For example, Ernest distinguishes six different types of mathematical knowledge and capabilities for the results/outcomes of mathematics education in school (Ernest, 2004, p. 317[119]). These are not intended to be seen as mutually exclusive, but as a set of different foci for mathematics education:

- utilitarian knowledge
- practical, work-related knowledge
- advanced specialist knowledge
- appreciation of mathematics
- mathematical confidence
- social empowerment through mathematics.

Apart from the third capability, 'advanced specialist knowledge', often a key focus for school mathematics, the other five categories are all compatible with and consistent with the above arguments about how adults might use mathematics in their lives and be numerate individuals, workers and citizens.

Educators working both with school students and adults increasingly aim to assist learners in developing mathematical concepts and skills in ways that are personally meaningful but also functional. Such approaches usually assume that there is often more than one right way to cope with a real-world functional task, and that adults require access to a repertoire of strategies for solving functional problems. Adults' personal methods of using mathematics are encouraged and valued. This is often a significant difference from traditional (pre-reform) school-based mathematics teaching, within which school students were often expected to solve a problem following the one correct method or algorithm, introduced by the teacher.

Several decades ago, ideas already began emerging in different countries that since mathematics is an essential aspect of society, mathematics education in schools should be derived from or prepare learners for broad real-life situations in family, work, community, and other contexts (National Council of Teachers of Mathematics (NCTM), 2000[120]; Willis, 1990[39]), beyond employers' desire to focus mostly on practical or job-specific numeracy skills. Two early influential examples are the recommendations of the Cockcroft Committee in the United Kingdom [Department of Education and Science/Welsh Office, (1982[44])], and Freudenthal's work in the Netherlands, which has led to the Realistic Mathematics Education movement (van den Heuvel-Panhuizen and Gravemeijer, 1991[121]). Over the last two or three decades, many countries have adopted adult education frameworks which give explicit attention to numeracy skills.

Indeed, the dialogue about the various demands on adults' knowledge has been reflected in part in the emphasis in PISA on the assessment of mathematical literacy and science literacy. Such constructs pertain, broadly speaking, to school students' readiness for entering adults' life contexts; it is indicative that they have been chosen to be the focus of assessment rather than more traditional notions of formal curriculum-based knowledge in mathematics or science areas which were assessed primarily in earlier studies.

A perspective on 21st century digital and technological implications

As outlined above, being numerate in the 21st century means being able to cope with the aspects of the world as we encounter it, which includes the digital and technological aspects of information and society—society generally already has all kinds of techno-mathematical aspects. The 2017 PIAAC numeracy framework review found that 21st century digital technologies provide tools and processes that mediate thinking as well as action and are not just devices that can be used to complete manual, hands-on tasks more efficiently. These tools and processes often change the numeracy task itself and so transform practices within adults' lives and within the workplace. The use and application of a range of techno-mathematical literacies underpins much of this.

This aspect of 21st century representations and tools was missing from much of the existing PIAAC Cycle 1 numeracy framework discussions, and not adequately reflected in the definition and then in the elaborations. This is explicitly addressed in the new refinements and enhancements to the numeracy framework and construct elaborated later in this paper.

However, it is important to acknowledge that in addressing this issue PIAAC is a survey of adult competencies across **all** aspects of life, not just about workplace and employment, and not just about engaging with numeracy and mathematics actions within technologically rich environments. It is essential that a balance be kept between numeracy activities in digital and technological environments and those embedded in other, non-digital media; between numeracy demands and situations met as an individual and those encountered as part of society; and between work and employment settings and home and family activities. From the PIAAC numeracy assessment perspective, this can in part be addressed by the need to keep for trend purposes some of the existing former ALL numeracy items, which were originally developed at the end of last century and are not as technologically based, along with a number of the Cycle 1 PIAAC items. The new Cycle 2 items can, hence, contain a set of new items that are more targeted at 21st century digital representations.

In addressing numeracy in adults' lives above there was reference to a set of formal based descriptions of numeracy for both adult and a youth curriculum, where numeracy use was described by four broad categories (Kindler et al., 1996[81]; Victorian Curriculum and Assessment Authority (VCAA), 2008[82]). The four categories are *Numeracy for practical purposes*; *Numeracy for interpreting society*; *Numeracy for personal organisation*; and *Numeracy for knowledge*. These categories were used to reflect on how these different purposes and uses might interact with digital information and technology. Table 3.1 below shows some possible connections between numeracy practices and 21st century digital information and technology.

Table 3.1. Four categories of numeracy use and their connections with technology

Category	Related to	Connections with digital information and technology
Numeracy for practical purposes	Aspects of the physical world that involve designing, making, and measuring	e.g., many aspects of measuring are now digital – theodolites, inclinometers, medical equipment/monitors, etc. e.g., design aspects are now available digitally, via software such as Computer-aided design (CAD) or online design software for kitchen/house planning
Numeracy for interpreting society	Interpreting and reflecting on numerical and graphical information in public documents and texts	e.g., much quantitative information is presented in digital and graphical formats, often dynamic in nature, including the use of spreadsheets for analysis. Even common software such as Word has sophisticated graphic and data options available e.g. use of data, statistics and probabilistic information through social and mass media for advertising, news and political information dissemination, etc.
Numeracy for personal organisation	Numeracy requirements for personal organisational matters involving money, time and travel	e.g., digital diaries, online banking, online shopping and planning, GPS and Google maps
Numeracy for knowledge	Mathematical skills needed for further study in mathematics, or other subjects with mathematical underpinnings or assumptions	The degree of technology inclusion is dependent on the programmes of study—some are technology intensive, others less so. But often it is expected to be able to use and work with sophisticated digital and technological tools, including calculators, software, etc.

The above reflection about numeracy use indicates a strong connection and entanglement of digital information and technology with literacy and numeracy use in adults' lives. The ubiquitous presence of social and mass media also carries implications for informed and participatory citizenship, particularly the need for citizens to be critical consumers of such media. This issue of the connection of numeracy with digital information and technology will be addressed explicitly in the later descriptions, elaborations and dimensions of numeracy in PIAAC Cycle 2.

Further research issues arising from the review paper

The research section in the review paper on the PIAAC framework (Tout et al., 2017[1]) raised a significant number of challenges, and pointed to the need for careful consideration in the revisions to the PIAAC numeracy framework and in the development of any new assessment items. The review considered not only new research but also looked at different descriptions and models for representing and describing numeracy. Some of these are considered below.

The 2017 review paper considered the PISA 2012 mathematical literacy framework and its descriptions (OECD, 2013[122]). It should be noted that the same mathematical literacy framework and assessment construct was used for the next two cycles of PISA in 2015 and 2018.

However, for PISA 2021, mathematical literacy is again the major domain for PISA, and hence the PISA framework and assessment construct is being updated and revised. This revision was happening in parallel with the development of this numeracy framework for PIAAC Cycle 2. The PIAAC Numeracy Expert Group was able to access a copy of the second draft of the PISA 2021 Mathematics Framework (OECD, 2018[123]) in November 2018. Because of the timing issues, most of the comparisons between PIAAC numeracy and PISA mathematical literacy have therefore been based on a comparison of the 2012 PISA framework and descriptions, but where possible the PIAAC NEG has also included comments and comparisons with the updated 2021 PISA mathematical literacy framework. It should be noted that it is therefore possible that information regarding 2021 PISA mathematical literacy may change from what was in the second draft of the framework paper.

The definitions of mathematical literacy in the 2012 and 2021 PISA frameworks are very similar and consistent, with some changes and updates to reflect some new perspectives. The two definitions are shown below.

Box 3.1. Definitions of mathematical literacy in PISA

PISA 2012-2018 definition of mathematical literacy

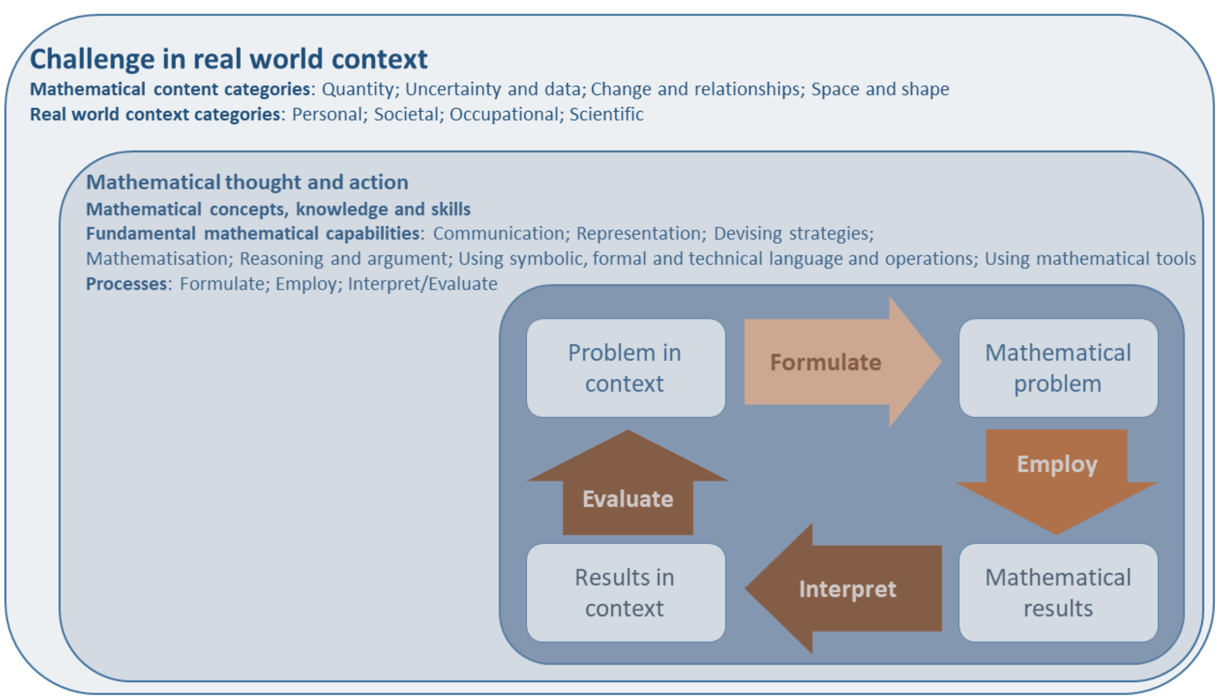
Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD, 2013, p. 25[122])

PISA 2021 definition of mathematical literacy

Mathematical literacy is an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens. (OECD, 2018, p. 8[123])

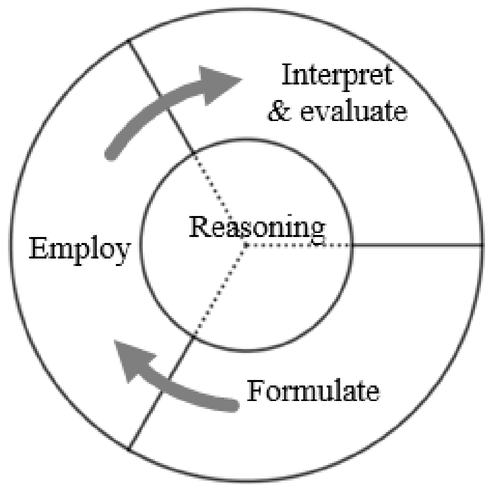
The PISA 2012 (OECD, 2013[122]) definition and description of mathematical literacy was based around a model that assumed that when individuals use mathematics and mathematical tools to solve problems set in a real-world situation, they work their way through a series of stages as depicted in Figure 3.2 (OECD, 2013, p. 26[122]).

Figure 3.2. The PISA 2012 model of mathematical literacy in practice



However, the PISA 2021 definition and description of mathematical literacy has extended the previous PISA 2012 model, and is based around a model that comprises two related aspects: mathematical reasoning and problem solving. When individuals use mathematics and mathematical tools to solve problems set in a real-world situation, they work their way through a series of stages as depicted in Figure 3.3 (OECD, 2018, p. 9[123]).

Figure 3.3. The PISA 2021 model of mathematical literacy: the relationship between mathematical reasoning and the problem solving (modelling) cycle



The *formulating*, *employing*, *interpreting and evaluating* processes are still key components of the mathematical modelling cycle that has underpinned the mathematical literacy construct in PISA since its beginnings. The mathematical reasoning process has been added as an explicit component in 2021 to highlight the centrality of mathematical reasoning to solving practical problems. The PISA mathematical reasoning aspect names these key understandings:

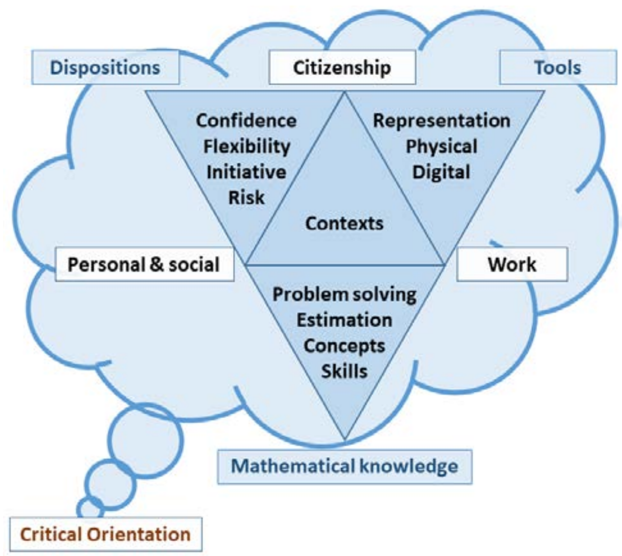
- understanding quantity, number systems and their algebraic properties
- understanding mathematics as a system based on abstraction and using symbolic representation
- seeing mathematical structure and regularities
- recognising functional relationships between quantities
- using mathematical models as a lens into the real world
- understanding variance as the heart of statistics (OECD, 2018, p. 16[123]).

As described in the draft PISA 2021 framework, these reasoning skills appear to mainly focus on reasoning skills *within* the world of mathematics, and *mathematical reasoning* is seen as a separate skill or process to the three problem solving processes of *formulating*, *employing*, *interpreting and evaluating*. As discussed further in the fifth section, this illustrates PISA's interest in the ability of 15-year-olds to use and apply curriculum-based mathematical skills and knowledge, whereas this type of more formal mathematical knowledge is not generally assessed in PIAAC.

The PIAAC description and definition of numeracy can learn from the PISA definitions, descriptions and models in relation to the need to highlight different problem solving skills and processes, including reasoning skills, and being critical (making well-founded judgements and decisions) framed around using mathematical models as a lens into the real world. The relationships between the PIAAC and PISA frameworks and their descriptions and constructs are discussed further in the fifth section: *Relationship between PIAAC and PISA*.

Another existing model for numeracy in the twenty-first century is illustrated in Figure 3.4 below, and attempts to capture the multifaceted nature, and especially the critical dimension, of using mathematics to act in the real world. This model incorporates four dimensions of settings/contexts, mathematical knowledge, tools, and dispositions that are embedded in a critical orientation to using mathematics (Goos, Geiger and Dole, 2014, p. 84[124]). These dimensions are described more fully in other publications [e.g., (Geiger, Goos and Dole, 2014[125]; Goos, Geiger and Dole, 2014[124])]. Although primarily developed for use in relation to teacher education programmes and numeracy across the curriculum, the model and its components has some consistency with both the PISA model and PIAAC's framework.

Figure 3.4. Model for numeracy in the twenty-first century



Both the PISA models and their sets of processes and this model for numeracy in the twenty-first century raise some issues to be considered in the redevelopment of the Cycle 1 definition and elaboration of numeracy for PIAAC Cycle 2.

Out of its research and review of conceptual and theoretical aspects of adult numeracy, the 2017 review recommended that there were four related issues to be explicitly addressed in updating and refining the existing PIAAC framework definition and description:

- disposition to use mathematics
- the ability to see mathematics in a numeracy situation
- critical reflection
- degree of accuracy.

Another issue that was raised in the review paper that is relevant here concerned the issue of authenticity, and this is also discussed below.

Disposition to use mathematics

The issue of a person's judgement on how to use mathematics (or not) in solving a numeracy problem is an important issue to address. The issue of choice or disposition when engaging with and solving a numeracy problem is an important factor to consider in an adult's use and application of mathematics in a real-world situation (Geiger, Goos and Dole, 2014[125]; Goos, Geiger and Dole, 2014[124]). Are individuals able to choose to use mathematics when it is relevant and appropriate? This can also relate to mathematics anxiety and individual's negative disposition to mathematics and their decision to avoid using mathematics, even when appropriate. Research about mathematics anxiety is well documented and demonstrates that it can have a significant impact on performance in mathematics [e.g., see: (Buckley, 2013[126]; Ma, 1999[127]; Tobias, 1993[128])].

There are three potentially related aspects behind this issue of disposition in relation to solving a numeracy problem where an adult is expected to use and apply some form of mathematical knowledge in a real-world situation:

- using other means than mathematics to solve a problem when mathematics should have been the obvious and most sensible approach;
- using formal mathematics when other sense-making methods would be more efficient; or
- avoiding doing anything at all and not attempting to solve the numeracy problem at hand.

This issue of disposition is addressed more explicitly in the discussions about the elaboration of numeracy and numerate behaviours and practices at the end of this section.

Seeing mathematics in a numeracy situation

Research indicates that an important aspect of a person's numeracy or numerate behaviour is their capability to "see" or notice when mathematics is embedded in a real-world situation—how to recognise the mathematics and to potentially take the next step and act on it. The ability to see the mathematics that surrounds adults in their everyday life is an important issue in relation to being numerate—to potentially link the mathematics they learned in school with mathematics embedded in a real-world situation (Maguire and Smith, 2016[129]; Roth, 2012[130]).This issue is also identified as important in research about workplace numeracy, for example in calculating medication dosage (Weeks et al., 2013[98]).

Seeing mathematics in a numeracy situation relates to aspects of two of the processes described as part of PISA's 2021 problem solving cycle for mathematical literacy: *Mathematical reasoning* and *Formulating.* In relation to *Mathematical reasoning,* before solving a problem, students need to "use their mathematics content knowledge to recognise the mathematical nature of a situation" (OECD, 2018, p. 9[123]). PISA also describes *Formulating* as: "seeing that mathematics can be applied to understand or resolve a particular problem or challenge presented" (p. 12).

As will be argued later, this aspect of being able to see and access the mathematics embedded in a numeracy situation and transposing the problem into a mathematical problem that can be solved is addressed explicitly in the revised numeracy framework and assessment construct through the new cognitive dimension.

Critical reflection and action

While the current framework mentions the notion of critical reflection under the facet *Responses* in its elaboration of numerate behaviour, having a critical orientation or reflection are aspects of numeracy that could be emphasised and described further. It is important for individuals in their lives as citizens and workers to critically review the mathematics used and the outcomes obtained to reflect on and question real-world implications, to be capable of following up with appropriate actions, and to make decisions and judgements. A critical orientation is also about supporting an argument or position with mathematical evidence or challenging the argument or position of another person or organisation.

This capability to reflect critically and to act is named and described explicitly in some other models and frameworks [e.g., (Geiger, Goos and Forgasz, 2015[54]; Goos, Geiger and Dole, 2014[124])]. The third problem solving process in the PISA mathematical literacy problem solving cycle, which is called *Interpreting and evaluating*, includes elements of critical reflection: the need to reflect and make contextual arguments, to evaluate the reasonableness of solutions, and to critique and identify the limits of any models used. As well the new *Mathematical reasoning* aspect of PISA 2021 includes evaluating and making arguments, to evaluate interpretations and inferences related to statements and problem solutions (OECD, 2013[122]; OECD, 2018[123]).

Degree of accuracy and tolerances

The Cycle 1 PIAAC numeracy framework did not explicitly address the issue of the degree of precision or accuracy that may be required in the solution of a numeracy problem. It is expected that a numerate person would use estimation and other skills to check the outcomes and decide on the appropriate degree of accuracy required when solving a problem. This is particularly true within a workplace environment, where precision, accuracy and working within specified tolerances can be critical. On the other hand in other situations and applications, there are instances in being numerate where accuracy is not a critical component (e.g., in relation to some spatial skills, in graphical/data interpretation and analysis, or in estimating quantities, where an order of magnitude estimate can often suffice).

Authenticity, embeddedness and text-related reading demands

Another issue raised in the review paper concerns the issue of authenticity, and as mentioned in the Introduction, the NEG did further research into the related issues of embeddedness, numerate behaviour and numerate practices. They relate to the connection between the real-world context in which mathematics is embedded and to their roles as individuals, citizens, family members or as workers. This can mean that the way mathematics is used to operate on a task is fundamentally shaped by the context in which it is employed (Turner et al., 2009[131]). This includes socio-cultural influences that afford or constrain action in civic, personal or workplace environments. In this view, there is a clear separation between school mathematical knowledge, how it is taught, learned and practised, and the use of this knowledge outside of schooling. As Harris (1991[132]) notes:

> *In work [. . .] mathematical activity arises from within practical tasks, often from the spoken instruction of a supervisor and always for an obvious purpose which has nothing to do with the numbers working out well. Thus, students taught to react to isolated, abstract and written commands in the specialist language and carefully controlled figures of a school mathematics class, find themselves confronted with the urgent spoken, if not shouted, instructions in a completely different context and code. (p. 138)*

Yasukawa, Brown and Black (2013[133]) make a clear connection between embeddedness and social practice arguing that numeracy practices cannot be understood independently of the social, cultural, historical and political contexts. They illustrate this point that make the comparison of students completing calculations individually, using paper and pen and perhaps a calculator against the use of mathematics in the supermarket, in which the same calculations completed at a checkout counter by the shop assistant using a cash register. In this situation the shopper might perform an estimation to avoid being overcharged. However, the shop assistant is equally concerned with charging the customer the correct price and recording accurate record of the items sold via the cash register. The calculations are fundamentally the same but the purpose—which is related to context—is different.

Authenticity of assessment tasks and word problems in mathematics education has been researched and documented [e.g., see (Hoogland et al., 2018[134]; Palm, 2006[135]; Palm, 2008[136]; Palm, 2009[137]; Stacey, 2015[138]; Verschaffel et al., 2009[139])]. In PIAAC it is important to have stimuli and questions that are based on authentic stimuli or scenarios with questions asked being ones that someone would want answered. While this is related to broader discussions about authentic assessment (Palm, 2008[140]) the focus in the PIAAC assessment programme is on the authenticity of the stimulus used and the questions asked. This matches what Palm describes as the “figurative context” where the context used in the assessment represents a situation taken from real life that has occurred, or might happen. PIAAC is interested in the ability of individuals to cope with tasks that are embedded in the real world, rather than assessing decontextualised mathematical tasks. This is in contrast to traditional school-based mathematical word problems which often disregard and challenge students’ sense making and only continue to distance students from the real world, and the usefulness and value of mathematics. The NEG believe that the assessment of numeracy in PIAAC is about promoting the belief that the value in mathematics is about its relationship with real-world things—whereas word problems often do the opposite. Another reason for PIAAC to utilise authentic situations in its questions is to encourage a more positive disposition towards solving relevant and engaging mathematics problems, not irrelevant, nonsensical word problems as can be met in school mathematics classrooms.

Hence, the PIAAC numeracy contexts and the items are developed by finding and identifying situations and tasks from different countries that provide authentic stimuli and then writing sets of questions using the information in the stimulus. Based on the description and understanding of numeracy in PIAAC, there have been deliberate attempts to avoid what are traditionally seen as school, curriculum-based word problems that are often contrived and have little real-world relevance or authenticity.

However, a challenge is that authentic situations and scenarios that involve mathematical concepts, and their related stimuli and materials, are often complex. In relation to textual components and reading demands, there are a range of issues in relation to the intersection between literacy and numeracy skills and the role that reading literacy aspects take in solving a numeracy problem. It is clearly acknowledged in the description of numeracy and its elaborations, and then reflected in the PIAAC complexity schema (PIAAC Numeracy Expert Group, 2009[2]; Tout et al., 2020[141]), that reading literacy is an integral and important aspect of numeracy. Certainly, in society and workplaces that adults occupy, tasks and challenges do not neatly divide into, or present as, discreet ‘literacy’ and ‘numeracy’ tasks. Real-world situations and demands cross those kinds of educationally defined boundaries.

The reality is that in using authentic situations as the basis for the numeracy assessment tasks where the mathematics is embedded in a real-world setting, the associated information and data can be very complicated, unfamiliar and involve a heavy reading load. This can create challenges in trying to focus the assessment on the mathematics and numeracy skills and knowledge. Hence a key goal in the item development process is to make the wording of numeracy items as simple and direct as possible, in order to help minimise the reading literacy demands.

Recent research that systematically compares descriptive mathematical assessment tasks with more depictive representations of problem situations through using illustrations and photos and minimising the use of words (Hoogland et al., 2016[142]; 2018[134]), gives an indication that even the use of simple supporting illustrations and images could improve performance by a small margin.

Big Ideas in mathematics

As introduced in the first section, *Big Ideas in mathematics,* is about describing powerful mathematical ideas central to the learning of mathematics, linking numerous mathematical understandings into a coherent whole [e.g., see (Charles, 2005[6]; Hurst, 2014[7]; Hurst and Hurrell, 2014[8]; Kuntze et al., 2009[9]; Steen, 1990[11])]. Initially, the term “Big Ideas” referred to how mathematical information can be classified in different ways compared with the traditional school mathematics curriculum content areas. Steen (1990[11]), for example, identified six broad categories pertaining to: quantity, dimension, pattern, shape, uncertainty, and change. Rutherford and Ahlgren (1990[143]) described networks of related ideas: numbers, shapes, uncertainty, summarising data, sampling, and Reasoning. Jones and his colleagues (2002[5]) provide a summary of the main contributions of the research to what they call *powerful mathematical ideas*, which included the following domains: whole number and operations, rational numbers, geometry, probability, data exploration and algebraic thinking and other underrepresented domains. It could be argued that being numerate means using the contents of all these domains not just as procedures (instrumental understanding) but in a critical/meaningful manner.

Charles (2005[6]) defines Big Ideas as “a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole.” This view is also shared by other authors such as Hurst and Hurrell (2014[8]). In their article, they track the notion of Big Ideas in mathematics back to the work of Bruner (1960[144]), who inspired Clark's (2011[145]) definition of a Big Idea as a "cognitive file folder" that we can file with "an almost limitless amount of information" (Clark, 2011, p. 32[145]). Big Ideas became conceptual structures (or schema) that can be used to provide a numeracy framework where content might be characterised by multiple connections. As Bruner (1960[144]), Hurst and Hurrell (2014[8]), Clark (2011[145]) and other authors claim, Big Ideas may become bridges for the transfer of learning.

Big Ideas in mathematics can also refer to processes (Kuntze et al., 2009[9]) where they include processes such as: Ordering; Classifying; Dealing with variation and uncertainty; Finding arguments and proofs; Formalising; Modelling; Generating and using algorithms, among others. Big Ideas have also been seen as a potential vehicle for making mathematics education a coherent and connected study. Descriptions of effective teaching of mathematics [e.g., (Ma, 1999[146]; Sullivan, 2011[147])] and related research (Askew et al., 1997[148]; Boaler and Humphreys, 2005[149]; Clarke and Clarke, 2004[150]) also consistently refer to the need for teachers to have a sense of how mathematics is a coherent and connected whole. In the *Effective Teachers of Numeracy* study (Askew et al., 1997[148]), this view was also supported, with highly effective teachers believing that being numerate requires "having a rich network of connections between different mathematical ideas." This is in contrast to ways in which mathematical content knowledge is often reduced to lists of specific dot points in curriculum frameworks that Askew terms "death by a thousand bullet points" saying that "too much effort goes into specifying the knowledge that teachers need to know" (Askew, 2008, p. 21[151]). Hurst and Hurrell (2014[8]), quoting Charles (2005[6]), state that "Big Ideas" allow us to see mathematics as a coherent set of ideas, encouraging a deep understanding of mathematics.

These perspectives of Big Ideas in mathematics provide an overarching and integrative idea of mathematics and how mathematics is used in the world—that is, they are about framing and viewing mathematics as making connections with the real world, which is what underpins numeracy in PIAAC. It could be suggested that being *numerate* as defined within the PIAAC numeracy framework links to the idea of being able to access, use, interpret and communicate mathematical information around what the international scientific community calls Big Ideas in mathematics.

Towards a definition and description of numeracy for PIAAC Cycle 2

Reaching a consensus on a definition of numeracy that can fit an international programme of assessment is a challenging undertaking. First, as noted earlier, there are various country-specific connotations for numeracy, if such a term at all exists in a local language. Second, there are overlapping or competing constructs such as quantitative literacy, mathematical literacy, functional mathematics, and so forth [e.g., see (Gal and Tout, 2014[3]; Hagedorn et al., 2003[152]; Tout and Gal, 2015[37]; Tout and Schmitt, 2002[38])]. Third, an attempt to discuss the definition and meaning of numeracy is complicated by the fact different stakeholders already view it from within a given lens imposed by the historical and cultural aspects, whether organisational, social, economic, or linguistic, of the systems within which they operate. For example, some of the existing conceptions of numeracy were developed by educators working in delivery systems for schoolchildren, while other stakeholders link the term numeracy only to adult-related competencies.

Full range of numeracy capabilities

As stated in the Introduction it is critical to note that the PIAAC numeracy assessment aims to describe the full range of numeracy capability in the adult population. This covers at one extreme, adults who have university level training and, at the other, adults who have very low levels of education (e.g. who left school at or before the age of 15). At the same time, it covers both young adults still in education and adults who completed their formal education 30-50 years prior to undertaking the assessment. As such, it incorporates a wide range of different mathematical and quantitative skills and knowledge, and is not based on a narrow view of numeracy that sees numeracy as only dealing with numbers and arithmetical operations. This will be expanded on later in the chapter.

Numeracy assessment construct in PIAAC Cycle 2

In this section, the various aspects that are to be assessed, and eventually reported on, as part of the numeracy assessment in PIAAC are defined, described and elaborated. These aspects or characteristics of the assessment were called facets in ALL and PIAAC Cycle 1, but in PIAAC Cycle 2 they are called dimensions, which is consistent with the terminology used in literacy in PIAAC.

The initial sub-sections look at the refinement of the definition and description of numeracy from ALL through to PIAAC Cycle 1 and then to the new definition and description for this, the second cycle of PIAAC. For a backwards look at the development of numeracy definitions and developments it is best to read the PIAAC Cycle 1 framework (PIAAC Numeracy Expert Group, 2009[2]) or refer to the OECD Working Paper that compared the PISA and PIAAC frameworks (Gal and Tout, 2014[3]).

Next, an updated and refined definition of numeracy for PIAAC Cycle 2 is presented based on the research and review detailed in the second section, followed by a discussion of the dimensions of numerate behaviour and practices, including the core dimensions that comprise the numeracy assessment. This leads on to the next sub-sections*,* where the assessment construct is elaborated, described and defined in full.

The 2017 review report made a number of recommendations regarding the definition of the construct of numeracy and the priorities for development of the assessment framework for numeracy in the second cycle of PIAAC. Many of the suggestions arose out of the concern that the existing Cycle 1 framework and assessment did not reflect some of the realities of the skills and knowledge adults needed to succeed in work, life, and citizenship in the 21st century. These have been documented in the discussions in the previous section. The review and this document and its resulting definitions and elaborations, while building on the two previous conceptual and assessment frameworks and all the cumulative wisdom developed in connection with prior surveys of adult skills, have been able to enhance the numeracy framework and construct for PIAAC Cycle 2. The resulting framework and its associated definition, elaborations and assessment construct is contained below.

Numeracy in the ALL survey

The conceptualisation of numeracy for the first international survey of adult numeracy, the Adult Literacy and Lifeskills (ALL) survey, was developed in 1998-2000 by an international team (Gal et al., 2005[28]). This was the first time the construct of numeracy had to be defined in a comparative assessment context and not purely in an educational context.

Numeracy was conceptualised and described in ALL as a much broader construct than Quantitative Literacy that was assessed in the earlier International Adult Literacy Survey (IALS). Quantitative Literacy was described in IALS as the knowledge and skills required to apply arithmetic operations to numbers embedded in printed materials. It was argued in ALL that numeracy requires more varied responses (order, count, estimate, compute, measure, interpret, explain) to a wider range of mathematical information (quantity, dimension and shape, pattern, change and relationships, and data and chance) that may be embedded in text in varying degrees.

Cognisant of the complexity and multifaceted nature of the numeracy construct, the ALL team developed a three-tier conceptualisation which attempted to reflect key perspectives of numeracy on the one hand, but also enable operationalisation of the construct in an assessment scale on the other. The three tiers are a brief definition of numeracy, a more elaborate definition of numerate behaviour, both presented below, and a detailed listing of components of the facets of numerate behaviour (Gal et al., 2005[28]).

> *Numeracy is the knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations.*

> *Numerate behaviour is observed when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, factors, and processes.*

Both the brief and elaborate definitions shown above were seen by the ALL numeracy team to be required, given the needs of a comparative assessment. A brief definition is essential to simplify communication with various stakeholders, such as policy makers and experts. However, as with most brief definitions of complex constructs, the language used is general and abstract, hence the definition cannot be explicit about what a numerate person can do in an assessment. With this in mind, a more detailed definition of numerate behaviour was developed as a way to emphasise different facets or dimensions that were seen by the ALL numeracy team as underlying numerate behaviour.

The advantage of using a more elaborate definition of numerate behaviour was that it is more explicit about what numeracy encompasses, and thus served as a springboard for developing an actual specification for an assessment. It is important to also note that the definition of numerate behaviour points to the presence of both cognitive and non-cognitive factors that underlie or enable effective numerate behaviour and practices. Ideally, coverage of both cognitive and non-cognitive aspects of numerate behaviour is essential in order to generate a full picture regarding the competence described by this view of numeracy. However, it needs to be acknowledged that the direct assessment component of PIAAC can only assess the cognitive aspects of numerate behaviour and that the non-cognitive aspects can only be addressed in proxy via responses to questions about skills use and data collected on respondents' backgrounds.

Numeracy in PIAAC Cycle 1

The development of the conceptualisation and definition of numeracy for PIAAC went through several stages of work and consultation. An expert panel appointed to develop the overall assessment design for PIAAC presented in summer 2006 tentative recommendations regarding all competencies to be assessed in PIAAC (Gal, 2006[153]; Jones, 2006[154]; Murray, 2006[155]; Tout, 2006[156]) and then proposed to define numeracy as: *"The ability to use, apply, and communicate mathematical information"*. Various perspectives on numeracy and its assessment were later examined by participants at the Canada-OECD Expert Technical Workshop on Numeracy, which met in November 2006 in Ottawa; a tentative working definition of numeracy was then proposed for PIAAC and included in a draft framework circulated for external review (Gal, 2006[153]). Further development of the numeracy framework was then undertaken by the Numeracy Expert Group for PIAAC appointed in April 2008, which released a revised framework for review by all participating countries in October 2008.

In general, work on the development of the numeracy framework for PIAAC Cycle 1, together with the assessment scale and related item pool, was conducted with two somewhat conflicting objectives in mind. One was the need to maintain compatibility with the conceptualisation of numeracy in the ALL survey, given the need for PIAAC to provide trend data related to ALL results. For this reason, PIAAC was designed with a specification that approximately 60% of the numeracy tasks that were to be employed in the final assessment would come from the item pool used in ALL. The other objective was the need to extend the ALL definition in light of PIAAC's overarching conceptualisation of "literacy competencies in the information age", and consider new or emerging uses of numeracy in the adult world.

Taking all the above into consideration, numeracy was defined for PIAAC Cycle 1 as follows:

> ***Numeracy*** *is the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life.*

This definition captured essential elements in numerous conceptualisations of numeracy in the extant literature; was compatible with the definition used for ALL and provided a solid basis from which to develop an assessment for PIAAC with its emphasis on competencies in the information age. The inclusion of "engage" in the definition signalled that not only cognitive skills but also dispositional elements, i.e., beliefs and attitudes, are necessary for effective and active coping with numeracy situations.

As with ALL, the definition of numeracy developed for PIAAC Cycle 1 was not to be considered by itself, but again was to be coupled with a more detailed definition of numerate behaviour and with further specification of what were called the facets of numerate behaviour. This pairing was seen as essential in order to not only describe numeracy but to also enable operationalisation of the construct of numeracy in an actual assessment, and in order to further broaden the understanding of key terms appearing in the definition itself. Consequently, a definition of numerate behaviour similar in general terms to the one used for the ALL survey, but shorter, was adopted for PIAAC Cycle 1:

> ***Numerate Behaviour*** *involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways.*

As with ALL, each of the different facets embedded within the definition of numeracy and the elaboration of numerate behaviour were defined and described. This included the same facets as ALL: *contexts*; *responses*; *mathematical ideas/content*; and *representations*.

Definition of numeracy for PIAAC Cycle 2

Based on the discussions in the previous section: *Conceptual and theoretical foundations*, the PIAAC Cycle 2 NEG developed and agreed on a new definition for Cycle 2 of PIAAC numeracy:

> ***Numeracy*** *is* ***accessing, using and reasoning critically*** *with* ***mathematical content****, information and ideas* ***represented in multiple ways*** *in order to engage in and manage the mathematical demands of a* ***range of situations*** *in adult life.*

In this updated definition and in the elaboration below, there are four core *dimensions* (previously called facets) described and used in PIAAC Cycle 2:

- cognitive processes
- content
- representations
- contexts.

The major changes

The key words or terms that have changed or been introduced into the new definition include:

- the use of the term ability has been deleted
- access, use and reason critically has replaced access, use, interpret, and communicate
- *represented in multiple ways* has been introduced into the definition.

The use of the term ability can imply that it is an “innate” ability that some people may not possess. The NEG firmly believe that adults (and children) all have the capacity to learn mathematics successfully and apply it successfully in their lives and hence be numerate.

Based on the views and research outlined in the previous section and raised in the review report, the NEG has substantially reworked the former *Response* facet (*access, use, interpret, and communicate*), and replaced with a more comprehensive description and elaboration of what is now called the core dimension of *Cognitive Processes*. This is named and described under three classifications:

- access and assess situations mathematically (assess, identify, access and represent)
- act on and use mathematics (order, count, estimate, compute, measure, graph and draw)

- evaluate, critically reflect, make judgements (evaluate, reflect, justify and explain).

This has been done in the light of a number of the outcomes of the above research, both by the new NEG, but also from the recommendations made in the 2017 review paper. The major influences have been about the need to incorporate the aspect of being able to “see” mathematics in a real-world situation, and to include a critical aspect and the ability to reason and make judgements. This dimension has been able to be more explicitly used to drive the assessment development and the NEG believe this has greatly enhanced the item pool, helping to address a number of the concerns expressed in the review paper. It will also help develop and write the scale descriptions once the data and results are available. This dimension, along with the other core dimensions are elaborated further in the following section.

The third facet of the previous elaboration in ALL and PIAAC that has been added in and made explicit by including it in the new definition is that of *representation*. Although this was included in the previous elaborations it was not part of the definition, and did not help drive item development. Again, to address issues raised above, and specifically in relation to 21st century changes in how mathematical and quantitative information is now presented, the inclusion of *represented in multiple ways* has been introduced. This is named and described under four classifications:

- text or symbols
- images of physical objects
- structured information
- dynamic applications.

The latter two classifications have enabled the NEG, and hence the item writers, to explicitly address the issue of 21st century digitisation and technologically based materials and representations such as interactive websites, infographics, online calculators, spreadsheets and more.

The other facets in ALL and PIAAC Cycle 1, *content* and *contexts* remain, although there have been changes made to their labels and descriptions. All these core dimensions are named and elaborated below and in the following section.

Elaboration of numerate behaviour and practices

As with previous cycles, the definition leads on to an elaboration of numerate behaviour, which is included in Box 3.2 below. This box first lists the direct assessment components of the four core dimensions of the definition, and these components are explained in more detail in the next sub-section. Additionally, the bottom part of Box 3.2 also lists several non-cognitive, enabling factors and processes, whose activation underlies numerate behaviour and successful numeracy practices. Most of these enabling factors and processes appeared in the ALL conceptual framework and PIAAC Cycle 1. Overall, the definition of numeracy and the description of numerate behaviour, with the details in Box 3.2 and the further explanations of the core dimensions following in the next section, provide the structure and roadmap for the development of the numeracy assessment as part of PIAAC Cycle 2.

Box 3.2. Numerate behaviour and practices – key facets and their components

Numeracy is an individual’s capacity to …

1. ***access, use and reason critically***

- access and assess situations mathematically (assess, identify, access and represent)
- act on and use mathematics (order, count, estimate, compute, measure, graph and draw)
- evaluate, critically reflect, make judgements (evaluate, reflect, justify and explain)

2. with ***mathematical content***

- quantity and number
- space and shape
- relationship and change
- data and chance

3. ***represented in multiple ways***

- text or symbols
- images of physical objects
- structured information
- dynamic applications

4. in order to engage in and manage the mathematical demands of a ***range of situations*** in adult life:

- personal
- work
- societal/community.

An individual's numerate capacity is founded on the activation of several enabling factors and processes:

- context/world knowledge and familiarity
- literacy skills
- disposition, beliefs and attitudes
- numeracy-related practices and experience.

Enabling factors and processes

Adults' numeracy competence is revealed through their responses to the mathematical information or ideas that may be represented in a situation or that can be applied to the situation at hand. It is clear that numerate behaviour will involve an attempt to engage with a task and not delegate it to others or deal with it by intentionally ignoring its mathematical content. Numerate behaviour, however, depends not only on cognitive skills or knowledge bases, but also on several enabling factors and processes as listed in Box 3.2.

As outlined in the second section, including in the discussion about *Numerate behaviour and practices*, the PIAAC conceptualisation of numeracy operates on two levels. It relates to numeracy as a construct describing an individual's capability to solve numeracy problems, and also to numerate behaviour and practice which is the way a person's numeracy is manifested in the face of situations or contexts which have mathematical elements or carry information of a quantitative nature.

We argue therefore that numeracy as described in PIAAC is comprised both of cognitive elements (i.e., various knowledge bases and skills) as well as non-cognitive or semi-cognitive elements (i.e., attitudes, beliefs, habits of mind, and other dispositions) which together help to shape a person's numerate behaviour and practices. Based on this, there are four non-cognitive or semi-cognitive enabling factors included in the elaboration of numerate behaviour:

- context/world knowledge and familiarity
- literacy skills
- disposition, beliefs and attitudes
- numeracy-related practices and experience.

Specifically, the enabling processes involve integration of mathematical knowledge and conceptual understanding with broader reasoning, problem-solving skills, and literacy skills. Further, numerate behaviour and numerate practices and autonomous engagement with numeracy tasks depend on the dispositions (beliefs, attitudes, habits of minds, etc.), and prior experiences and practices that an adult brings to each situation. These are briefly summarised below. Most of these enabling factors and processes have also been described by Kilpatrick, Swafford and Findell (2001[157]) as part of their analysis of the construct of mathematical literacy, and further examined and deemed relevance for description of adult numeracy in the analysis by Ginsburg, Manly and Schmitt (2006[29]).

It should be noted that the direct assessment via the numeracy test component in PIAAC has, as its primary emphasis, the cognitive aspects of numerate behaviour as framed in the first part of Box 3.2, namely the numeracy and mathematical knowledge and skills that underpin answering the test questions, which are mediated by written materials, without oral support, in the context of a formal assessment. The non-cognitive aspects of numerate behaviour, are addressed indirectly through other components of the PIAAC assessment, namely through the skills use questions and the comprehensive background questionnaire.

Context/world knowledge and familiarity

Proper interpretation of mathematical information or quantitative messages by adults depends on their ability to place messages in a context and access their world knowledge, as well as rely on their personal experiences and practices. World knowledge also supports general literacy processes and is critical to enable “sense-making” of any message. For example, adults’ ability to make sense of statistical claims or media-based graphs will depend on information they can glean from the message about the background of the study or data being discussed. When interpreting statistical claims made by journalists, advertisers and the like, context knowledge is the main determinant of the reader’s familiarity with sources for variation and error, and helps to imagine why a difference between groups can occur (as in a medical or educational experiment), or what alternative interpretations may exist for reported findings about an association or correlation between certain variables. Likewise, world knowledge is a prerequisite for enabling critical reflection about statistical messages and for understanding the implications of the reported findings.

Different people will have very different settings and applications in which they may comfortably and more confidently use and apply their mathematical knowledge, often related to their familiarity with the actual context and the mathematics that is embedded in the situation at hand. This is related to the discussion in the previous section about numeracy practices, authenticity and the embedded nature of mathematics within a numeracy context. Finding the right problem situation or setting for each individual so that they can demonstrate their understanding of mathematics concepts will be a challenge in such an assessment as PIAAC. Hence, it is important to not have contexts, especially workplace contexts, which are too technical or so uncommon that most adults faced with that stimulus and questions will not be at all familiar with the context, and would be potentially locked out from engaging with and answering the question. This is one of the key challenges in writing items for PIAAC – to make them all relatively accessible and realistic for all the respondents, but still with a wide range of difficulty and complexity.

Literacy skills

It is clearly acknowledged in the description of numeracy and its elaborations, and then reflected in the PIAAC Complexity schema (PIAAC Numeracy Expert Group, 2009[2]; Tout et al., 2020[141]), that literacy is an integral and important aspect of numeracy. Certainly, in society and workplaces that adults occupy, tasks and challenges do not neatly divide into, or present as, discrete ‘literacy’ and ‘numeracy’ tasks. Real-world situations and demands cross those kinds of educationally defined boundaries.

The reality is that in using authentic situations as the basis for the numeracy assessment tasks where the mathematics is embedded in a real-world setting, the associated information and data can be very complicated and potentially involve a heavy reading load. In cases where “mathematical representations” involve text, one's performance on numeracy tasks will depend not only on formal mathematical or statistical knowledge but also on reading comprehension and literacy skills, reading strategies, and prior literacy experiences. For example, following a computational procedure described in text (such as the instructions for computing shipping charges or adding taxes on an order form) may require special reading strategies, as text is very concise and structured. Likewise, analysing the mathematical relationships described in words requires specific interpretive skills, as in the simple case of recognising the similarity of “the price doubled” and “the price was twice as high”, but the different meanings in “production levels were constant over the last five years” and “production levels constantly increased over the last five years.”

This creates challenges in trying to focus the assessment on the mathematics and numeracy skills and knowledge. Hence a key goal in the item development process is to make the wording of numeracy items as simple and direct as possible, in order to help minimise the reading literacy demands. Recent research that systematically compares descriptive assessment tasks with more depictive representations of problem situations through using illustrations and photos and minimising the use of words (Hoogland et al., 2016[142]; 2018[134]), gives an indication that even the use of simple supporting illustrations and images could make the contexts and questions more accessible. Lowrie and Diezmann (2009[158]) also researched the impact of supporting graphics and illustrations in numeracy test items, and argued that the design of mathematics assessment items is more likely to be a reliable indication of student performance if graphical, linguistic and contextual components are considered both in isolation and in integrated ways as essential elements of task design.

Disposition, beliefs and attitudes

The issue of choice or disposition when engaging with and solving a numeracy problem is an important factor to consider in an adult’s use and application of mathematics in a real-world situation, and was addressed in the second section, including in the sub-section about *Numerate behaviour and practices*. Research literature suggests that the ways in which a person responds to a numeracy task, including overt actions as well as internal thought processes and the adoption of a critical stance, depend not only on their knowledge and skills but also potentially on their disposition and attitude towards mathematics. Negative attitudes towards mathematics, beliefs about one's mathematical skills, habits of mind, and prior experiences involving tasks with mathematical content are all key influencers on mathematics engagement and performance, alongside beliefs about mathematics and what it is for and who it is for (Geiger, Goos and Dole, 2014[125]; Goos, Geiger and Dole, 2014[124]; Lave, 1988[53]; Saxe, 1992[109]; Schliemann and Acioly, 1989[111]). Are individuals able to choose to use mathematics when it is relevant and appropriate?

This also relates to mathematics anxiety. In some cultures, some adults, including highly educated ones, decide that they are not “good with numbers” or have other sentiments or self-perceptions usually attributed to negative prior experiences they have had as pupils of mathematics. Such attitudes and beliefs stand in contrast to the desired sense of “at-homeness with numbers” (Cockcroft, 1982[44]) and can interfere with one’s motivation to develop new mathematical skills or to tackle math-related tasks, and may also affect test performance (McLeod, 1992[159]). Research about mathematics anxiety is well documented and demonstrates that it can have a significant impact on performance in mathematics [e.g., see: (Buckley, 2013[126]; Ma, 1999[127]; Tobias, 1993[128])].

In real-world contexts, adults with a negative mathematical self-concept may elect to avoid a problem with quantitative and mathematical elements, address only a portion of it, or prefer to delegate a problem, e.g., by asking a family member or a salesperson for help. Such decisions or actions can serve to reduce both mental and emotional load (Gal, 2000[27]). Yet, such actions may fall short of autonomous engagement with the mathematical demands of real-world tasks, carrying negative consequences, e.g., not being able to fully achieve one’s goals.

Numeracy-related practices and experiences

The discussion in the previous section about the research and issues about numerate behaviours and practices, and about the relationship between school mathematics and workplace mathematics, demonstrates that numerate behaviour and practices do not rely only on mathematical knowledge or related reasoning and problem-solving skills acquired as part of formal learning in a school context. Both attitudes and beliefs as well as numeracy-related practices and world knowledge are important enabling processes and may influence adults' ability to act in a numerate way. Therefore, scales assessing selected attitudes and beliefs about mathematics, and numeracy-related practices in work, everyday, and other settings, have been developed for PIAAC's background questionnaire. Information collected by such scales can help to explain differences in performance among adults, further informing our understanding of factors that affect skill acquisition and retention or motivation for further learning. They can be used to help explain the links between numeracy performance and covariates such as participation in a range of numeracy practices in their lives including at work, participation in further learning or employment/unemployment status.

Further, the frequency of engaging with mathematical tasks or of exposure to mathematical or statistical information or displays, whether at work, home, when shopping, or in other contexts, is of much interest. Engagements or practices in this regard can be both the result of a certain skill level, but also the cause of observed skill levels, or at a minimum a factor influencing observed skill level apart from prior formal schooling.

Summary

The above enabling factors address the issue of the non-cognitive aspect of numerate behaviour and practice which is the way a person’s numeracy is manifested in the face of situations or contexts which have mathematical elements or carry information of a quantitative nature. They are addressed indirectly in PIAAC through the other components of the PIAAC assessment, namely the skills use questions and the comprehensive background questionnaire.

The fourth section of the framework discusses the operationalisation of the construct of numeracy in PIAAC and how this is affected by many factors which determine and shape the extent to which the theoretical construct can be fully addressed by the actual collection of test items used in the direct assessment.

The dimensions in PIAAC Cycle 2

This sub-section elaborates on the dimensions incorporated into the definition of numeracy and the elaboration of numerate capacity, as outlined in the first part of Box 3.2. Elaborations on the original facets of the previous two cycles were based on previous research and materials documented in both the ALL and PIAAC Cycle 1 framework. Key in that work was the analysis of the components of adult numeracy by Ginsburg, Manly and Schmitt (2006[29]) which was based on an integrative review of multiple numeracy frameworks from several countries. It also benefited from the positions presented in a report of the UK’s National Research and Development Centre for Adult Literacy and Numeracy (NRDC) (2006[35]), background papers prepared for the OECD-Canada Expert workshop on numeracy (November 2006, Ottawa) and suggestions made by workshop participants. Input was also received from external reviews of early drafts of the PIAAC Cycle 1 framework, and professional perspectives of PIAAC's Cycle 1 Numeracy Expert Group.

For Cycle 2, these facets, or as they are now called, dimensions, have been further developed and substantially enhanced, mainly based on the 2017 review paper (Tout et al., 2017[1]). A significant factor to reworking the core dimensions was the comparison with the PISA 2012 mathematical literacy framework and its classifications. The comparison with PISA mathematical literacy is addressed explicitly and in detail in the fifth section: *Relationship between PIAAC and PISA*. Specific issues arising from this are incorporated into the discussions below.

There are four core dimensions named and described in numeracy for PIAAC Cycle 2, namely:

- cognitive processes
- content
- representations
- context.

Each of these four core dimensions are elaborated below.

Cognitive processes

This dimension is new to PIAAC Cycle 2 and replaces the previous *Response* facet of PIAAC and ALL. It also incorporates to some extent the first of the facets described under the category of *Mathematical knowledge and conceptual understanding* in the enabling processes elaboration in both ALL and PIAAC Cycle 1. This facet addressed the notion of conceptual understanding. This referred to *an integrated and functional grasp of mathematical ideas* (Kilpatrick, Swafford and Findell, 2001, p. 118[157]). Ginsburg, Manly and Schmitt (2006[29]) suggest that the two aspects of conceptual understanding, i.e., it being integrated and functional, frame the ability to think and act effectively as a numerate adult, and that across different numeracy frameworks in different countries, equivalent terms are used such as "meaning making," "relationships," "model," and "understanding." Conceptual understanding can help produce reasonable estimates that can help adults catch computational errors, or realise that an exact product is not necessary, but an estimate is enough for the purpose. Ginsburg, Manly and Schmitt (2006[29]) further explain that conceptual understanding permits one to be free from relying on memory for all methods and procedures, i.e., an adult can think about the meaning of the task and "construct or reconstruct" a representation that both illustrates what it means and suggests a method for solution.

The Cycle 1 framework described and elaborated how in different real-life situations, adults may have to react to a numeracy problem with different types of responses or actions. The Cycle 1 framework grouped those under three broad headings: *identify, locate, or access*; *act upon or use*; and *interpret, evaluate/analyse, communicate*.

The PISA 2012 to 2018 mathematical literacy framework described and used three processes—*formulating*, *employing*, *interpreting and evaluating*—as key components of their mathematical modelling cycle. The *mathematical reasoning* process has now been added as an explicit component in PISA 2021 to highlight the centrality of mathematical reasoning to solving practical problems. As described in the draft PISA 2021 framework, these reasoning skills appear to mainly focus on reasoning skills *within* the world of mathematics, and *mathematical reasoning* is seen as a separate skill or process to the three problem solving processes of *formulating*, *employing*, *interpreting and evaluating*. This highlights PISA's interest in the ability of 15-year-olds to use and apply more formal mathematical skills, knowledge and representations, whereas this type of more formal mathematical knowledge is not generally assessed in PIAAC. For PISA 2021, it is acknowledged that the assessment items will be assigned to either mathematical reasoning or one of the three mathematical processes associated with real-world based mathematical problem solving. The PISA 2021 goal "is to achieve a balance that provides approximately equal weighting between the two processes that involve making a connection between the real world and the mathematical world (formulating and interpreting/evaluating) and mathematical reasoning and employing which call for students to be able to work on a mathematically formulated problem" (OECD, 2018, p. 33[123]).

Based on the views and research outlined earlier and raised in the review report, the NEG has substantially reworked the former *Response* facet, and replaced it with a more comprehensive description and elaboration of what is now called the dimension of *Cognitive processes*. This was to more explicitly describe and address the way adults have to deal with solving a problem embedded in an authentic context. The skills adults need in the 21st century cover not only a range of specific mathematical knowledge and problem solving skills, but include the ability to recognise and identify how and when to use mathematics; to be able to understand, use and apply mathematical concepts and procedures; along with strategic, reasoning and reflective skills to use when using and applying the mathematics.

This is also derived in part from the comparison with the PISA processes as part of the PISA problem solving and modelling process. Unlike a number of the other facets of numerate behaviour in PIAAC Cycle 1 and their related descriptions, this facet of responses or actions had the least in common between PISA and PIAAC. It is the view of the NEG, that for the assessment of numeracy skills of adults the mainly intra-mathematical aspect of mathematical reasoning, as added to the PISA 2021 mathematical literacy construct, needs to be embedded within the real-world problem solving aspect for PIAAC, and not assessed as a separate part of the construct. Therefore mathematical reasoning understanding is integrated into the relevant aspects of the three cognitive processes.

The revisions and enhancements to this facet or dimension also more closely met the need to address a range of factors to do with both 21st century skills and the need to be more reflective and be able to reason and think critically, and make judgements. The NEG believes that this enhanced and more explicitly defined and described *Cognitive process* dimension has supported the test developers to write new types of items and has greatly enhanced the item pool, helping to address a number of the concerns expressed in the review paper. It will also help develop and write the scale descriptions once the data and results are available.

Table 3.2 below compares the terms used for the cognitive process or response-related descriptions in PIAAC Cycle 1 with the three processes used in PISA 2012 and the four processes used in PISA 2021 including against the new *Cognitive process* of PIAAC Cycle 2.

Table 3.2. Cognitive processes labels in PIAAC and PISA

PIAAC Cycle 1	PIAAC Cycle 2	PISA 2012	PISA 2021
Identify/locate/access	Access and assess situations mathematically	Formulating situations mathematically	Formulating situations mathematically
Act on/use (order, count, estimate, compute, measure, model)	Act on and use mathematics	Employing mathematical concepts, facts, procedures, and reasoning	Employing mathematical concepts, facts, procedures, and reasoning
Interpret/evaluate/communicate	Evaluate, critically reflect, make judgements	Interpreting, applying and evaluating mathematical outcomes	Interpreting, applying and evaluating mathematical outcomes
			Mathematical reasoning

Description

Solving problems in real-world contexts requires a range of capabilities and cognitive processes. When engaging with a real-world problem, one of the decisions to be made is whether the use of mathematics is relevant and then if it is best way to solve a problem. If the use of mathematics is deemed appropriate, the essential features of the problem will need to be identified in order to turn the real-world situation into a mathematical problem. From this point, relevant mathematical content, procedures, processes and tools needed to solve the problem must be identified and accessed by the problem solver. Once accessed, these procedures and processes will need to be employed correctly and decisions made about the appropriate degree of accuracy required to yield a mathematical solution. The solution needs to be reflected on and evaluated against the original problem situation in terms of its reasonableness and relevance to the real-world context and a decision made about whether to accept the solution or to revisit aspects that require refinement. In cases where decisions or judgements are being made on the basis of the solution, other factors might also be considered such as social or economic consequences.

So the first core dimension described in the PIAAC definition and elaboration of numeracy is about the cognitive skills and processes required to engage with and solve the task or problem at hand. These have been named as:

- access and assess situations mathematically
- act on and use mathematics
- evaluate, critically reflect, make judgements.

It is important to understand that these activities are not mutually exclusive of one another or that they take place in a rigidly linear manner. For example: the identification of a problem's essential features will have consequences for the identification of relevant mathematics to be engaged; an inability to access a particular area of mathematics may result in the selection of mathematical procedures and processes that are less effective; or the evaluation of the solution against the original problem situation may indicate those features, identified as essential, were not as relevant as first thought and so backtracking through the steps of the solution is necessary. Thus, while the cognitive processes outlined in this sub-section are described separately, the activity of addressing a real-world problem via mathematical means should be considered first and foremost as a holistic process.

It will be the combination of these three processes and their components that drive the difficulty and complexity of each numeracy problem being solved and each question asked in PIAAC numeracy units and items. After the description of each cognitive process below, there are a number of key questions outlined that describe the issues and factors that will influence the complexity of each process.

Note: for the purpose of guaranteeing a spread of types of items across PIAAC that focus on or emphasise the different aspects of these cognitive processes, each item has been prioritised against one of the three processes.

Access and assess situations mathematically (assess, identify, access and represent)

When adults encounter problems within real-world contexts they must first decide if mathematics is an appropriate means to engage with the situation. Once they deem the use of mathematics will provide advantage in addressing the problem, they need to identify the essential features to be accommodated when transforming the real-world situation into a mathematical problem. This transformation requires adults to look forward and identify and access the mathematics and mathematical representation embedded in the specifics of the situation, and make decisions about how the task can be represented and solved mathematically. The direction of the thinking and reasoning in this process is going from the real world to the mathematical world.

The actions that underpin assessing situations and accessing the mathematics in order to solve a real-world problem include:

- identifying the essential features of a real-world problem that can be represented mathematically
- identifying and describing/defining the mathematical operation(s), processes and tools needed to solve the problem
- simplifying a situation or problem in order to represent it mathematically, using appropriate representations, for example, variables, symbols, diagrams, and models
- representing a problem in a different way, including organising it according to mathematical concepts and making appropriate assumptions
- anticipating the real-world restrictions on the possible outcomes of decisions made while defining and representing the problem.

Key questions that drive the complexity of this process:

- How is the mathematics represented and embedded within the real-world situation? Through words and language? Through numbers and symbols, diagrams, pictures, graphs and charts? How informal, formal or complex are the mathematical representations and the mathematical information?
- Is a mathematical approach suitable for the presented situation – is the use of mathematics a sensible way to address the real-world problem? If so, what is the degree of transformation required of the real-world situation to move it into a mathematical problem? How implicit or explicit/obvious is it to decide on the mathematical problem solving solution? Is the question presented in an unambiguous way so that necessary mathematical processes and procedures can be identified?
- What literacy skills are required to make this transformation – what are the reading demands, how much distracting information is there?
- Will a decision need to be made about how well the solution generated by solving the mathematical representation of the problem matches the contexts of the original real-world situation? How complex is that decision?

Act on and use mathematics (order, count, estimate, compute, measure, graph and draw)

Adults utilise mathematical processes, facts and procedures in order to derive results and solve real-world problems, and will need to select and use appropriate tools, including technology. For example, they may need to perform arithmetic computations; select, create, solve equations; make logical deductions from mathematical assumptions; perform symbolic manipulations; create and extract information from mathematical tables and graphs; represent and manipulate geometrical objects in 2D and 3D; and analyse data. Mathematical processes and procedures used to solve real-world problems include:

- applying mathematical facts, rules and structures
- performing arithmetic computations and applying routine algorithms
- undertaking measurements
- looking for a pattern
- using symbolic, formal, and technical language and mathematical conventions
- using mathematical tools, including technology
- manipulating numbers, graphical, statistical and chance-based data and information, algebraic expressions and equations, geometric representations
- collecting, organising, structuring and representing information
- generating estimations and approximations
- making and extracting information from mathematical diagrams, graphs, infographics and constructions
- reviewing and reflecting upon initial or part solutions
- generalise from a more complex mathematical situation to a simpler mathematical problem/situation that can be more easily solved.

Key questions that drive the complexity of this process:

- How difficult and complex are the mathematical concepts, facts, processes and procedures that need to be used and applied?
- What level of mathematical reasoning, arguing, manipulating and computing is required for an effective response to the problem?

- How many steps and types of mathematical steps/processes are required to solve the problem? Is it one operation, action or process or does it require the integration of several steps covering more than one different operation, action or process?

Evaluate, critically reflect, make judgements (evaluate, reflect, justify and explain)

Responses to real-world tasks, including any mathematical solutions, judgements, decisions or conclusions, require reasoning and critical reflection and evaluation. Any solution of a real-world problem needs to be evaluated against the original problem situation in terms of its reasonableness and relevance to the original context and a decision made about whether to accept the solution or to revise and adjust the solution—often referred to as contextual judgement. In cases where decisions or judgements are being made on the basis of the solution, other factors might also be considered such as social or economic consequences. This will require that responses include explanations and justifications for decisions, judgements and conclusions that are reasonable and make sense within the context of the original situation. Critical reflection and evaluation within real-world contexts requires:

- evaluating the reasonableness of a solution or part solution to a problem. This includes consideration of the appropriateness of estimations and/or the degree of accuracy required
- understanding the real-world implications of solutions generated by mathematical methods, in order to critically reflect and make judgements about how the results should be adjusted or applied
- using mathematical arguments to construct, defend or challenge decisions and/or judgements
- considering social norms and influences, in addition to physical constraints, when considering the validity or effectiveness of a mathematical solution to a real-world problem
- reflecting on mathematical processes and arguments used and explaining and justifying results
- identifying and critiquing the limitations inherent in solving some real-world problems.

Key questions that drive the complexity of this process:

- How complex is it to evaluate, reflect, justify, and explain and connect the mathematical outcomes to the real-world context? Does the task require a choice from a number of provided solution options? Or does the task require an explanation to be derived or decided upon with no provided solutions?
- How complex is it to justify the validity of the mathematical outcomes and evidence with the essential elements of the original real-world problem? To what extent does the task require judgement about the quality of a mathematical argument used to defend or challenge a proposition within a real-world context?
- How complex is it to connect the mathematical evidence to the essential elements of the real-world problem? To what extent does the task require judgement about the appropriateness and reasonableness of a proposed result to the real-world context? To what extent does the mathematical result need to be adapted to fit in with the original real-world context? Does it require consideration of the appropriateness of estimations and/or the degree of accuracy required?

These three *Cognitive processes* are linked to the Numeracy Complexity Schema described further in the fourth section: *Operationalisation of the PIAAC Numeracy Assessment* and detailed in Tout et al. (2020[141]). It is believed that the cognitive processes will drive much of the item difficulty and that together with the descriptions and scores described in the Complexity Schema, these will help to describe performance when it comes to elaborating the different levels in PIAAC.

Mathematical content

Mathematical information can be classified in several ways and on different levels of abstraction. One approach is to refer to fundamental "Big Ideas in mathematics" (see discussion in the second section). Steen (1990[11]), for example, identified six broad categories: *Quantity*, *Dimension*, *Pattern*, *Shape*, *Uncertainty*, and *Change*. Rutherford and Ahlgren (1990[143]) described networks of related ideas: *Numbers*, *Shapes*, *Uncertainty*, *Summarising data*, *Sampling*, and *Reasoning*. Dossey (1997[160]) categorised the mathematical behaviours of quantitative literacy as: *Data representation and interpretation*, *Number and operation sense*, *Measurement*, *Variables and relations*, *Geometric shapes and spatial visualisation*, and *Chance*. More broadly, many curriculum frameworks around the world in one way or another refer to these key areas, albeit using somewhat different terminologies and with somewhat different groupings [e.g., National Council of Teachers of Mathematics (NCTM) (2000[120])].

This dimension remains similar to PIAAC Cycle 1 and is similar to the equivalent facet in PIAAC and ALL. There are some name changes, partly to make them more consistent with the PISA mathematical literacy descriptions and labels for *Content* (see Table 3.3).

Table 3.3. Mathematical content labels in PIAAC and PISA

PIAAC Cycle 1	PIAAC Cycle 2	PISA 2012 and 2021
Quantity and number	Quantity and number	Quantity
Dimension and shape	Space and shape	Space and shape
Pattern, relationships and change	Change and relationships	Change and relationships
Data and chance	Data and chance	Uncertainty and data

Description

Four key areas of mathematical content, information and ideas are described and used in the numeracy assessment in PIAAC:

- Quantity and number
- Space and shape
- Change and relationships
- Data and chance.

For an individual item in PIAAC numeracy, these four content areas are not mutually exclusive and any item may involve one or more of these mathematical content areas. For example, a unit and item in *Data and chance* will necessarily also include data that will be expressed as a quantity or number, and similarly a measurement item in *Space and shape* will be expressed as a quantity or number. The classification of such items is based on what content area the key conceptual understanding and skill is directed at.

Quantity and number

The notion of quantity and number is a fundamental and essential mathematical aspect of engaging with, and functioning in, our world. The *Quantity and number* content area involves understanding ordering, counts, place value, magnitudes, indicators, relative size and numerical trends. This will encompass aspects of quantitative reasoning, such as number sense, multiple representations of numbers, computation, mental calculation, estimation and judging the reasonableness of results. This content area requires knowing and applying integers, rational and irrational numbers, positive and negative numbers and equivalence. It also requires understanding and applying number operations, including order of operations, in a wide variety of settings.

Illustrative examples:

- Identify and counting the number of items shown in a photo of a set of items or object.
- Calculating the cost of one can of soup, given the cost of 4.
- Calculating the cost when buying 0.283 kg of cheese at a given price per kg.
- Another example could be deciding whether given decimal numbers are within a given range.

Space and shape

The *Space and shape* content area encompasses a wide range of phenomena that are encountered everywhere in our visual and physical world. It includes an understanding and use of: measurement (informal and standardised) systems, measurement formulas; dimensions and units; location and direction; geometric shapes and patterns; angle properties; symmetry; transformations and 2D and 3D representations and perspectives. This content area requires understanding and interpreting measurements and scales, position and orientation, plans, models, maps and diagrams, and navigation (including understanding travel distances, speeds and times, and using tools such as Global Positioning Systems).

Illustrative examples:

- The identification of a shape or matching an image of a real object to the correct plan/diagram.
- Reading the weight/mass of an object off an analogue scale.
- Interpreting an online map in relation to travel distances, speeds and times.
- Working out quantities required for a task such as wallpapering or tiling or painting given particular dimensions.

Change and relationships

The *Change and relationship* content area includes the ways to describe, model and interpret mathematical relationships, quantitative patterns, and change, where they occur in the real world. Real-world variables can be based around linear and non-linear relationships. Such relationships can be represented by descriptions, picture or images, tables, graphs or formula. In the latter case it could require the understanding and use of algebraic expressions and related methods of solution. This content area requires understanding, using and applying proportional reasoning and rates of change, including the use and application of ratios. It also requires recognising, describing, and/or using a relationship between different variables derived from a real-world situation.

Illustrative examples:

- Comparing the different proportional discounts on a shopping item in two different sales where the discounts are displayed in different ways.
- Understanding and using formulae such as for calculating interest or inflation rates, or one's BMI (Body Mass Index).
- Understanding and applying proportional reasoning to calculate values based on existing percentage or proportions of quantities/ingredients.
- Understanding and applying linear growth in order to predict future growth or decline.

Data and chance

The *Data and chance* content area encompasses topics such as data collection, data displays, charts and graphs, measures of central tendency and variance, alongside understanding appropriate approaches to data collection and sampling. The representation and interpretation of data are key concepts in this category. This content area also includes understanding and knowing about chance and probability. Chance and probability encompass subjective probability, certainty and uncertainty, likelihood and unlikelihood, prediction, and decision making. For example, attaching a numerical value to the likelihood of an instance is a ubiquitous phenomenon no matter whether it has to do with the weather, the stock market, a medical prognosis or the decision to board a plane.

Illustrative examples:

- Interpreting and identifying particular information on a simple bar graph or pie chart.
- Using an interactive online data tool and chart to interpret and analyse provided data.
- Use and understand averages (mean) to calculate required targets.
- Sort and interpret a set of data to test a number of opinions about the set of data.

Context

Context is the parameter or term used in both PISA and PIAAC for naming and classifying the settings or situations where people use and apply their mathematical knowledge to solve a realistic problem. The main purpose behind the use of the chosen context categories is to ensure a mixture or blend across the different categories to help guarantee some degree of balance in the assessment, with no particular context overwhelming the others (and therefore advantaging or disadvantaging respondents with greater or lesser daily interaction with some settings/contexts).

In PIAAC Cycle 1, the contexts used were:

- Everyday life
- Work-related
- Societal or community
- Further learning.

The sets of descriptors used in both PISA and the PIAAC Cycle 1 frameworks regarding the first three contexts (*Everyday life/Personal*; *Work-related/Occupational*; *Societal or Community/Societal*) were highly consistent with each other. One of the review team's recommendations was that the PISA label *Personal* is preferable to the PIAAC label of *Everyday*. “Everyday” suggests some “sameness” in what people do which is not particularly illuminating, whereas the term *Personal* aims to indicate that the issue at hand bears most directly just on that individual. This has been implemented.

Further Learning in PIAAC Cycle 1 was another context that the review project recommended for reconsideration. *Further learning* has some similarity and consistency with the term intra-mathematical that PISA refers to within its description of *Scientific*:

> *… Particular contexts might include (but are not limited to) such areas as weather or climate, ecology, medicine, space science, genetics, measurement and the world of mathematics itself. … Items that are intramathematical, where all the elements involved belong in the world of mathematics, fall within the scientific context. (OECD, 2013, p. 37[122])*

This "Scientific" context of PISA has two elements to it. Some items classified as *Scientific* in PISA are in fact "intra-mathematical", that is, situations which are within the world of mathematics, explicitly related to knowing about formal aspects of mathematics, with no, or little, real-life connections. There were, in fact, no questions in PIAAC that are purely intra-mathematical, as there can be in PISA. There was a second set of questions in PISA that were classified as *Scientific*, where the situation or context related to the natural world (e.g., climate or ecology). In PIAAC Cycle 1 this context of *Further learning* was described as being related to adults needing to solve problems that may arise when participating in further study, whether for academic purposes or for vocational training, and was explicitly related to knowing about the more formal aspects of mathematics, including the conventions used to apply mathematical rules and principles. But the actual items could also be classified against the other three contexts. The sample PIAAC Cycle 1 item from this context that is discussed in the PIAAC reports was the item "Candles" (OECD, 2013, p. 77[161]). However, this item could also have been classified as *Everyday life*, *Work-related*, or even *Societal or community*.

For this, and a number of other reasons, the 2017 review paper recommended that the NEG review this fourth context of *Further learning*. The NEG considered this and decided that it was best to remove the classification named *Further Learning.* As a result, only the first three contexts continue in PIAAC Cycle 2, as these were the most relevant to adults, and that any existing *Further learning* items in PIAAC Cycle 1 should be reclassified against one of the other three contexts. The need to have items that were about knowing about the more formal aspects of mathematics, including the conventions used to apply mathematical rules and principles, would be covered through the inclusion of those requirements through the content knowledge area of *Change and Relationship*.

Hence there are some name changes, partly to make them more similar to the PISA mathematical literacy descriptions and labels for *Context*. The three versions of the *Context* labels are described in Table 3.4.

Table 3.4. Context labels in PIAAC and PISA

PIAAC Cycle 1	PIAAC Cycle 2	PISA 2012 and 2021
Everyday life	Personal	Personal
Work-related	Work	Occupational
Societal or community	Societal/community	Societal
Further learning		Scientific

Description

People try to manage or respond to a situation involving numeracy and mathematics because they want to satisfy a purpose or reach a goal. Three types of contexts that may require the use and application of numeracy skills are described below:

- Personal
- Work
- Societal/community.

These are not mutually exclusive and may involve the same underlying mathematical themes. The capability of being critically reflective about the use and application of mathematics is important in the 21st century, and adults need to be able to make decisions and judgements, and defend or support arguments. The different contexts provide the different areas of their lives where adults may encounter numeracy situations and which therefore provide the purpose for engaging in, solving and being reflective about real-world problems involving mathematics.

Personal

Numeracy tasks are often encountered in personal and family life, or revolve around hobbies, sports and games, personal development and personal interests. The personal context focuses on activities for an individual and in their interactions with immediate family. Representative tasks include (but are not limited to): handling money and personal or family finances and transactions, health and well-being, activities with family and friends, shopping, personal time management, travel and holiday planning, including reading maps, and using measurements in home situations such as cooking, gardening, administering medicines, or doing home repairs.

Work

Adults often encounter mathematical situations at work that are more specialised than those in everyday life. Today's workplaces often require increasing levels of techno-mathematical literacy. Representative tasks include (but are not limited to): completing purchase orders, maintaining inventories, totalling receipts, calculating change, managing schedules, budgets and project resources, payroll/accounting, using spreadsheets, completing and interpreting production and control charts, managing production inputs and outputs, tracking costs and expenditures, interpreting results from technological devices, and applying formulas. Work-related tasks can also include reading plans, blueprints and workplace diagrams, having spatial awareness for best storage options and organising and packing goods, and planning the most efficient delivery journey. This context can also include making and recording measurements such as lengths, weights, temperatures, dosages, areas, volumes or other work-related measurements, and using and applying measurement ratios and formulas. Occupational contexts may relate to any level of the workforce, from unskilled work to the highest levels of professional work.

Societal/community

Adults need to know about quantitative data and statistics and their representations, and be able to interpret trends and the consequences of a range of activities and actions happening in the world around them at the local, national or global level. Adults need to know about and be able to understand different mathematical relationships, such as proportional reasoning, when reading and interpreting information presented by a range of community or government authorities. Adults also may take part in a range of social events or community activities, including social and political participation, organising and participating in community-based functions and fundraising. Representative tasks include (but are not limited to) understanding and interpreting financial, statistical and numerical information and graphs about public transport, crime, health, education, politics, demographics, pollution, community events, etc. This information is increasingly being presented by the media, government services, financial institutions, utilities, and by a range of community services and organisations.

Representations

The third facet of the previous elaboration in ALL and PIAAC that has been added into the actual definition and hence made more explicit is that of *Representation*. Although this was included in the previous elaboration, it was not part of the definition, and did not help drive item development. Under *Facet 4: Representations of mathematical information*, the PIAAC Cycle 1 numeracy framework stated that mathematical content/information/ideas can be represented in multiple ways: objects and pictures; numbers and mathematical symbols; formulae; diagrams and maps, graphs, tables; texts; and finally, technology-based displays. However, none of these was expanded in much detail (PIAAC Numeracy Expert Group, 2009, p. 28[2]) and although the issue of different forms of representation of information is raised, digital or dynamic formats are not addressed.

The nature of information graphics is only now being unpacked within the field of mathematics education. Diezmann and Lowrie (2008[162]), for example, have argued for the importance of becoming proficient in interpreting information graphics (e.g., graphs, tables, maps) as these are increasingly used to manage, communicate, and analyse information. Societies are becoming increasingly reliant on representing information both diagrammatically and graphically. The new, more dynamic representation of data and information needs to be addressed. It is now no longer a matter of interpreting static images, as in the existing PIAAC Cycle 1 item pool, but also how new scenarios and different problems can be posed by interpreting and manipulating dynamic representations.

The 2017 review recommended that PIAAC Cycle 2 harness the potential of technology to support a more effective and representative 21st century assessment, through greater use of different technology, media and associated representations to make the assessment more relevant to the 21st century. This is discussed further in the fourth section.

Description

Quantitative and mathematical information in real-world situations and contexts is always represented and embedded in some format or other, whether that be in words and text, or diagrammatically or graphically, or dynamically. Mathematics, per se, does not exist in the real world by itself in its own isolated abstract form, such as 80% x €7.80 – such mathematics will be most likely embedded in an advertisement saying "20% discount" and the reader will need to read the information and decide that the solution is to take off 20% of the original price of €7.80. Hence the PIAAC framework needs to elaborate on the different ways that mathematics can be represented in the real world in a numeracy situation.

Mathematical information in a situation may be available or represented in many forms. It may appear as concrete objects to be counted (e.g., people, buildings, cars, etc.) or as pictures of such things. It may be conveyed through symbolic notation (e.g., numerals, letters, and operation or relationship signs). Sometimes, mathematical information will be conveyed by formulae, which are a model of relationships between entities or variables. Mathematical information may be encoded in visual displays such as a diagram or chart; graphs and tables may be used to display aggregate statistical or quantitative information (by displaying objects, counting data, etc.). Similarly, a map of a real entity (e.g., of a city or a project plan) may contain information that can be quantified or mathematised. Last but not least, textual elements may carry much mathematical information or affect the interpretation of mathematical (and statistical) information, as explained further below.

A person may have to extract mathematical information from various types of texts, either in prose or in documents with specific formats (such as in tax forms). Two different kinds of text may be encountered in numeracy tasks. The first involves mathematical information represented in textual form, i.e., with words or phrases that carry mathematical meaning. Examples are the use of number words (e.g., "five" instead of "5"), basic mathematical terms (e.g., fraction, multiplication, percent, average, proportion), or more complex phrases (e.g., "crime rate increased by half") which require interpretation, or coping with double meanings (or with differences in mathematical and everyday meanings of the same terms). The second involves cases where mathematical information is expressed in regular notations or symbols (e.g., numbers, plus or minus signs, symbols for units of measure, etc.), but is surrounded by text that despite its non-mathematical nature also has to be interpreted in order to provide additional information and context. An example is a bank deposit form or interactive device (e.g., on a mobile device or an *automated teller machine*, ATM) with some text and instructions in which numbers describing monetary amounts are embedded, or a parking ticket specifying an amount of money that has to be paid by a certain date due to a parking violation, but also explaining penalties and further legal steps that will be enacted if the fine is not paid by a certain date.

With the 21st century digitisation of information and processes, the types of representation now explicitly include technology-based displays and visualisations on websites, in infographics, in online calculators, spreadsheets and other software and apps on mobile devices and more.

Four classifications for the representation of real-world numeracy situations are described:

- Text or symbols
- Images of physical objects
- Structured information

- Dynamic applications.

For an individual item in PIAAC numeracy, these four descriptions of different representations are not mutually exclusive and any item may involve one or more of these dimensions.

Text or symbols

The stimulus is primarily based on running text that describes the problem situation and can include symbols and numerical information integrated into the text.

Images of physical objects

The stimulus is primarily based on photos or images of physical objects which depicts the problem situation. The image contains the crucial information to solve the problem (e.g. ruler or measuring instrument/scale, 3D objects). Sometimes some text is added to specify or narrow down the problem situation.

Structured information

The stimulus is primarily based on data or information that is represented in tables, graphs/charts, maps, plans, calendars, schedules, timetables, infographics, etc. In most cases, these are computer-generated representations of data, which are becoming more ubiquitous in all news and social media, and in information from government services, financial institutions and utilities. Text will often be used to help specify and describe the information and the problem situation.

Dynamic applications

The stimulus is primarily based on interactive applications, animations, calculation applications (for instance planning and designing software, structured spreadsheets, drawing programmes, online applications and calculators such as loan calculators, currency converters, etc.), which are designed to support users to perform calculations or plan or design activities. This category could also contain: (simulations of) handheld devices and measurement instruments. Sometimes text is used to specify or narrow down the problem situation.

Operationalisation of the PIAAC numeracy assessment

The operationalisation of the construct of numeracy in a large-scale assessment such as PIAAC is affected by many factors which shape the extent to which the theoretical construct can be fully addressed by the actual collection of items used in the direct assessment.

The 2017 review (Tout et al., 2017[1]) undertook a review of assessment developments, including in relation to numeracy and mathematics assessments. The review recommended that, because the numeracy description and construct in PIAAC is a multifaceted view and definition of numeracy, it requires a multimodal way of assessing the concept and construct, and because of the availability of new developments in technology and communication, new assessment developments could provide opportunities to enhance the assessment of numeracy in PIAAC Cycle 2.

The review also recognised and acknowledged up front that the existing PIAAC Cycle 1 items are based predominantly around static images and associated responses and are more like paper-based assessments transferred onto a computer, partly due to the transfer of many of the paper-based items from the previous ALL assessment to the computer-based assessment in PIAAC. As well, the platform used for PIAAC Cycle 1 was quite restrictive in terms of modalities and interactions that were available to house the stimulus and for the responses that could be automatically scored.

This section first begins with a general introduction about assessment developments in the 21st century, especially in relation to assessing adults' mathematical skills, followed by a sub-section outlining a possible process and structure for enhancing the assessment of numeracy in PIAAC Cycle 2. This is followed by a discussion of the constraints that affect the development of the direct assessment of numeracy in PIAAC. Based on these discussions, an outline is then presented of the principles that guide the assessment of numeracy in PIAAC, including specifying the blueprint for the proportions of items against each of the core dimensions in the construct. This discussion and the consequent blueprint specifications of the test content are critical in ensuring that the direct assessment of numeracy in PIAAC Cycle 2 meets both construct and content validity requirements.

Finally, there is a brief discussion about, and further details on, a supporting scheme regarding factors that affect task complexity (or item difficulty) which is of importance both for task design as well as interpretation of results regarding numeracy in PIAAC.

Assessment developments

There have been technologically-driven advancements in the educational measurement and assessment field in the 21st century, some of them based around the need to assess 21st century skills. There is much research about such developments [e.g., see (Bennett, 2015[163]; Geisinger, 2016[164]; Parshall et al., 2002[165]; Shute et al., 2016[166])]. Bennett (2015[163]), who has been researching and mapping educational assessment for a considerable period, describes three generations of assessment. He described first-generation technology-based testing as largely an infrastructure-building activity, laying the foundation for tests to be delivered in a new medium, where much of the testing closely resembled traditional tests. In his description of second-generation tests, he argued that qualitative change and efficiency improvement become the driving goals (Bennett, 1998[167]; 2010[168]), and where the tests use less traditional item formats, moved towards new multimodal formats and where there were attempts to measure new constructs. The driving force was often the technology. Bennett describes a third generation assessment as one of reinvention occurring on multiple fronts where these assessments were able to serve both institutional and individual-learning purposes. They are designed from both cognitive principles and theory-based domains, and where the assessments utilise "complex simulations and other interactive performance tasks that replicate important features of real environments, allow more natural interaction with computers, and assess new skills in more sophisticated ways" (Bennett, 2015, p. 372[163]). This includes the use of Augmented and Virtual Reality [e.g., see (Bower et al., 2014[169]; Sommerauer and Müller, 2014[170])].

While the review paper and the current expert group acknowledge that it is not yet possible in a large-scale international assessment such as PIAAC to implement and use the potential of Augmented and Virtual Reality, there are considerations to take on board for the future development and potential enhancements to the assessment of numeracy in PIAAC. There are many different computer-based models and options available to inform how numeracy might be more effectively assessed in future iterations of PIAAC, including in Cycle 2. The following sub-sections describe some of these possible enhancements, but also conclude with a discussion about the reality and constraints of an assessment such as PIAAC and especially the need to have all the materials and questions made available across a large number of different languages.

Computer-based assessment of mathematics and numeracy

The literature specifically on computer-based assessment of mathematics (CBAM) and multimedia learning of mathematics [e.g., (Atkinson, 2005[171])] mostly focuses on the multimodal representation of mathematical concepts: calculating, graphs, diagrams, computer algebra systems, spreadsheets, statistical programmes, etc. However, in the computer-based assessment of numeracy another focal point is also of importance, namely the role of the representation of the problem (the situations and settings in which the mathematics is embedded). More general research on representations of real-life situations in education and assessment should also be considered [e.g., see (Schnotz, 2002[172]; Schnotz, 2005[173]; Schnotz and Bannert, 2003[174]; Schnotz and Kürschner, 2007[175]; Schnotz et al., 2010[176])], as well as research on more general multimedia learning (Mayer, 2005[177]; 2009[178]), while being aware of the cognitive load discussion (Sweller, 2005[179]; 2010[180]; van Gog, Paas and Sweller, 2010[181]).

In an analysis and review of the optional computer-based assessment of mathematics items developed for PISA 2012, the Australian Council for Educational Research (ACER) test developers created a list of features that benefited and advantaged CBAM test items over traditional paper-based assessments of mathematics:

- Their appeal to students' interactive learning styles increases the engagement of students with the task.
- Items are less dependent on text and reading skills, which means students can access an item from visual cues, and then use the text to confirm the required response details.
- Response modes are more flexible and less daunting. Students can easily edit a response, so they are more inclined to "have a go".
- Relevant calculations can be automated, which means answers are correct, and less time is taken. This allows items to address higher-order mathematical reasoning.
- Items can assess spatial and visual skills using accurate simulations and manipulatives that are not readily available in pencil-and-paper formats.
- Items can test ability to use a wider range of problem solving strategies, such as observation of patterns and trends, and of the effect of manipulations and actions.
- Items can simulate computer-based processes, such as spreadsheets, drawing tools and graphing tools, and handling information in an online environment.
- Systems can collect data about what the student did within an item, such as the time taken, number of clicks, processes followed and the final state (PISA Mathematics Expert Group, 2009[182]).

These are also applicable to an adult numeracy assessment such as PIAAC. A useful classification of the CBAM item types in PISA 2012 was also developed by the ACER test development team for the set of items (PISA Mathematics Expert Group, 2009[182]; Tout and Spithill, 2015[183]). These included, for example, items that were based around automatic calculation, where calculation could be automated "behind the scenes" to support assessment of deeper mathematical skills and understanding; animations, and/or manipulations; drawing, spatial, visual cues and/or responses; simulation of computer applications (e.g., using the data sorting capability of an 'imitation' spreadsheet); interactive graphing allowing automatic mathematical function graphing and statistical graphing; and simulation of web-based applications or contexts, with or without computer-based interactivity (e.g., buying goods online).

Another advantage of computer-based (or tablet-based) assessments is that the responses and item types available and that were used and can be automatically scored is quite extensive and can include options such as: selected-response formats (e.g., multiple choice, complex multiple choice such as true/false type questions); short numeric responses; click-on and hot-spots; drag-and-drop; pull-down menus; matching and ordering; and manipulation of images to a correct, final position and solution.

Enhancing the assessment of numeracy in PIAAC Cycle 2

In the first cycle of the PIAAC assessment, there was a gap between the sophistication of the concept of numeracy used and the functionality of the assessment platform. As acknowledged above, the assessments that exist in the current item pool are relatively simple and one-dimensional. This analysis is corroborated in the literature [e.g., (Bennett, 2015[163])].

More sophisticated assessments utilising some of the possibilities outlined above are not necessarily aimed at more complex or higher order skills, but focus more on the multifaceted and multimodal nature of numeracy problem situations encountered in real life. To assess a sophisticated concept of numeracy there is the need for multimodal options to better represent reality, in which the respondents can show their competence (or not).

It is important to make a distinction and achieve a balance between the drive that stems strongly from development of technologies, that can be used in assessments (technology-driven) and the drive to design an assessment that is closely related to the concepts that are designed around the construct of numeracy (concept-driven).

Specifically, there is the need to frame any assessment development enhancements to PIAAC numeracy around two underpinning aspects of the PIAAC numeracy construct:

- That PIAAC is based on a multifaceted concept of numeracy and has an associated multimodal assessment.
- That PIAAC is an assessment of how well individuals can use their mathematical knowledge and skills to solve problems stemming from pragmatic and authentic (i.e., real-world) situations, needs or demands.

The PIAAC definition and description of numeracy falls into the category that Maguire and O'Donoghue (2002[184]) called the "integrative phase." They classified the development of definitions and ideas about numeracy into three phases: formative, mathematical, and integrative. In this "integrative" classification, as with PIAAC, numeracy is viewed as a complex, multifaceted and sophisticated construct incorporating each individual's mathematics, communication, cultural, social, emotional, and personal aspects in a real-world situation. Numeracy, as with mathematical literacy in PISA, is seen as a sophisticated capability requiring more than just arithmetic calculations and basic mathematics. These more integrative approaches to numeracy have become influential over the last few decades, as illustrated by projects that define numeracy instructional content standards and assessment frameworks such as in PISA, ALL, and PIAAC and national adult curriculum frameworks/standards [e.g., see (Department for Education (DfE), 2014[185]; McLean et al., 2012[77]; Quality and Qualifications Ireland (QQI), 2016[78]; Tertiary Education Commission, 2008[79])]. The assessment of such a multifaceted phenomenon therefore requires a multifaceted and multimodal set of assessment items that are authentic, as described above in relation to assessment developments and possibilities.

Hoogland and Tout (2018[186]) in looking at the pressures and challenges on Computer-based assessment of mathematics (CBAM) into the twenty-first century, argued that technology has the potential to support the assessment of higher-order thinking skills in mathematics, and also to represent authentic problems from the world around us to use and apply mathematical knowledge and skills. However, they also argued that the challenge is to not allow the technological capabilities, supported by psychometric analysis, to focus too much on assessment of lower order goals, such as the reproduction of procedural, calculation-based, knowledge and skills. These aims are consistent with the aims of the assessment of numeracy in PIAAC. Assessing a multifaceted concept based only on simple assessment tasks has two negative implications:

- The capabilities of individuals to cope with complex and multifaceted mathematics problems from real life are not assessed in full when the items are too straightforward and one-dimensional.
- The rollback effect of an international assessment of adult competencies on adult numeracy education practices is not to be underestimated. There is a responsibility for assessment developers such as those in PISA and PIAAC that their framework and assessment items are in sync with and reflect the complex, multifaceted constructs and concepts being assessed.

As described in the earlier sections of the framework, it is critical in the assessments of adults' numeracy to make the situations, representations and the responses authentic and make them as similar as possible to the way adults encounter mathematics in different life contexts, and not use questions such as the typical school-based word problems described earlier. Arguably the problem of authenticity and cultural appropriateness is lessened when testing pupils in schools, such as in PISA, because test designers can use conventional mathematical terminology, formulae, symbols, and so forth; this helps school-age assessments to standardise the demands from respondents by conveying the mathematical information embodied in different situations in consistent ways regardless of the cultural context. However, testing of adults' numeracy presents more challenges because many will not remember formal school-based notations or terminology. In countries where a sizeable proportion of the population are immigrants or speak multiple home-based languages, the gaps between mother tongues and school-based mathematical linguistic conventions may further affect performance on some numeracy tasks. Thus, attention has to be given to linguistic and cultural factors when adapting items for adult assessments.

21st century representation and interactivity

As developments in the 21st century impact on how mathematical and numerical information is represented, the PIAAC Cycle 1 facet, *Representations of mathematical information*, in the PIAAC framework has been significantly updated to reflect these changes. The revised framework and definition for numeracy in PIAAC Cycle 2 and the resulting new items harness the potential of technology to support a more effective and representative 21st century assessment, for example, through greater use of visual and interactive media, such as the use of infographics, interactive websites and online calculators, spreadsheet processes, graphing and measurement tools, etc. in assessment items. However, a balance has been kept between numeracy and mathematics tasks and actions embedded in 21st century digital and technological environments with those embedded in modes that are more traditional. This balance can partly be maintained by the necessary use of the existing PIAAC Cycle 1 numeracy item pool as linking items as these were based mainly around static images and the items are more like paper-based assessments transferred onto a computer.

A dimension for reviewing assessment possibilities

In order to facilitate the development of newer assessment content and delivery mechanisms, the review team developed a dimension of assessment possibilities – see Figure 3 in Tout et al. (2017, p. 31[1]), that could form a starting point for monitoring and balancing the range of PIAAC numeracy item formats and types. Hoogland and Tout (2018[186]) further developed this dimension of assessment possibilities, and argued that it could be used to reflect on, discuss and research the relevance, usefulness and effectiveness of mathematical assessment tasks, especially in relation to twenty-first century skills. Having a spread of stimuli and items developed and selected from across this spectrum would enable PIAAC numeracy to be better representative of the framework and construct, add to the issue of authenticity, and hence better assess adults' capabilities and competencies around numeracy practices in the 21st century.

The existing PIAAC Cycle 1 numeracy items which will continue to be used as link items tend to be at the left hand, more traditional, end of the dimension of assessment possibilities as described. Therefore, the focus on new item development has been to complement the existing pool by taking on board the potential enhancements and innovations due to 21st century capabilities as described in the dimension of assessment possibilities spectrum. This has provided the ability to create a balance across the spectrum of assessment possibilities.

Based on the above, the Numeracy Expert Group set the item writing teams the task of requesting that the new item development for Cycle 2 of the PIAAC numeracy assessment build on some of the assessment developments described above and to introduce new assessment content, representations and item formats that better reflect 21st century related digital representations, stimuli and numeracy tasks and assessment responses.

Outcomes

Fortunately, the PIAAC Cycle 2 platform and delivery system has been able to support many such types of stimuli, items and response types, and the items are in the process of being created and implemented for use on a tablet ready for the Field Trial. For PIAAC Cycle 2 these item types, responses, representations and interactive stimuli have included:

- illustrations and photos of authentic contexts/items/objects
- interactive calculators and online tools
- tap on area or multiple areas of the screen/image
- drag-and-drop responses
- drawing a graph
- use of a keypad to enter responses
- single and multiple selection multiple choice questions
- access to an online calculator
- interactive online charts and data
- online map
- online ruler
- simple spreadsheets.

Constraints and challenges to enhancing the assessment of numeracy in PIAAC Cycle 2

Despite the advances incorporated into the Cycle 2 assessment, there are constraints and challenges to what can be achieved in such an international assessment like PIAAC, and these need to be acknowledged. These include constraints related to the capabilities of adults aged 16-64 who will be undertaking the assessment, the practicalities and costs of developing an assessment across such a wide range of cultures and languages, and the limitations of the available computer platform.

First, the results from the Problem Solving in Technology-Rich Environments (PS-TRE) assessment in PIAAC Cycle 1 need to be considered in relation to the review and implementation of the PIAAC numeracy construct. The first cycle of the PIAAC survey provided two different pieces of information regarding the capacity of adults to manage information in technology-rich environments: the proportion of adults who had sufficient familiarity with computers to use them for PIAAC tasks, and the ability of adults with at least some basic ICT skills to solve the PS-TRE tasks.

The PIAAC Cycle 1 PS-TRE assessment results showed that there were adults with no, or extremely limited, ICT skills in all of the participating countries. The assessment found that:

> *From around 7% to 27% of the adult population reported having no experience in the use of computers or lacked the most elementary computer skills, such as the ability to use a mouse. In addition, there are also adults who appear to lack confidence in their ability to use computers, primarily because they use them infrequently. Of the adults undertaking the assessment, most were proficient at Level 1, which involves the use of familiar applications to solve problems that involved few steps and explicit criteria, such as sorting e-mails into pre-existing folders. (OECD, 2013, p. 98[161])*

These results from the Problem Solving in Technology-Rich Environments component of PIAAC Cycle 1 with its warnings regarding the high proportion of adults with no or extremely limited ICT skills needed to be taken into account in deciding on the balance of items incorporating the new technological and digital aspects of the PIAAC numeracy framework and its associated assessment.

Second, on a pragmatic level, some of the innovations and developments arising from technologically-driven advancements in educational measurement and assessment in the 21st century needed to be carefully reviewed and considered as to the feasibility of their use and practical implementation for PIAAC Cycle 2. Some issues that needed to be considered include:

- The costs—some technologies, media and tools would potentially be expensive to use and implement—both from a content development point of view (the production of videos/animations/etc.) and from a delivery and implementation perspective (conducting such assessments in people’s homes).
- The time available for testing—would the use of such innovations in assessment take substantially more time for the delivery of the PIAAC assessment?
- The feasibility of producing and using any animations, simulations, video or audio support in potentially 30 different languages, which would be challenging, costly and require substantial quality assurance processes of translations.
- The performance of the use of any such innovations, especially the use of simulations (e.g., the use of games is highly unlikely to be relevant in such an adult assessment at this point in time), and hence the cost of the mandatory trialling and psychometric checking for performance, reliability and validity.

The aim has been to have a pragmatic and balanced set of items that meet as many of the demands as possible for the enhancement of the numeracy assessment in PIAAC Cycle 2 while taking into account the constraints outlined above, including the ICT-related capabilities of adults across the PIAAC age-range. However, it needs to be remembered that there will always be a substantial set of link items from the previous assessments and that the Field Trial can be utilised to check how any new items work compared with existing items.

It should also be noted that the change in platform for the delivery of Cycle 2—from a laptop computer requiring the use a mouse, to a tablet where respondents can use a stylus or finger—and the fact that 10 years have elapsed since Cycle 1 suggest that more participants will be able to use the platform. The Field Trial will allow the opportunity to test whether this is the case or not.

Constraints of the assessment design, platform and certain response types

One also needs to distinguish between the conceptual framework (second section) and the assessment construct (third and fourth sections). Not all real-life numeracy tasks can necessarily be simulated well in a specific assessment. Further, the ability of an assessment to actually *capture, evaluate, and score* responses associated with the full spectrum of numeracy as defined in PIAAC ultimately depends on the technical aspects of that assessment. While the computer-based assessment platform chosen for PIAAC Cycle 2 offers many advantages over Cycle 1, there are still restrictions and limitations, and this has constrained the ability to develop many sophisticated interactive items or use audio or video that required translation, for example.

Firstly, the overall testing time per respondent does not allow inclusion of extended problems or lengthy simulations of complex authentic numeracy tasks, although it is recognised that ability to solve complex or extended numeracy problems is an inherent part of the numeracy competency. In order to cover all facets of the numeracy construct in the limited time available, the use of a larger number of short tasks is prescribed.

Secondly, the need to score all responses automatically limits the type of assessment tasks that can be used. The traditional divide in item format is between selected-response (sometimes called forced-choice or multiple choice) format versus a constructed-response format. Selected-response items require the choice of one or more responses from a number of response options. Responses to such questions can usually be automatically processed and scored when presented on a computer/tablet, and can include a range of interactive responses, such as tap-on, drag and drop, etc. Within constructed response items there are closed constructed-response or open constructed-response items. Closed constructed-response items provide a more structured setting for answering, and they aim to produce a response that can be automatically scored, against a scoring rubric, to be either correct or incorrect. Open constructed-response items require respondents to communicate in their own words the answers to tasks or questions, and such items require trained experts to manually code responses.

While the platform allows respondents to provide an answer in several different modes (e.g., numeric entry, tapping on an area of the screen, choosing from different options, etc.), in its present stage of development it cannot accept most types of open constructed-response or free-form items because of the huge possible diversity in how respondents may enter their answers. The limitations stem from the difficulty to automatically code (i.e., designate an answer as correct or incorrect) free-form responses in dozens of languages while accommodating various grammatical and syntactical structures, as well as overcoming typing mistakes which are naturally expected when people type text into a computer. Examples are when respondents:

- write number ranges or estimates which have multiple mathematically equivalent representations, such as "a quarter", "0.25", "1/4", "1 in 4", or “around five to six”, "1.00 to 6.00"
- describe their interpretation of given information such as in a simulated media statement
- write justifications for their answers, or list arguments supporting their conclusions.

Specifically, tasks requiring communication-based responses, such as when adults have to explain interpretations of given information, or describe their evaluation or analysis of a situation *or their thinking about that situation*, are difficult to implement in the direct assessment of the skills targeted by PIAAC. Such tasks do comprise an important, inseparable part of the landscape of adult numeracy situations and are an inherent part of the conceptual framework of adult numeracy, yet few could be included in the item pool for the second cycle of PIAAC.

Money/currency issues

Consumer and shopping issues are important components of numeracy and are represented in the numeracy tasks. Since the currencies of the participating countries vary greatly, consumer-related items present a challenge to item developers and translators. It is crucial to try to keep any questions relating to money at the same time realistic and mathematically comparable. Hence, PIAAC specifies strict guidelines about how countries can change the magnitude of any monetary amounts in order to maintain comparability. These are published as part of the *Translation and Adaptation Guidelines*, and will be based on current exchange rates for currencies.

PISA copes with this by having all its monetary-related items set in a fictional country, Zedland, with a fictional currency of zeds and zedcents. This approach has been deemed as not suitable to be applied to PIAAC due to its need to apply to adults across a wide age and educational range, where it is believed that some adults may not relate to a very unfamiliar currency nor to fictional prices and costs.

PIAAC’s tasks are therefore designed to allow countries whose currencies are somewhat similar in value to the dollar to keep exactly the same number and change only the currency sign.

As a general practice, when we refer to a monetary value that can be written in different currencies, we note it as _45, for example. It signifies that 45 can be considered as a number of dollars, Euro, krona, guilders, pesos, or whatever the local currency is.

The following options for making changes to monetary values are listed in order, from the least to the greatest impact on the equivalence of the cognitive demand of the item.

- Option 1: Change the currency sign only. Keep the numbers the same and change only the currency sign to the local currency sign. (e.g. change the US Dollar sign, $, to Euro, € or GBP, £) This will be the option of choice for the European Community countries, since the Euro and the Dollar are close in value.

- Option 2: Change the numbers by multiplying or dividing them by powers of 10. When changing the currency sign does not seem to work and the object's value seems unrealistic, the translator will have to change the numbers or amounts in the item.

> ***The rule:*** *If numerical values must be changed to retain the realism of an item, they can only be multiplied or divided by powers of 10 (i.e., by 10, 100, 1000, etc.). This restriction aims to keep the cognitive demands of the item (such as the nature of the mathematical steps and mental operations) similar in all countries. Consider, for example, the Raincoat item (PIAAC unit 603), priced at _80. If this price is unreasonable for any type of raincoat, the translator can choose to multiply the number 80 by 100 to be 8000, if this is a reasonable price for a raincoat in the currency of the country where this change is seen to be unavoidable. [In Hungary, for example, 1 US dollar is now equivalent to 250 HUF (Hungarian Forint). The raincoat could be priced at 8000 HUF. Yet, a change to 20000 HUF (80 x 250) is not permitted, even though it is the "true" value of the raincoat in HUF, since it significantly changes the mental operations required by the task.]*

Different measurement units

Another challenge with an international assessment such as PIAAC relates to the fact that countries can also have different measurement systems, although most countries are now solely metric, with the exception of the US, which still (mainly) use the imperial measurement system.

The following is the approach taken to creating mathematically equivalent items for metric and imperial-based measurement units. There were different solutions that could be applied, depending on how dependent the question was on understanding the measure system embedded in the context/task.

The first issue was to decide whether a conceptual understanding of the measurement system is critical to answering the question. If not, then it is possible to leave the measurement units in their original, authentic units that fitted the context. However, if a question requires "intimate" knowledge of the metric system to answer it then parallel metric and imperial units are required. For example, when an estimation of a length or height is required, as in the existing PIAAC *Tree* and *Path* questions where the respondent is expected to have sense of size in familiar contexts–the height of a tree relative to a person, and the length of a path–all shown in photographs). In these situations you could not consider using a metric unit in the US, nor an imperial unit in a metric country. People need to have a feeling for the measurement units.

Similarly, a US version of a question that included doing a conversion between metric units as part of the solution, would **not** work—knowledge of conversion factors could be very unfamiliar to an older generation American. The other point here is that the conversions between the various measurements in the US are not consistent and all based around powers of ten like in metrics, and many people routinely need to look them up – like how many ounces in a pound, inches in a foot, yards in a mile, fluid ounces in a pint, pints in a gallon and more.

A starting point was to research and see what is common and authentic in the US—the US **does** use metric units now for a number of common commodities, such as for popular soft drinks, medication doses, etc. In some cases it was therefore possible to select particular situations/objects that are measured in metric units and hence were suitable and compatible for use in questions in the US.

If a unit needed to be created in both metric and imperial, the best solution was to maintain the same dimensions and just change the units – from meters to yards, for example, or kilometers to miles. This was more easily able to occur where the units have some similarity—like cm to inches, or meters to yards. This approach has been used in link items such as the *Path* and the *Tree*.

However in some cases there was no alternative but to change the dimensions/measurements to make them both authentic. For example, this was done in the existing link items, such as the BMI formula item in ALL survey and then PIAAC. The necessity is to then keep the degree of difficulty of any arithmetical calculations as similar as possible, so the difficulty level is maintained. For example, in a possible item based on photos—in metric it was possible to have the dimensions of 8 x 12 cm, and in inches have it as 4 x 6 inches—they are authentic, similar small photo sizes and the dimensions are in the same ratio.

The PIAAC approach to assessment

The PIAAC assessment design involves using a household survey methodology which assumes that overall testing time per respondent is around 60-80 minutes. In that time, study participants will be asked to complete:

- A background questionnaire, which collects information on possible outcomes and antecedents of key skills, as well as on demographic and structural indicators that are needed to describe the distribution of such skills within participating countries.
- A tablet tutorial and orientation to the assessment.
- A short locator test, which will be used to direct respondents to the appropriate section of the direct assessment and will also provide information about the literacy and numeracy skills of adults who may not be able to continue on to the direct assessment.
- A brief measure of component skills in reading and numeracy.
- The direct assessment, where each respondent will take two of three domains of literacy, numeracy and adaptive problem solving.

The direct assessment will utilise an adaptive design that will optimise the match between respondent ability and the difficulty of administered items. Such a design provides more reliable information about respondents' skills within the available testing time.

Item pools and scale scores

The items for the assessment are expected to enable reporting of respondents' performance in a manner similar to the one used in ALL and PIAAC Cycle 1, which scaled raw scores in the range 0-500, but focused on reporting performance on six ability levels with the following (tentative) boundaries:

- Below Level 1: below 176 (lowest level)
- Level 1: raw score 176 – 225
- Level 2: raw score 226 – 275
- Level 3: raw score 276 – 325
- Level 4: raw score 326 – 375
- Level 5: raw score 376 – 500 (highest level).

Usage of calculators and other tools or objects

The assessment of numeracy, whether by paper-and-pencil tasks or computer-based, has to take into account that the practice of numeracy in everyday or work situations also involves the use of certain objects and artefacts. First is the use of calculators, either handheld or now also available on smartphones and tablets, which are now widely available to adults from all walks of life in many countries. Thus, calculators are tools which are part of the fabric of numeracy life in many cultures. Increasingly, respondents in large-scale tests are allowed, sometimes even expected, to use calculators. It follows that adults in PIAAC should be given access to a calculator as part of an assessment of numeracy skills, and they can then choose if and how to use it. An online, basic calculator will be available on the tablet-based delivery of PIAAC, and as well a handheld basic calculator will be made available and can be used if requested. There are no numeracy questions that require the use of a more sophisticated type of calculator than a basic, four function calculator.

The use of an online ruler is also presented in one unit, in both a metric and imperial (inches) system, as rulers/measuring instruments are part of contexts where adult numeracy competence is manifested. The use of other technologies, such as a computer spreadsheet, also fit the assessment of numeracy, and in PIAAC Cycle 2 some items assess this skill.

It should also be noted that it is intended that the interviewer will provide access to a pen and paper in order for respondents to make notes, write down and undertake calculations, etc.

Basis for assessing numeracy in PIAAC Cycle 2

The development of the numeracy assessment for PIAAC has been based on a number of general principles or guidelines listed below. These principles reflect the cumulative literature on large-scale assessment of mathematical skills and adult numeracy (Gal et al., 2005[28]; Gillespie, 2004[187]; Murat, 2005[188]), and various background documents and positions prepared as part of the planning of PIAAC [e.g., (Gal, 2006[153]; Gal and Tout, 2014[3]; Jones, 2006[154]; Murray, 2006[155]; PIAAC Numeracy Expert Group, 2009[2]; Tout, 2006[156]; Tout et al., 2017[1])]. This also incorporates the general ideas discussed earlier, as well as any known technical limitations in the delivery of PIAAC Cycle 2.

Some general approaches include:

- *Items should cover as many aspects as possible within each of the four core dimensions of the numeracy definition and elaboration*. Items should require the activation of a broad range of skills and knowledge included in the construct of numeracy, as portrayed in the conceptual framework depicted in Box 3.2. The specifications and targets for the item development and spread against each of the core dimensions are spelt out in the next sub-section.
- *Items should aspire to maximal authenticity and cultural appropriateness*. Tasks should be derived from real-life stimuli and pertain to the full range of contexts or situations (i.e., everyday life, work, societal) that can be expected to be of importance or relevant in the countries participating in PIAAC. Item content and questions should appear purposeful to respondents across cultures, although it must be acknowledged that in a large-scale assessment such as PIAAC, not all items and contexts can be personally familiar to all adults within any one country, let alone across all countries.
- *Items should have different response formats*, to the extent feasible by the computer platform used for administering the direct assessments in PIAAC. Items should be structured to include a stimulus (e.g., a picture, drawing, visual display) and one or more questions, the answers to which the respondent communicates via the modes available within the platform. Numeric entry is limited to the set of 10 digits and common separators (, and .) or other mathematical symbols where relevant.
- *Items should spread over different levels of ability*. Items should span the range of ability levels anticipated within PIAAC participants, from low-skilled individuals (who are of particular interest in countries where policies and educational programmes may be earmarked for low-skill populations), all the way to those with advanced competencies. The ALL and PIAAC Complexity Schema (Tout et al., 2020[141]) was used to provide an initial estimate of the spread of item difficulties in order to assist in the selection of the items for the Field Trial.
- *Items should vary in the degree to which the task is embedded in text*. Some items should be embedded in or include relatively rich texts, while others should use little or no text. This distribution aims to reflect the different levels of text involvement in real-world numeracy tasks, as well as reduce overlap with the literacy scale.
- *Items should be efficient*. To allow for coverage of many key facets of the numeracy competency, the inclusion of a large number of diverse stimuli and questions will be needed. However, in light of testing time constraints, the use of short tasks is necessitated, precluding items that can simulate extended problem-solving processes or that require a lengthy open-ended response.

- *Items should be adaptable to unit systems across participating countries*. Items should be designed so that their underlying mathematical demands are as consistent as possible across countries, regarding language and mathematical conventions. For example, items should be designed so that different currency systems or different systems of measurement (metric or imperial) could be applied to the numbers or figures used. Items should retain equivalency with respect to their mathematical or cognitive demands after being translated.

Blueprint for assessing numeracy in PIAAC Cycle 2

Based on the definition and elaboration of numeracy described in previous sections and on the above discussions on assessment enhancements and constraints attached to delivering PIAAC Cycle 2, this sub-section specifies the blueprint for the proportions of items against each of the core dimensions in the construct. For a comparison, the specifications for each dimension is compared with the previous targets for Cycle 1 and for PISA 2012 and 2021 too.

This blueprint specifies the test content for the direct assessment of the cognitive aspects of numeracy as defined for PIAAC Cycle 2, taking on board the above constraints and limitations, and the enhanced opportunities provided for Cycle 2 compared with Cycle 1 of PIAAC. These help establish the construct and content validity requirements for the cognitive assessment aspects of numeracy, with this being confirmed and refined through the quality assurance (QA) processes and psychometric item analysis and review following the Field Trial. The QA processes include feedback from participating countries and a formal translation and review process with language experts. These QA processes pick up issues to do with the language structure and meaning of items, and also content and cultural issues. A Field Trial is undertaken with a sample of the target population in each participating country before the final assessment is implemented. Field trial data is collected and analysed psychometrically and from these detailed analyses, ‘misbehaving’ items are rejected on a number of levels including for reliability, fairness and validity. Then for the remaining successful items, any fine-tuning is undertaken, and a representative set of items are chosen based on the blueprint and placed into final forms.

This blueprint is specified against the four dimensions incorporated into the definition of numeracy and the elaboration of numerate capacity, as outlined earlier:

> *Numeracy is **accessing, using and reasoning critically** with **mathematical content**, information and ideas **represented in multiple ways** in order to engage in and manage the mathematical demands of a **range of situations** in adult life.*

The four core dimensions named and described in numeracy for PIAAC Cycle 2, namely:

- cognitive processes
- content
- representations
- context.

Cognitive processes

For the three new *Cognitive processes* the spread across the dimension is very similar to PISA 2012 processes which have a similar structure, but quite different from PIAAC Cycle 1 due to the revised and different structure of this response classification. There is also an attempt to be less focused on the traditional process of doing the mathematics (*Act on and use mathematics*), and have a good representation in the two other processes too, which are seen by the Numeracy Expert Group as significant aspects of how adults engage with and solve a numeracy problem where mathematics is embedded in an authentic situation. This target might be difficult to achieve, but these were the aspirational targets set by the NEG.

Table 3.5. Representation of cognitive processes in PIAAC and PISA

PIAAC Cycle 1	PIAAC Cycle 2	PISA 2012	PISA 2021
Identify/locate/access (10%)	Access and assess situations mathematically (25-35%)	Formulating situations mathematically (25%)	Formulating situations mathematically (25%)
Act on/use (order, count, estimate, compute, measure, model) (50%)	Act on and use mathematics (30-40%)	Employing mathematical concepts, facts, procedures, and reasoning (50%)	Employing mathematical concepts, facts, procedures, and reasoning (25%)
Interpret/evaluate/communicate (40%)	Evaluate, critically reflect, make judgements (25-35%)	Interpreting, applying and evaluating mathematical outcomes (25%)	Interpreting, applying and evaluating mathematical outcomes (25%)
			Mathematical reasoning (25%)

Content

For the four content areas the spread across the dimension was similar to Cycle 1 and similar to PISA. One difference is that PIAAC does not aim to have as many items in the more formal mathematics area of *Change and relationships*, which includes algebraic thinking, which is more of an interest to 15-year-olds in a school-based assessment such as PISA. For PIAAC Cycle 2 there has been a slightly higher focus placed on *Data and chance*. This is seen as a more common and important area that adults now have to negotiate with in the 21st century, and it is higher use and reliance on presenting numerical, quantitative and other data and related analyses in a range of ways and often in ways that are critical to people's lives.

Table 3.6. Representation of content areas in PIAAC and PISA

PIAAC Cycle 1	PIAAC Cycle 2	PISA 2012 and 2021
Quantity and number (30%)	Quantity and number (20-30%)	Quantity (25%)
Dimension and shape (25%)	Space and shape (20-30%)	Space and shape (25%)
Pattern, relationships and change (20%)	Change and relationships (15-25%)	Change and relationships (25%)
Data and chance (25%)	Data and chance (25-35%)	Uncertainty and data (25%)

Representations

For the four new *Representation* classifications, again the NEG was aspirational in its targets and aimed to set relatively high goals for 21st century type representations, which are covered under both *Structured information* types of materials (infographics etc.) and also *Dynamic applications* which includes online interactive websites and applications alongside more standard software applications and tools. This will in the end be balanced by the existing link items from PIAAC Cycle 1 and ALL, where the representation may be more traditional and less 21st century in their style and format. It should be noted that in the previous cycle, the type of representations was not explicitly monitored in terms of its proportion across the pool of items, and it is felt that incorporating representation into the definition and the dimensions will enhance the quality of items across PIAAC numeracy.

Table 3.7. The representation classifications in PIAAC

PIAAC Cycle 1	PIAAC Cycle 2
Objects and pictures (not specified)	Text or symbols (15-25%)
Numbers and mathematical symbols (not specified)	Images of physical objects (15-25%)
Formulae (not specified)	Structured information (35-45%)
Diagrams and maps, graphs, tables (not specified)	Dynamic applications (15-25%)
Texts (not specified)	
Technology-based displays (not specified)	

Context

For the three remaining contexts in PIAAC Cycle 2, the aim is to have an equal spread as per previous cycles, as also occurs in PISA.

Table 3.8. Representation of context in PIAAC and PISA

PIAAC Cycle 1	PIAAC Cycle 2	PISA 2012 and 2021
Everyday life (25%)	Personal (30-35%)	Personal (25%)
Work-related (25%)	Work (30-35%)	Occupational (25%)
Societal or community (25%)	Societal/community (30-35%)	Societal (25%)
Further learning (25%)		Scientific (25%)

Factors explaining item/task complexity

In planning an assessment, it is of course important to be able to understand what it measures. Assessment designers assume that when engaged with the assessment items (including tasks, questions, stimuli, etc.), respondents activate cognitive processes and rely on stored knowledge and learned skills which are part of the construct being measured. Thus, differential performance levels can be accounted for by the underlying cognitive knowledge bases and other enabling processes. It follows that it is useful to have a theoretical model or set of assumptions regarding what factors cause certain tasks to be harder or more complex than others, so that the assessment results can be correctly interpreted. A model or scheme of factors affecting task complexity can also help when linking the assessment results to possible social (or educational) interventions, i.e., point to the skills that are lacking and have to be further developed in the population (Brooks, Heath and Pollard, 2005[189]).

Prior seminal work by Kirsch and Mosenthal [e.g., (Kirsch and Mosenthal, 1990[190]; Kirsch, Jungblut and Mosenthal, 1998[191])] and earlier projects have pointed to several key factors which account for task difficulty when considering arithmetic items or items involving text comprehension. These include readability, type of match, plausibility of distractors, operation specificity ('transparency'), and type of calculation and number of steps. The Kirsch and Mosenthal work has informed the design of assessment tasks for IALS and other surveys, and the interpretation of their results. In designing the ALL numeracy scale, the ALL Numeracy team attempted to advance the Kirsch and Mosenthal complexity scheme and develop tentative assumptions regarding factors which affect difficulty of multiple types of new tasks introduced to measure the numeracy construct which were beyond those encompassed by the more focused construct of *quantitative literacy* in IALS. Examples are items involving percents, knowledge of measurement and spatial reasoning, statistical concepts, and so forth.

The developers of the *mathematical literacy* scale for PISA (2006) also recognised multiple factors affecting item difficulty, such as the kind and degree of interpretation and reflection required by the problem, the kind of representation skills required, or the kind and level of mathematical skill required, e.g., single-step vs. multi-step problems, or more advanced mathematical knowledge, complex decision-making, and problem solving and modelling skills, or the kind and degree of mathematical argumentation required. Further factors that are assumed to affect difficulty both in PISA, ALL and other surveys relate to the degree of familiarity with the context, and the extent to which tasks require reproduction of known procedures and steps or present novel situations requiring non-routine and perhaps more creative responses. It should be noted that the PISA description of complexity factors seems quite compatible with that of ALL, although some of the terminology is different, and published PISA reports do not explain in detail how it was used to guide the design of specific items.

The complexity scheme for numeracy used in ALL (Gal et al., 2005[28]) has been instrumental for the item development and scale construction stages of that study, especially in that it helped to evaluate in advance if items will span different difficulty levels. Given that PIAAC's numeracy assessment is founded on the principles developed for ALL and that the PIAAC numeracy assessment scale uses linking items used in ALL, the ALL complexity scheme has been adopted and updated as an analytic tool for item development and interpretation for PIAAC as well. The details about this updated PIAAC numeracy complexity schema are provided in Tout et al. (2020[141]).

Relationship between PIAAC and PISA

This section discusses the relationship between the PIAAC numeracy framework and the PISA mathematical literacy framework and assessment. Many of these aspects have been discussed earlier when considering the framework construct and its parameters, and the assessment blueprint. This section summarises the similarities and differences.

Note: in this section the references used are as those written and documented in the original, full PIAAC Cycle 1 numeracy framework (PIAAC Numeracy Expert Group, 2009[2]). Similarly, the references to PISA are mainly to the PISA 2012 mathematical literacy framework and its descriptions (OECD, 2013[122]). This is because in 2012, mathematical literacy was the major domain for PISA, when the relevant framework was revised and updated. The same mathematical literacy framework and assessment construct was used for the next two cycles of PISA in 2015 and 2018. For PISA 2021, mathematical literacy is again the major domain for PISA, and hence the framework and assessment construct is being updated and revised. This was happening in parallel with the development of the numeracy framework for PIAAC Cycle 2. The PIAAC Numeracy Expert Group was able to access a copy of the second draft of the PISA 2021 mathematics framework (OECD, 2018[123]) in November 2018. It was prepared by the Expert group for mathematics under the guidance of RTI International as the international contractor who led this work for the OECD. Most of the comparisons between PIAAC numeracy and PISA mathematical literacy have therefore been based on a comparison of the 2012 PISA framework and descriptions, but where possible the PIAAC NEG has also included comments and comparisons with the updated 2021 PISA mathematical literacy framework.

The commonalities between PIAAC and PISA

The following sub-sections look at the commonalities and links between PISA and PIAAC across the features and parameters of PIAAC.

Mathematical content

Both numeracy in PIAAC and mathematical literacy in PISA use a non-school-curriculum focused approach to naming and describing the content areas covered in their assessments. The purpose behind both frameworks is describing mathematics for use and application outside of the classroom, and so the organisational structure for mathematical content knowledge is based on how mathematical phenomena are encountered in situations in the outside world. While the PISA and PIAAC frameworks were developed by independent teams, they use very similar descriptors for their content classifications, introducing and describing these in terms of the *Big Ideas* behind mathematics.

The two frameworks are highly consistent in terms of their descriptions and structures of the mathematical content covered in their assessments. There are very similar spreads across each content area. As discussed earlier, PIAAC has less interest in the more formal mathematics area of *Change and relationships*, which is more of an interest to 15-year-olds in a school-based assessment such as PISA.

Contexts

The sets of context descriptors used in both frameworks regarding the first three contexts (*Personal*; *Work/Occupational*; *Societal* or *Societal/community*) are highly consistent with each other, with a similar spread of items.

The NEG reviewed the fourth context, *Further learning* in PIAAC Cycle 1 and after comparing it with *Scientific* in PISA, decided that it was best to remove the classification named *Further learning* which in PISA incorporated items that are considered intra-mathematical. The need to have such items in PIAAC that were about knowing about the more formal aspects of mathematics, including the conventions used to apply mathematical rules and principles, are to be covered through the inclusion of those requirements through the content knowledge area of *Change and relationship*s.

The difference here with the PISA context classifications reflects the different interests in the more formal mathematical understandings of 15-year-olds within a school setting with those of adults out-of-school. Any other differences in the item coverage here are due to the age of the two target groups, with some of the PIAAC situations described being more relevant to adults, and some of the PISA situations being more appropriate for 15-year-olds.

Responses/actions

This facet of the original PIAAC Cycle 1 numeracy structure was an aspect of the numeracy framework that was recommended as needing major review by the 2017 review paper, which recommended that PIAAC could potentially learn from the processes described in the PISA 2012 mathematical literacy framework. Hence for the three new *Cognitive processes* developed for PIAAC Cycle 2 there is a significant amount of similarity and consistency with the PISA processes of *Formulate, Employ and Interpret/Evaluate*. This is because the NEG took on board the intent and structure for the three PISA 2012 process cycles in their development of the new cognitive process dimension for PIAAC Cycle 2. In relation to the new mathematical reasoning process included in PISA 2021, it is the view of the NEG, that for the assessment of numeracy skills of adults the mainly intra-mathematical aspect of mathematical reasoning needs to be embedded within the real-world problem solving aspect for PIAAC, and not assessed as a separate part of the construct. Therefore mathematical reasoning and its understanding and application is integrated across the relevant aspects of the three other cognitive processes.

Item formats

In their review and comparison of the two numeracy frameworks for PISA and PIAAC, Gal and Tout (2014[3]) concluded in relation to the issue of item formats that:

> *PISA 2012, with its more comprehensive range of item types and more interactive computer-based assessment, will enable richer and extended descriptions of sub-components of mathematical literacy compared to the information that can be generated by the numeracy assessment in PIAAC. (Gal and Tout, 2014, p. 52[3])*

Furthermore, the review paper commented that much of the existing PIAAC Cycle 1 and ALL item pool was based around static images and was more like paper-based assessments transferred onto a computer, and that this also does not now seem to reflect the way numeracy tasks and actions are now situated and practised in the 21st century.

As stated earlier, the next delivery of the PIAAC Cycle 2 numeracy assessment is much more capable than Cycle 1 was in terms of allowing the use of new and more interactive, 21st century style item formats. Such items have been developed and will be trialled in the Field Trial.

General comments

Based on detailed comparisons of the two numeracy frameworks for PISA and PIAAC by the review team, by Gal and Tout (2014[3]), and from two of the 2017 review team who were familiar with both the full sets of PISA 2012 and PIAAC items and not just the publicly released items, it is apparent that both assessments describe and cover very similar territories.

On the conceptual level, numeracy and mathematical literacy are closely related constructs in terms of their core, underlying ideas. In relation to the definitions and descriptions of the constructs and what they are assessing, Gal and Tout, in their comparison of the two programmes, summarised the similarities:

> *Both constructs refer to the ability of individuals to cope with tasks that are likely to appear in the real world, and that contain mathematical or quantitative information, or that require mathematical or statistical skills and knowhow.*
>
> *Both constructs focus on how well individuals can use their mathematical knowledge and skill to solve problems stemming from pragmatic (i.e., real-world) needs or demands, and to 'engage', manage, and understand various tasks in the world around them—rather than addressing decontextualised mathematical tasks.*
>
> *Both PISA and PIAAC describe mathematical literacy or numeracy as not synonymous with a minimal or low level of mathematical knowledge and skills. That is, both assessments view the constructs as describing competencies lying on a continuum, i.e., individuals could be placed on a scale from low levels to high levels. (Gal and Tout, 2014, pp. 47-48[3])*

They concluded that both the PIAAC and PISA frameworks, definitions and assessments have substantial conceptual similarities and also practical commonalities in their test items and design principles, as well as in the range of content areas and skills they cover (Gal and Tout, 2014[3]). However, there are some differences between the two assessments, related to the diversity in the experiential backgrounds and the distances from schooling for adults compared to children. As Gal and Tout wrote:

> *Because many adults may not remember more formal school-based representations or technical language, the design of PIAAC items has taken into account from the outset the need to establish authenticity while reducing the use of formal notations and 'school-like' appearance. (Gal and Tout, 2014, p. 39[3])*

An examination of the item sets of both PISA and PIAAC shows that PISA is more interested in the ability of 15-year-olds to use and apply curriculum-based mathematical skills and knowledge embedded in a real-world situation. On the other hand, PIAAC is somewhat less focused on how respondents use formal mathematical skills when solving a real-life-type mathematical problem. For example, in some of the PISA mathematical literacy items 15-year-olds are asked to use information from a real-life situation to calculate and identify specific formal characteristics of linear equation graphs, such as the gradient and the y-intercept. This type of more formal mathematical knowledge is not assessed in PIAAC, as generally PIAAC respondents are not required to show evidence of their knowledge of the use and understanding of formal school-based mathematical notations, which are often forgotten from not having been in current or recent use.

Drivers/indicators of mathematical proficiency

One of the important features of both the PISA and PIAAC frameworks is the way that each has independently developed a schema that describes aspects of test items that drive item difficulty, and which indicate the mathematical proficiency of tested individuals and populations.

PIAAC does this in considerable detail in the Appendix to the framework (PIAAC Numeracy Expert Group, 2009, pp. 44-56[2]). As well as classifying test items according to the mathematical content knowledge required to complete each item, the Annex presents a detailed scheme designed to show the complexity of test items. Five 'complexity factors' are defined, and a scheme is presented for rating mathematical tasks according to the extent each factor is present in the test items. Examples are given to show how the rating scheme would be applied, and the assumption is that a total score across the factors for an individual item (20 score points could be generated) would be strongly related to item difficulty; and by implication, successful completion of particular items can be used as indicators of levels of mathematical proficiency.

The PISA 2012 framework specifies a set of ‘fundamental mathematical capabilities’, the activation of which is assumed to collectively provide indicators of mathematical literacy. A scheme designed to rate individual items based on the extent to which each of those capabilities is needed to respond to PISA questions has been developed as part of research activities documented by Turner, Blum and Niss (2015[192]). This research has shown that the scheme predicts the difficulty of PISA test items. Evidence of activation of the capabilities is fundamental to PISA’s descriptions of growing mathematical literacy.

Alignment of the two scales

The 2017 review team agreed that given the general consistency in what is being assessed by both PIAAC and PISA, and based on their review and knowledge of the two frameworks and item pools, there could be much to gain from making significant and more explicit connections and links between the PIAAC numeracy framework and the PISA mathematical literacy framework and assessment. The best way to do this would be by establishing an empirical relationship between the two scales through a mapping/linking study where adults and 15-year-olds would sit common items from across both assessments. This would make analysis and comparison across the two assessments stronger and more useful for research purposes, both within countries and internationally. It would also enable research into how items are approached differently by those in school versus those that are not, and support provision of stronger data to enable research into progress from school into adult life.

Numeracy components

The implementation of an equivalent to the PIAAC reading components assessment in PIAAC’s numeracy assessment is outlined in this section.

Introduction

The second cycle of PIAAC will include a new set of low-level items, called numeracy components, that are aiming to shed more light on the numeracy competencies of low-scoring adults (below Level 1). In analysing the first cycle of PIAAC results it was felt that information was missing to make valid inferences on what numeracy skills adults below Level 1 possessed or lacked. It should be noted that the NEG and the 2017 review recognised that there were two solutions to achieve this. The first was to write some new easier items to complement the existing three below Level 1 items, along with this second solution of developing the numeracy components.

In the review of the PIAAC Cycle 1 numeracy framework, possible building blocks for the components were investigated and arising issues and constraints were discussed (Tout et al., 2017[1]).

During the development of this numeracy framework for PIAAC Cycle 2 and the design of the second cycle numeracy items, a further investigation was undertaken to establish which kind of numeracy assessment items would be suitable for assessing some of the identified numeracy components, given the constraints of the delivery modalities.

Reading component skills

In its first cycle, PIAAC included for the first time an assessment of ‘reading component skills’ often abbreviated to ‘reading components’ to evaluate how well individuals with low levels of proficiency master the basic building blocks of reading (Sabatini and Bruce, 2009[193]). Three types of tasks were included in Cycle 1:[2]

- Print vocabulary, where respondents were asked to identify which one of four words matched a picture.
- Sentence processing, where a single sentence was presented and respondents were asked to identify if it made sense or not by selecting "Yes" or "No".
- Paragraph comprehension, which were cloze tasks where respondents selected one of several words to make sentences within a paragraph make sense.

The delivery of the reading components assessment in the first cycle of PIAAC included a level of oral support by the test interviewer, and this is under consideration for Cycle 2. Both accuracy and timing information were captured for these items, allowing the analysis of both skill and fluency.

In Cycle 2, the reading components measure will include sentence and paragraph comprehension items and will be administered on the tablet. This will make an automated presentation of items possible, allowing respondents to better demonstrate fluency, and will also allow the collection of comparable timing information.

Rationale for the numeracy components assessment

The overall performance in PIAAC Cycle 1 showed that 5% of adults surveyed in the first round of 24 countries were at below Level 1. When including the second round of countries, the results showed 6.7% were at below Level 1 across 33 countries. This compares with a performance in reading literacy of 3.3% across the original 24 countries, and 4.5% for the second round of 33 countries. Hence, the percentages of adults performing at the lowest level in numeracy are significantly higher when compared with literacy (OECD, 2013[161]; OECD, 2016[194]). Therefore, there is a very strong argument from the empirical data for developing an equivalent to the reading components assessment in the PIAAC numeracy assessment based on the higher numbers of adults performing at that level compared with reading.

Numeracy component skills – conceptual issues

The purpose of defining, constructing, and administering items for a numeracy components assessment have the same aims: to develop a set of "fine-grained tasks" so that "at least some of these adults would demonstrate some level of **numeracy** knowledge and skills".

In numeracy, such component skills for adults have been much less researched, theorised, and examined quantitatively compared with component skills for literacy [e.g., see (Grotlüschen et al., 2016[195]; Sabatini and Bruce, 2009[193])]. Therefore, conceptualising and developing the numeracy components assessment in the second cycle of PIAAC was a challenging task. There was recognition that much more research and discussion needed to be undertaken to establish the sensible and meaningful content of such numeracy skills component for adults, the scope of those skills, and how they relate to the existing PIAAC below Level 1 items and their descriptions. However, time constraints related to the need to develop the test items within 6 months of the NEG's first meeting in March 2018, meant that the NEG had to proceed in the best way it could.

The NEG was aware, however, that it had a unique opportunity to create an assessment that had not been developed or administered before and that would potentially provide valuable research data and insights into adults with low levels of numeracy and mathematical skills. The opportunity to utilise the PIAAC Field Trial to test how such a numeracy components assessment would work was taken on board and work proceeded to research what would work best and to develop some trial items.

A range of potential sources of content were investigated by the review team (Tout et al., 2017[1]) and consequently by the NEG, but their research time was limited. In the following years, it is the NEG's recommendation this issue should be researched and trialled more thoroughly.

Numeracy component skills - prerequisites or fundamentals?

The reading components are described as fine-grained foundational reading skills which precede more complex reading skills. In numeracy, such fine-grained foundation skills are not yet clearly defined. The numeracy development of individuals starts 'in the crib', where newborns have their first experiences with numbers, shapes and sizes of objects and spatial orientation. The exact nature of these numeracy and mathematical foundation skills is still under-researched.

To complicate things further, in research literature, the term 'components of numeracy' is also used for the fundamental elements which constitute the concept of numeracy. This is a different perspective from thinking of them as the foundations or prerequisites for the development of more complex numerate skills. For instance, Ginsburg, Manly and Schmitt (2006[29]) did a comprehensive investigation in existing numeracy frameworks to discern any reoccurring aspects in a range of existing numeracy frameworks. They labelled these elements as the 'components of numeracy' and called them "those fundamental elements that are inherent in proficient numeracy practice" (p. 2). They listed the components as: *Content*, *Context* and *Cognitive and affective*. This is clearly another definition of a 'component of numeracy' compared with the perspective of describing and defining some assessable foundational aspects of low-level adult numeracy skills.

Another clarification is also necessary. Components of numeracy are not, as sometimes assumed in laymen's opinions, the "basics"—knowing by rote the arithmetical operations like addition and subtraction up to 20 and multiplication and division with 1-digit numbers. The NEG views these as basic arithmetical facts in the domain of operations with decontextualised numbers, which only covers a minute part of the PIAAC content dimension. These "basic" skills are in no way basic or elementary to many of the low level performing adults in PIAAC, as these skills make use of abstract, school-based notations and conventions and lacks the key dimension of "meaningfulness", which is an essential fundamental pillar of the numeracy framework as a whole.

So, there are two major challenges to consider in the development of a numeracy components assessment for the second cycle of PIAAC. One is the breadth and the level of the mathematical content that should be included as some of the foundational skills. Second is how the meaningfulness of the items could be maintained, for instance whether the use of real-world problems embedded in authentic situations is feasible in an assessment of the components of numeracy, or at least what considerations need to be accommodated in order for the components assessment to work and be relevant to the adult respondents undertaking the assessment.

Delivery and other constraints

Another major challenge for the NEG is that the items must fit within the delivery options of the whole PIAAC assessment, including issues to do with the time available and the uncertainty about the level of oral support available for respondents undertaking the Components Assessment.

The constraints imposed by the practicalities of delivering such an assessment internationally in multiple languages need to be considered, and will impact on what can be achieved. Furthermore, given the likelihood of an interaction between low numeracy and low literacy skill levels, delivery of a numeracy components assessment should take special account of the reading demands of the assessment. Other factors to consider include: the time available, which will impact on the number and range of items that can be utilised in terms of content areas and difficulty levels; the delivery and item types (oral instructions and support by administrator; online delivery; interactive or not) and more. A number of these factors are discussed below.

The NEG therefore believes that this second cycle of the PIAAC numeracy framework is only taking the first steps into gaining insights into the nature of low-level adult numerate behaviour and performance. However, these are important and very valuable first steps.

Representations and reading demands

It will be essential to make the reading demands as minimal as possible for this assessment of numeracy components, while maintaining the connection to real life. The review team (Tout et al., 2017[1]) suggested to offer oral/spoken support in some form or other, either from the administrator, or if conducted on a laptop or tablet, through audio or video support. Consideration could also be given to the administrator recording oral answers for the respondent.

Another recommendation is that the stimuli should be based on photos or videos of realistic representations of real-life objects, which would help to make them accessible, more familiar and more realistic and authentic, while potentially helping to reduce reading load. Another suggestion was to use real items or objects for some test items. These could be used for tasks such as comparing, sorting, or classifying. This would make the numeracy components assessment more accessible, practical and hands-on. Additionally, or alternatively, technology could be used so that similar actions could be done on screen, such as using drag-and-drop items on a laptop or tablet using touch screen capabilities. For example, respondents could be asked to order objects representing quantities by dragging and dropping rather than writing down an ordered list.

The conclusion of the NEG is that the use of the tablet allows the use of photos and realistic representations of real-life objects, which can help make them accessible and more familiar. The existing below Level 1 item on counting the number of bottles is a good example of how this can be done using a photo and little text; in fact, even without being able to read the question text it is highly likely that respondents would be able to assume what the question was asking. If some level of oral guidance or support was also made available, then this would make such questions even more accessible.

Time

Given the overall time constraints for the assessment, the NEG was informed that there would be a restriction in time to a maximum of approximately 3 minutes for the duration of the numeracy components assessment, as is the case for the reading components assessment. This obviously presents significant restrictions on what can be included in the components. However, it needs to be noted that this restriction in time will not be revealed to the participants to avoid unwarranted stress and failure anxiety. The NEG has argued that for the Field Trial, more time be allowed (up to a total of 5 minutes) in order to trial and test the new items. This then will provide the necessary empirical data and information for a more informed decision about how to best implement the numeracy components assessment in the Main Study.

The time restriction for the delivery means that after that time has expired the set of items will be terminated and no more new items will disappear. The respondent will not be told how many items are to be presented—they will continue until either all the items have been presented, or until the time limit expires. The interviewers will be instructed to ask the participant to keep working on during the set.

Oral instructions and support

As mentioned above, the Review paper suggested to offer oral/spoken support in some form or other, either from the interviewer, or if conducted on a laptop or tablet, through audio or video support. However, the constraints on the delivery of PIAAC and the need for approved translations of any spoken texts in any video or audio files, made it unfeasible to build in any oral directions into the assessment itself. However, as mentioned earlier, consideration is being given to the possibility of some level of oral support or instructions by the test interviewer.

Hence, it was very important to address the issue of minimising the use of text, and the best way to do this was through the use of photos and realistic representations of real-life objects, which can help make items more accessible and more familiar.

Using money

One other issue is that it would seem obvious that some of these numeracy components assessment items should be based around recognising and working with money, which appears to have the advantage of being a) number-based, and b) important in most adults' lives, and also relatively familiar. Money is, however, highly country-dependent: its very familiarity is grounded in its localisation in a particular set of relationships, financial and otherwise, and these are not necessarily consistent across countries. Monetary systems across participating countries vary significantly, and although PIAAC specifies strict guidelines about changing the magnitude of monetary amounts in order to try to keep them at the same time realistic and mathematically comparable, at this lowest level of mathematical complexity this may be difficult to achieve. The NEG believes that the number sense construct underpins an understanding of currency and working with money.

Item formats

Based on the delivery constraints for the components, some of the recommended item type options which were seen to best support a numeracy components assessment, would include:

- use of photos and realistic representations of real-life objects
- minimise the need to read written instructions – use a simple, single stem to introduce the sets of questions/items (note that this approach will also contribute to the fluency measure as respondents will not need to spend time reading changing item instructions)
- not expecting any written responses – so use "tap-on" style of responses.

Numeracy component skills – possible content

As with the PIAAC reading components assessment, the aim is to better understand the numeracy and mathematical skills of adults scoring below Level 1. These will be the individuals who in previous surveys essentially could not answer any, or many, of the numeracy items correctly.

The current, lowest level in PIAAC is below Level 1, and the description of this level of numeracy performance in PIAAC is:

> *Tasks at this level require the respondents to carry out simple processes such as counting, sorting, performing basic arithmetic operations with whole numbers or money, or recognising common spatial representations in concrete, familiar contexts where the mathematical content is explicit with little or no text or distractors. (OECD, 2013, p. 76[161])*

The existing three below Level 1 PIAAC numeracy items are:

- the counting or estimating of the number of objects shown in a photo where the objects are in layers and therefore not all visible (total is under 100)
- adding up three whole numbers listed in a short text (total is just over 200)
- identifying the item that was packed first from four supermarket price tags, each of which includes the date packed.

Therefore, the skills that need to be assessed in a numeracy components assessment would preferably need to be at a lower level than those three questions.

In a recent review of options for developing a low level assessment of numeracy for adults in low- and middle-income countries (UNESCO, 2016[196]), the authors said:

> *It is therefore necessary to distinguish between people with no formal skills (those who have relatively few mental calculation skills beyond counting simple quantities and who cannot understand the meaning of written digits) and with low formal skills (those who can engage in some mental calculations using indigenous number systems or measurement techniques but know few print-based or formal numeracy symbols and systems, even if they may be able to complete very simply written math problems). (UNESCO, 2016, p. 284[196])*

These issues are at the heart of the development of a numeracy components assessment.

National and international frameworks

There are existing adult numeracy standards and frameworks in different countries that have described relatively low levels of numeracy competence, and these could be used as starting points for descriptions of possible numeracy components questions and tasks. One challenge is that many such frameworks, as with PIAAC numeracy Cycle 1, do not detail or describe a level below PIAAC's existing below Level 1.

What is common at the lowest levels of existing adult numeracy frameworks is that they describe mathematical content across a number of content areas, as with PIAAC's four content areas of *Quantity and number*; *Dimension and shape*; *Change and relationships*; and *Data and chance*.

For example, Ireland has five areas described: *Quantity and number*; *Data handling; pattern and relationship*; *Problem solving*; and *Shape and space* (Quality and Qualifications Ireland (QQI), 2016[78]). New Zealand has three areas described: *Make sense of number to solve problems*; *Reason statistically*; and *Measure and interpret shape and space* (Tertiary Education Commission, 2008[79]). The Netherlands has described an entrance level for adults around four domains: *Numbers*, *Proportions*, *Measurement and geometry*, and *Relations*, stressing the concrete nature of the content with a few data, a minimum of text, rounded numbers and problems taken directly from everyday life and the work environment (Centre for Innovation of Education and Training (CINOP), 2013[197]).

As examples of what is described at the lower levels approximating below Level 1 of PIAAC or lower, Box 3.3 below includes some sample statements from a number of different national **adult** curriculum frameworks/standards, organised against the PIAAC content areas [excerpts from: Quality and Qualifications Ireland (QQI), (2016[78]); McLean et al. (2012[77]); Tertiary Education Commission, (2008[79])].

Box 3.3. Sample statements from national adult curriculum frameworks/standards, organised against the PIAAC content areas

Quantity and number

- Recognise the relationship between numerical value and groups of objects, up to and including 10.
- Recognise the language of mathematics in everyday situations using elementary language, e.g., greater than, less than, bigger than, farther than.
- Solve addition and subtraction problems by counting all of the objects.
- Solve addition and subtraction problems by counting on or counting back, using ones and tens.
- Solve multiplication problems by skip-counting, often in conjunction with one-to-one counting and often keeping track of the repeated counts by using materials (e.g., fingers) or mental images.
- Read and write personally relevant numbers, e.g., street number.

- Recognise and write money as symbols (e.g., $12.50) up to $100.
- Recognise and use ordinal numbers from first to tenth.

Dimension and shape

- Identify key characteristics of shapes and forms, e.g., number of sides, corners and curves.
- Use the language of measurement in relation to shape and form, e.g., longer, shorter, wider, narrower.
- Sort and describe objects by their shape attributes.
- Describe, name and interpret relative positions in space.
- Compare and order objects directly, using attributes of length, area, volume and capacity, weight, angle, temperature and time intervals in order to understand the attributes.
- Read digital time (not including concept of am/pm).
- Identify dates in a calendar.
- Recognise common time sequences; e.g., the order of the days of the week.
- Identifies differences and similarities between common 2 dimensional (2D) shapes.

Pattern, relationships and change

- Make a pattern; e.g., a sequence of images, symbols or sounds with two variables (different colour, same shape, etc.).
- Data and chance.
- Identify the use of data in everyday life; e.g., the numbers of people who want tea/coffee.
- Sort objects according to their attributes, organise data about the objects and represent data, using concrete objects or pictures.
- Identify all possible outcomes in situations involving simple (single-stage) chance.
- Compare information and data within highly familiar simple texts, lists, charts, diagrams and tables.

In the review of the PIAAC Cycle 1 numeracy framework (Tout et al., 2017[1]), the review team acknowledged that there is a potential issue with using national **adult** curriculum frameworks/standards directly, because some national adult numeracy frameworks and standards have been either developed formally to align with established, hierarchical levels in child-focused curricula or are at least built on notions of children's learning. This can be illustrated in a number of ways, for example, by the inclusion in adult curriculum frameworks of simplistic, bounded statements such as 'can count to 20'; by specific, school-based terminology such as the 'place values of digits in whole numbers up to 100'; or where percentages are not named and included until higher levels of performance. Such statements do not acknowledge the empirical data that exists from PIAAC or other empirical data, as it does not match the knowledge of adults nor represent the day-to-day tasks that many adults can in fact successfully undertake, but who may nonetheless be performing at below Level 1 numeracy in PIAAC (Tout et al., 2017[1]).

Another perspective on possible content for the numeracy components is the growing body of research on number sense.

Number sense

Number sense appears increasingly in literature as one of the main components of "numeracy." Being numerate means having a certain sense of quantities and numbers and how we use numbers—orally, vocally and in writing—to represent, inform, predict, and estimate phenomenon from real life.

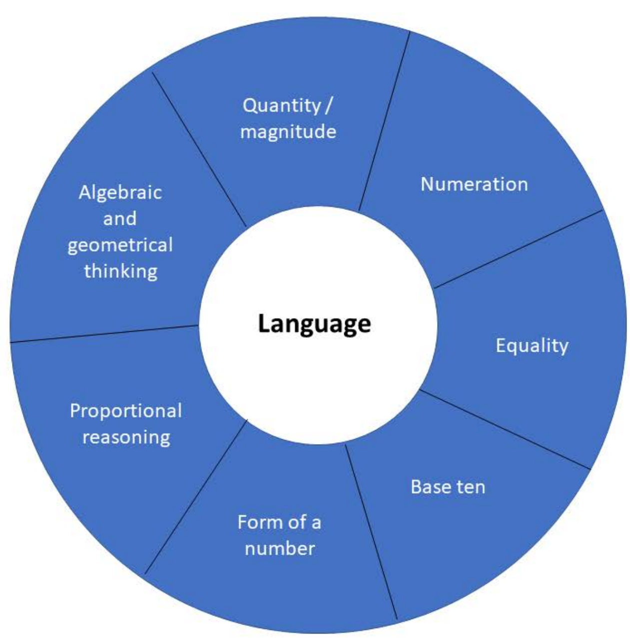
The term number sense was coined in the 1930s by Dantzig: "Man, even in the lower stages of development, possesses a faculty which, for want of a better name, I shall call Number Sense. This faculty permits him to recognise that something has changed in a small collection when, without his direct knowledge, an object has been removed from or added to the collection." (Dantzig, 1934, p. 1[198]). In the

1990s the concept became more visible [e.g., (Greeno, 1991[199]; Mcintosh, Reys and Reys, 1992[18])]. McIntosh, Reys and Reys developed a framework for number sense including three components: *Numbers*, *Operations* and *Computational settings*, which are interconnected. According to them, number sense involves being able to use numbers, operations and their applications in different computational settings. They talk about the meaningful understanding of the Hindu-Arabic number system, the development of a sense of orderliness of the number, the multiple representations for numbers (including the idea of composition / decomposition), the understanding of mathematical properties, and the relationship between operations. For them, having "number sense" means being able to solve problems in the real world, providing suitable answers, using (or creating) effective strategies to compute, count, etc. It is not just reproducing instrumentally a certain algorithm, but being able to use the mathematical knowledge and components in a flexible manner. At the same time Dehaene (1997[200]) published his best-selling book *Number Sense – How the Mind Creates Mathematics*, which made a connection of number sense with the structure of our brains.

Yang, Reys and Reys (2009[17]) defined number sense as "a person's general understanding of numbers and operations and the ability to handle daily life situations that include numbers. This ability is used to develop flexible and efficient strategies (including mental computation and estimation) to handle numerical problems" (2009, p. 384[17]). Regarding the components of number sense, these authors argue "Number sense is a complex process involving many different components of numbers, operations, and their relationships" (Yang, Reys and Reys, 2009, p. 384[17]). Among these processes, they highlight two aspects, 1) the use of benchmarks in recognising the magnitude of numbers, and 2) the knowledge on the relative effects of an operation on various numbers.

Faulkner and Cain (2009[201]) claim that "the characteristics of good number sense include: a) fluency in estimating and judging magnitude, b) ability to recognise unreasonable results, c) flexibility when mentally computing, d) ability to move among different representations and to use the most appropriate representations" (Faulkner and Cain, 2009, p. 25[201]). Cain et al. (2007[202]) described a set of components of number sense as shown in Figure 3.5, where the different components of number sense all relate to and are underpinned by language:

Figure 3.5. Components of number sense



As stated by Thompson (1995[19]), using numbers is more than reasoning about number and more than skilled calculations. It is about making sense of the situation to which we apply numbers and calculations" (Thompson, 1995, p. 220[19]). It also involves a critical way to make decisions and solve problems in *Personal*, *Work*, and *Societal/community* contexts (Peters, 2012[15]).

Summary and where to next?

Based on the above research about the potential content of a new numeracy components assessment, the NEG also considered the possibility of developing a spatial sense components assessment, as spatial sense was also seen as a key foundational skill for the target cohort of adults. However, mainly due to a number of the constraints on the delivery of the numeracy components discussed below, especially in relation to the time available for administering the components, alongside the greater difficulty of reducing the literacy/reading demands of items based around an assessment of spatial sense, the NEG decided to move forward with using number sense as the content base for the new numeracy components assessment.

In its broadest definition, 'number sense' overlaps to a great extent the 'big ideas' and the domains of the (inter)national frameworks mentioned before. In a smaller and more fundamental interpretation, number sense relates to the sense of quantities and the sense of how numbers represent quantities. This latter interpretation turned out to be the most suitable basis for the further development of the items that must operationalise the numeracy components.

Numeracy component skills – the scope

Another significant issue that needs to be addressed in developing a numeracy components assessment is that of the embedded nature of the mathematics in real-world settings and situations and the role that this plays. This is often called context-based in contrast to non-context-based tasks or contextualised in contrast to decontextualised. Individuals acquire mathematical knowledge through both formal and informal learning, and informal learning is as valuable as formal, school-based learning. The field of ethnomathematics richly documents this issue of "street maths versus school maths" and as this components assessment will often target adults with little formal schooling but who are functioning as adults in society, this issue needs to be taken on board and addressed. For example, D'Ambrosio (1985[203]) theorised the concept of ethnomathematics. Carraher, Carraher and Schliemann's (1985[101]) research with street children in Brazil found they could operate in quite sophisticated ways when using mathematics to survive in a commercial sense, although they had been previously adjudged as being incapable of doing mathematics in schools. This was discussed in more detail in the second section under the topic *School mathematics versus everyday or workplace mathematics*.

Matthijsse (2000[204]) specifically addressed the issue of how adults cope with mathematical knowledge in practical daily situations and the gap between school mathematics and its formal algorithms and the mathematics that adults use in their daily lives. He looked at the informal methods adults used in daily life, and found they were often anchored and embedded in familiar knowledge and real-life settings and situations. Although his focus was on instructional methods to use with learners, his research, like the other ethnomathematical research, indicates that this proposed PIAAC numeracy components assessment cannot be constrained by only offering non-context-based tasks with the mathematics being like formal, school-based questions. However, a significant risk, and challenge, exists with regard to cultural and the possible national specificity of particular rule of thumb or informal methods, and how these differences could be overcome in an international assessment. Given this, a low-level components assessment could aim to find out about adults' informal/common sense ways of doing mathematics—what mental models and processes do adults use when solving a numeracy problem? In addition, can data and information be collected about the connections (or non-connections) between the school ways of doing mathematics (and the use of algorithms) versus the way adults solve such problems in everyday life?

Different people will have very different settings and applications in which they may comfortably and more confidently use their mathematical knowledge. Finding the right problem situation or setting for an individual so that they can demonstrate their understanding of mathematics concepts will be a challenge. At this more basic level of mathematical knowledge, the familiarity of the setting and situation could be critical. A potential solution could be to use a form of adaptive delivery to allow respondents to be able to select from a range of settings and situations where the same content and level of mathematics content is embedded.

In relation to the three named PIAAC numeracy contexts (*Personal*, *Work*, *Societal/community*), it would make sense for the numeracy components assessment to focus on the more common, generic and familiar settings and contexts which would appear to be *Personal*, *Work*, *Societal/community*. The three existing below Level 1 numeracy items are located within those contexts. Again, a challenge exists in how to use work-related situations, given that research shows that adults with poor formal skills are often able to function 'perfectly well' in particular jobs where they have learned rule of thumb or other methods that enable them to get by.

One challenge with context-based items is that where the mathematics is embedded within texts and stimuli, some of the targeted cohort will not be easily able to read, interpret and hence engage with and understand the mathematics required to be used due to their potential low level of literacy skills.

These considerations strengthen the idea of keeping items for assessing numeracy components in line with the fundamental definition of number sense, focusing on the connection with quantities in real life and the way numbers are used to represent quantities. This seems feasible without necessarily using long or complex verbal descriptions to present contexts of items or to ask the questions.

The proposed numeracy components for PIAAC Cycle 2

Given the constraints of level, reading demands, time, and the available representation of tasks, the NEG decided to implement a modest set of number sense items that would be the main ingredients in the landscape of relevant numeracy components. These items will ask the participants to estimate quantities from real-life pictures and furthermore estimate the relative magnitude of several numerical representations of quantities.

It was intended that the respondent would be able to quickly view the stimulus without needing to read much text at all, tap on a response and immediately be sent to the following question that would be based on the same stem, requiring no need to read anything further.

The content is limited to a fundamental perspective on number sense and more specifically to:

- A set of 12 items where the respondent must select the quantity (<20) of a number of objects displayed. The representations are limited to pictures of real-life objects.
- A second set of 15 items on the relative magnitude of quantities or phenomena, partly from real life and partly more decontextualised.

Based on the above decisions, the NEG:

- Consulted the translating partner for PIAAC, cApStAn, to assist in identifying the best question wording that could be used that would utilise a simple, single stem and reduce the need to read each question separately.
- Developed a draft of the components assessment and ran two brief pilots.
- Revised the draft prior to release to countries for feedback and comment.
- Reviewed and revised the draft again at the NEG meeting held in October 2018.

Linguistic issues

In discussions with cApStAn it was soon realised that when translated into other languages, what seemed simple solutions and wording in English often become complicated when translated into a range of other languages. This often depended on what the objects or images were that would be presented to respondents.

After a period of discussion with cApStAn about what wording, and what content and images would work best without creating the need to change the stem throughout, it was decided that the best solution was fourfold:

- To have an introduction up front that would foreground the assessment items to follow.
- Include some simple practice questions that would model what was required.
- To have the two sets of items – the first using the stem **How many ...?** for identifying a quantity.
- The second set was using the stem **The biggest?** for identifying the relative magnitude of different quantities/values.

It did mean that some of the NEG's ideas *re* items and their images or stimuli to include could not be used, such as temperatures, charts, people etc. Some items and images were gender-sensitive and this restricted some of the options.

Feedback from pilots

Two pilots of the first drafts of the Numeracy Components assessment were held, utilising access to adults participating in adult literacy and numeracy classes, and who were known to be low-performing adults. One pilot was in Belgium with 10 adults, and the other in Spain with 29 adults. The main outcomes from the pilots included:

- The participating adults were positive about the experience—they were normally used to not getting mathematics/numeracy questions correct.
- They liked the real-life images to quantify/count.
- They could all answer all of the **How Many?** questions correctly, but some took a long time.
- Some of the adults took up to 30 minutes to answer the sets of questions.
- There were clear boundaries in knowledge with **The biggest?** set of questions. Difficulty started with the understanding and comparing of decimal numbers and fractions.

The results from the pilots indicated that the assessment, for the most part, worked successfully. A number of issues resulted that enabled the NEG to make further decisions and refinements to the draft assessment. These included the following:

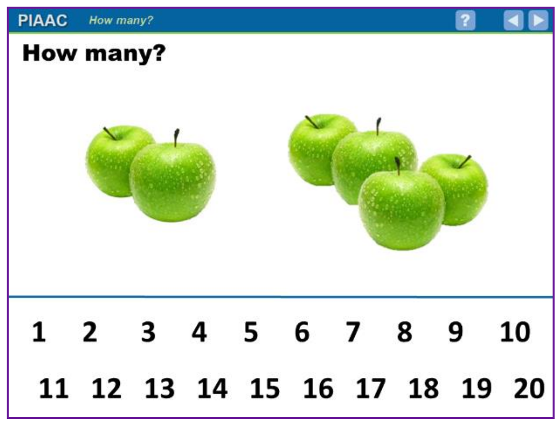
- For the most part, the content was appropriate for the target group, and the wording and presentation seemed to be accessible—some questions were able to be refined based on the observations made as the adults undertook the assessment.
- The items that assessed the ability to recognise and answer some decontextualised basic sums where the adults needed to recognise the meaning of some standard arithmetical operations were confusing and not appropriate for these learners—this reinforced the beliefs of the NEG about the relevance and meaning of such types of test items for adults. They have been removed for the Field Trial.
- Given the length and range of times taken, the timing and the estimate of fluency will be crucial to measure.

- The number of items to be included in the Field Trial have been reduced, both due to the time taken, but also because it was felt that the extra number of items was not collecting extra data or information.

The NEG has refined the sets of questions and have developed two different forms for the Field Trial, with linking items. The NEG will select best performing items from the Field Trial for the Main Study.

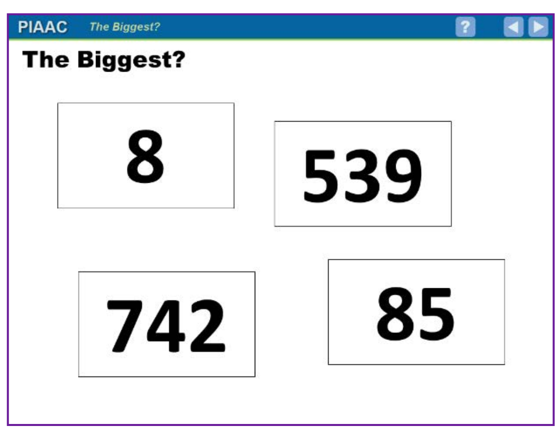
The questions and items being asked—How many ...?

For the Field Trial there will be 12 questions of this type, with a maximum time allowance of 2 minutes. Here is a mock-up of an example of the sort of item asked in this set of questions. The respondent is to tap on the matching number.



The questions and items being asked—The biggest?

For the Field Trial there will be 15 questions of this type, with a maximum time allowance of 3 minutes. Here is a mock-up of the sort of item asked in this set of questions. The respondent is to tap on the biggest item/number of the set shown.



Timing

There will be a restriction in time for the Field Trial to a maximum of 5 minutes – 2 minutes for the set of *How many?* items; and 3 minutes for the set of *The biggest?* items. However, this restriction in time will not be revealed to the participants to avoid unwarranted stress and potential anxiety about failure. Respondents will just be told to do their best to work both accurately and quickly. The timeframe for the delivery will be limited to the number of minutes and after that time has expired the set will be terminated and no more new items will appear. The respondent will not be aware that there were unanswered items in the set, if that is the case. The time parameter will be recorded for each participant and will be analysed in relation to a measure of fluency. However, a decision how to incorporate the time issue and the estimate of fluency in the reporting will only be decided after the Field Trial data is analysed.

Automaticity/fluency

Given that in the delivery of the reading components assessment each part was timed in order to be able to get an estimate of speed and automaticity, this should also be made available for the numeracy components assessment. Collecting the timing information and the ability to create measures of fluency in number sense will provide the capability to look for any correlates of interest, such as with the respondent's performance in numeracy overall and/or with particular dimensions of the numeracy assessment.

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Notes

[1] The term "mathematical" is used here as inclusive of situations where *statistical* or *probabilistic* information may appear or where statistical thinking or statistical literacy are required as well. Such usage is made for brevity and convenience only. It is acknowledged that statistics is not a branch of mathematics, and that statistical reasoning and statistical literacy have unique elements, concepts and processes which are not mathematical in nature (Moore and Cobb, 2000[205]).

[2] For more discussion and examples of the reading components tasks see OECD (2016[51]), *The Survey of Adult Skills: Reader's Companion, Second Edition*.

4 PIAAC Cycle 2 assessment framework: Adaptive problem solving

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This chapter defines the concept of adaptive problem solving (APS) in the second cycle of PIAAC. The concept of APS accounts for the fact that we need to be vigilant, adaptive, and willing to modify our plans when interacting with the social, physical, and technological world of the 21st century. In this framework chapter, the cognitive and metacognitive processes that successful people engage into when solving problems and when adapting to changing conditions are described. In this, the PIAAC assessment of APS draws from a large set of information contexts and task dimensions that drive overall APS performance and individual proficiency levels. Several example items, considerations on item scoring and data capturing as well as a thorough discussion of the relation between APS and other competencies provide a comprehensive overview of the APS measurement framework for PIAAC.

Introduction

Rapid changes in the social, physical, and technological world require individuals to be more vigilant to changes, more adaptive, and more willing to modify their plans in pursuit of their goals. It is therefore indisputable that the competence to solve problems and to adapt to changing conditions is of crucial importance in the 21st century, where citizens are faced with increasingly complex technologies, social systems and subject matters (Levy and Murnane, 2006[1]; National Research Council, 2012[2]). The need for problem solving is ubiquitous in the workplace, as well as everyday life, for most adults. For instance, Felstead et al. (2013[3]) conclude that problem solving skills are more important than numerical or communication skills for a worker to be successful, a finding that is likely to generally apply to economies that are service-oriented. Problem solving is therefore generally important to assess as an overarching construct.

The Programme for the International Assessment of Adult Competencies (PIAAC) included a measure of problem-solving proficiency in its first cycle in 2011. In addition to core dimensions of adult skills, i.e., reading component skills, literacy, and numeracy, the survey assessed problem solving in technology-rich environments (PS-TRE) for individuals aged 16 to 65. PS-TRE focused on goal setting, monitoring, and planning in technology-rich environments (OECD, 2012[4]) and assessed proficiency in the use of specific digital applications to access, search, manage, interpret, and evaluate information. The second cycle of PIAAC in 2022 will focus on adaptive problem solving (APS). “Adaptive” underlines that problem solving is a process that takes place in complex environments and that this process is not a static sequence of a number of pre-set steps but rather a constant attempt to solve a problem. Hence, while problems themselves can either be static (i.e., with no changes in the given states or the goal states) or dynamic (i.e., with changes occurring in the problem situation), the process of problem solving when confronted with dynamic problems is adaptive (i.e., problem solvers need to adapt to the dynamic nature of such problems).

There are three important core aspects that distinguish APS from previous large-scale assessments of problem solving, such as PS-TRE or as implemented in the assessment of the Programme for International Student Assessment (PISA):

- First, the competence to handle dynamic and changing problem situations has become increasingly important in today’s society, and therefore the need for skills that enable adults to adjust their thinking and reasoning to novel and changing information has grown crucially. The assessment of APS will therefore focus on dynamic problems that require problem solvers to monitor their problem solving and to adapt their initial solution to new information or circumstances.
- Second, the characteristics of the typical problems that individuals encounter at work and everyday life have been changing over the last five decades, in part because of radical changes in digital technologies and communication media (Autor, Levy and Murnane, 2003[5]). The solutions to particular problems are also more distributed over time as people take advantage of social and digital resources that have particular constraints in access and timetables. This new wealth of information, and the shift in the information environment that people are confronted with, will be reflected in the characteristics of the tasks included in the APS assessment, i.e., the information environments (physical, social, and digital) and problem contexts (personal, work, and social community) in which tasks will be situated.
- Finally, cognitive processes are inherently bound to the problem-solving process and have always been an important aspect of the problem-solving assessment. However, especially in highly adaptive and higher difficulty problems, problem solvers also need to strongly engage in metacognitive processes (i.e., the ability to calibrate one’s comprehension of the problem, evaluate potential solutions, and monitor progress towards the goals). Consequently, the assessment of APS in the second cycle of PIAAC will put emphasis also on metacognitive processes.

The purpose of this document is to provide an assessment framework following the conceptual framework paper for APS (Greiff et al., 2017[6]) to guide the construction of APS items to be used in the second cycle of PIAAC as well as the definition of the proficiency scale for APS.

Adapting to dynamically changing situations: The importance of adaptive problem solving

The ability to quickly and flexibly adapt to new circumstances, learn throughout life, and turn knowledge into action has always been important for full participation in labour markets and society (National Research Council, 2012[2]). However, in a world that has become increasingly and dynamically changing, and which provides a plethora of information from different resources, the need to flexibly adapt to unexpected changes has become more and more important. Over the course of a single day, an individual can be a purchaser of consumer goods, an organiser of local transportation, a holiday planner (searching for flights and accommodation arrangements in hotels or house swaps), a financial planner, and a home decorator. These various activities address multiple goals in non-routine ways that require APS skills. People need to adjust, for example, to prices of commodities that change overnight, a strike of transportation workers, internet sites that go down, and people who cancel appointments. Adapting to these unexpected changes in these various environments requires problem solvers to consider different resources in the physical, social, and digital environments, in addition to their own mental activities. Therefore, APS is particularly important to assess as problems often dynamically change during the course of problem solving, which then requires constant monitoring and, if necessary, adaptation of the original problem solution. These changes occur because of unexpected physical and/or social events in the environment and because of unintended consequences of the problem solver's actions.

It is important to emphasise that the assessment of APS in the second cycle of PIAAC goes beyond what was assessed in previous OECD international assessments of problem solving. For one, the problems assessing individual problem solving in PISA 2009 were entirely static (i.e., the given states and goal states did not change) and preceded the collection of data on computers. In PISA 2012, the assessment of problem-solving competency was computer-based and allowed the implementation of interactive problem situations in addition to static ones (OECD, 2014[7]). The items became dynamic in the sense that the problem solver needed to interact with the problem environment in order to find all the relevant information to solve the problem. PISA 2015 then focused on collaborative problem solving with computer agents that interacted with a problem solver through chat facilities and actions performed in shared workspaces (OECD, 2017[8]). It is important to stress that the term *dynamic* is broadened in the assessment of APS as it refers not only to the exploration of the environment, i.e. the interaction between the problem solver and the information context, but also to changes in the problem situation to which the initial solution needs to be adapted to. When we refer to "dynamic" in the following, we always use the term in this broadened manner.

As mentioned before, problem solving was already assessed in the first cycle of PIAAC. The PS-TRE assessment was conceived to monitor the problem solver's information-processing skills when operating in technology-rich environments using information and communications technology (ICT) skills. Core to the PS-TRE assessment therefore was the understanding and evaluation of meaningful information available in technology-rich environments, including simulated websites, e-mail and spreadsheet environments (OECD, 2012[4]). The assessment of APS will also use technology-rich environments. However, these environments will rather form the context in which the problem unfolds dynamically and to which the problem solvers need to adapt their initial problem solution.

The cognitive and metacognitive components of adaptive problem solving

As mentioned before, successful problem solving requires the problem solver to engage in cognitive as well as metacognitive processes. Previous assessments of problem solving have incorporated core cognitive theories of problem solving (Funke, 2010[9]; Mayer and Wittrock, 2006[10]). They start with the definition of a problem as having a given state, a goal state, a set of legal operators to get from the given to the goal state, and plans for solutions to subtasks. The PISA 2012 and 2015 assessments identified the problem-solving components as 1) exploring, understanding, and representing the problem, 2) searching, planning, and executing potential solutions, and 3) monitoring and reflecting on the progress towards solving the problem. The assessment of APS in the second cycle of PIAAC will have the following *cognitive* problem-solving components that are similar but not exactly the same: *defining the problem* – the same as 1) –; *searching for information,* and *applying a solution* – these latter two components mapping onto 2) –, whereas the explicit assessment of metacognition will incorporate 3).

The cognitive processes become more complicated in APS where the problem solution might need to be adapted in reaction to dynamically changing situations. That is, physical, social, and digital worlds are frequently undergoing changes that an adaptive problem solver must accommodate. The problem solver faces the additional challenge of having to continuously monitor, often through conscious effort, whether the current problem state remains the same or changes throughout the course of problem solving, whether operators that are already known from similar problem-solving attempts are still available or whether new ones need to be identified, and which plans can be executed using the available resources at a given point in time. The second cycle of PIAAC will contain items that measure metacognitive processes in addition to cognitive processes. The role of metacognitive processes becomes more important to the extent that problems are more complex and difficult to comprehend (requiring comprehension calibration), the problems change dynamically (requiring evaluation and re-evaluation of the suitability of operators and plans), and progress towards the solution becomes more difficult to discern (requiring monitoring and reflecting on progress towards the goals).

Both cognitive and metacognitive processes will be assessed at three stages of problem solving: defining the problem, searching for a solution, and applying a solution. There are cognitive processes and metacognitive processes required at each stage, with some items tapping both processes and others focusing on either cognition or metacognition.

In a nutshell, in the second cycle of PIAAC, the APS assessment will put greater emphasis on individuals' capacity to a) flexibly and dynamically adapt their problem-solving strategies to a dynamically changing environment, b) identify and select among a range of available physical, social, and digital resources, and c) monitor and reflect on their progress in solving problems through metacognitive processes. The assessment tasks will therefore reflect the fact that solutions to problems in the modern world require a reflexive, flexible, and adaptive mind.

In the following, we will first define APS and introduce two tasks to exemplify how APS can be assessed. We then detail the task dimensions that define each APS tasks and describe the required cognitive and metacognitive processes. The next section describes the factors that may be used to describe the APS proficiency levels and is followed by a summary of the assessment of APS. We close with a comparison of APS with other core competencies, i.e., literacy, numeracy, and digital competency.

Definition of adaptive problem solving

Explanation of the definition of adaptive problem solving

As mentioned above, there are three core aspects that are represented in the conceptual framework (Greiff et al., 2017[6]) and in the assessment framework of APS. First of all, in a dynamically changing world, it is essential to react to unforeseen changes and new information in a flexible and adaptive way. This is represented in the term "adaptive" in APS. Second, as the amount of information available in the world of the 21st century is ever increasing, we are faced with a wealth of information from different sources. This expansion of information environments needs to be taken in account and will be reflected in the tasks developed for APS, which will be situated in a range of information environments and contexts. Finally, whereas cognitive aspects have always been an important part of problem solving, the necessary change of plans and approaches to a problem and the adaptability and flexibility coming along with this require a stronger focus on metacognition in addition to the existing focus on cognition. Thus, APS puts a strong focus on metacognitive aspects throughout the process of problem solving.

The definition of adaptive problem solving in the second cycle of PIAAC is as follows:

> "Adaptive problem solving involves the capacity to achieve one's goals in a dynamic situation, in which a method for solution is not immediately available. It requires engaging in cognitive and metacognitive processes to define the problem, search for information, and apply a solution in a variety of information environments and contexts". (Greiff et al., 2017[6])

Each part of this definition is explained in more detail below.

Adaptive problem solving…

The term "adaptive" stresses the adaptive nature of problem solving irrespective of the environment or the context in which the problem solving takes place. This underlines that problem solving is a process that takes place in complex environments and that this process is not a static sequence of a number of pre-set steps. Rather there could be an adaptive nature to the problem-solving process in each step. Put differently, problem solvers need to remain open and pay attention to changes in the situation and adapt their problem-solving approach accordingly. The term "adaptive" readily connects to notions such as cognitive flexibility or plasticity, but is broader in its meaning and encompasses the entire set of cognitive and non-cognitive components involved in APS.

"Problem solving" was chosen as a core term for the focus on situations that require non-routine solutions (as opposed to tasks, see below) independent of the specific content domain. Problem solving is generally regarded as one of the most ubiquitous activities that is necessary to successfully master challenges in unforeseen situations, be it in educational contexts, on the job, or in private life. Because problems can occur in a number of settings, the process of problem solving, including its different components, can be applied across different domains. In fact, a transversal understanding of problem solving has recently been included in several large-scale assessments, such as PIAAC and PISA, but those assessments differed in that they did not focus on the "adaptive" nature of problem solving in the 21st century.

…involves the capacity to achieve one's goals in a dynamic situation…

The broad term of "capacity" is meant to convey that APS is a complex proficiency that is composed of a number of more specific sets of skills, most notably cognitive and metacognitive aspects that are explicitly targeted in the assessment. APS also includes the motivation to deal with the problem situation and to face the challenges of the problem situation and its unforeseen changes. Through this, the motivational aspect is implicitly part of the assessment, but it is not an explicit part of the core APS assessment.

Problem solving is a goal-directed activity, in which the problem solver is embedded into a situation that needs to be mastered successfully and this situation may be dynamic. That is, as opposed to static problem solving that takes place exclusively in situations that have no dynamic component, which implies that all relevant information is available at the outset and that there is no change in the problem setup, the constraints, or the goals have to be foreseen. When engaging into APS, problem solvers need to anticipate, incorporate, and deal with the many types of dynamic changes that might happen while moving from an initial state to a desired goal state. APS therefore refers to the process of problem solving in dynamically changing situations. More precisely, the dynamic aspect of the problem situation implies on the one hand that relevant information from different sources might need to be acquired throughout the process, something that has been considered relevant in previous assessments of problem solving (cf. the assessment of problem solving in PISA 2012). However, in addition to the capacity of exploring a problem situation, the problem solver also needs to deal with various types of changes in the situation and needs to react to these changes. Put differently, problem solvers need to monitor their progress, the problem state, and the environment and context in an attempt to pay tribute to the dynamic nature of the overall problem situation that might exhibit constant change or hardly any change at all. From an assessment perspective, the inclusion of the dynamic component relies on the use of technologically based assessments that allow for the type of items in which such dynamic changes can be implemented. In this, the second cycle of PIAAC is a technology-based assessment that allows a broadening of the scope of the proficiencies through the technical means and, through this, new item formats available to test developers.

…in which a method for solution is not immediately available.

This part of the definition alludes to a core component of virtually any problem-solving definition: at the outset, the path to the solution and the solution itself are not immediately clear and require that the problem solver initiates a process that, ultimately, leads to the goal state. This distinguishes problem solving from a mere task, in which a solution usually is readily available. It also shows that, even in specific domains such as mathematics or science not all items are problem solving items as some of them could be solved merely by knowing the correct answer, and it also stresses the non-routine aspect of the problems in this domain. In this, there is a direct link between existing frameworks of problem solving (e.g., problem solving in PISA 2012 or collaborative problem solving in PISA 2015), but the notion of a solution that is not immediately accessible is even more central to APS because changes in the problem setup or the problem situation require a re-examination of initial solutions and, in some cases, new approaches to solve the presented problem.

…It requires engaging in cognitive and metacognitive processes…

Cognitive and metacognitive components are both critical aspects of APS. Problem solving always requires some cognition such as organising and integrating information into a mental model or evaluating operators as to whether they are relevant for reaching the desired goal state. But metacognition, such as setting a goal or reflecting on progress, is equally important. In fact, both components are often intertwined in a way that makes it difficult to separate them and it will be a challenge in the assessment to do so. While the role of metacognition has been acknowledged in previous assessment frameworks, it has often not been targeted explicitly but rather been considered as a part implicitly included into the assessment. Here, APS differs in the sense that dealing with a dynamic situation in an adaptive way always requires a certain level of metacognition. For instance, if the situation changes, without a sufficient level of metacognitive awareness, this change will go by unnoticed and will not lead to a solution of the problem. Thus, the conceptual framework (Greiff et al., 2017[6]) stresses that the world of the 21st century cannot be successfully mastered without a certain level of metacognition. The assessment of APS will be designed in a way that it clearly reflects the need for metacognition and will also develop items that primarily target the problem solver's metacognitive proficiency.

…to define the problem, search for information, and apply a solution…

The APS framework defines three broad problem-solving stages that are logically ordered from first defining the problem, second searching for information, and, finally, applying a solution. However, this is a schematic description and any problem-solving activity switches between the different stages or might even employ them simultaneously. The description here is meant to convey that usually one of those activities prevails. The assessment will aim to elicit problem solvers' cognitive and metacognitive proficiencies along these three stages in a comprehensive way.

In each of the three stages, both cognitive and metacognitive processes are relevant and while there is some overlap, many of the processes are distinct for a specific stage. In fact, the delineation of the problem-solving process into different stages is ubiquitously found in the problem-solving literature even though there is some disagreement as to the number and the nature of the stages. In APS, the problem solver is faced with the challenge that a change in the setup might occur at any time, requiring constant monitoring and a readiness to react throughout these stages. That is, as compared to other problem-solving approaches, a once derived definition, a set of information, or a chosen path towards solution might become obsolete, but instead, a new definition, new information, or a new path towards the solution needs to be derived.

…in a variety of information environments and contexts.

This final part of the definition stresses that in information-rich environments – and virtually all of today's problems are embedded into such – the different sources from which the information originates and the different contexts are of high relevance. Information can be gathered from physical, social, or digital environments, which is meant to cover the ubiquitous nature from which the problem solver derives the knowledge about a problem in today's world. In this, APS differs from previous problem-solving assessments that focused on specific sources of information such as the social environment in collaborative problem solving in PISA 2015 or on knowledge gathered on websites in the assessment of problem solving in technology-rich environments in the first cycle of PIAAC. In addition, as situations that require APS may occur throughout different contexts, there can be problems that are embedded into a personal, a work, or a social community context because good adaptive problem solvers must be able to apply their proficiency across contexts and derive their information from a comprehensive set of sources.

The next section outlines two example tasks, "Dinner Preparation" and "Stock Market", to give an exemplary understanding of what is meant by APS in terms of real-world situations. We then proceed with a more detailed description of the problem characteristics underlying APS tasks, the associated difficulty drivers, the cognitive and metacognitive processes involved, and define the assumed proficiency levels that determine the quality of the derived solution. We will link this formal description to both of the example tasks throughout this framework document to illustrate the process of APS.

Example tasks "Dinner Preparation" and "Stock Market"

The APS assessment in the second cycle of PIAAC will contain scenario-based tasks, that describe every day and working-life problems. In the following, we describe two examples of APS tasks, in order to illustrate how the principles of APS are transposed into practice. It is important to note that participants will learn how to interact with the provided environments before starting with the assessment. Also, the two units listed below are examples of how APS tasks can look like. None of the examples will be part of the final APS assessment.

- The first example, *Dinner Preparation*, covers an everyday life scenario in which the problem solver has to plan and accomplish different goals over the course of a day. Because of the often encountered need to adapt initial plans by reacting flexibly to changing circumstances and upcoming impasses, and by incorporating and dealing with new information, navigating through everyday life might be seen as the prototype of an APS task.

- The second example, *Stock Market*, describes a financial simulation in which the problem solver has to make buying and selling decisions for a number of companies, depending on their market evolution, in order to maximise profits. The problem is highly dynamic as the problem setup constantly changes and the problem solvers have to continuously adapt their solutions to the latest evolution of the problem environment.

Example task: Dinner Preparation

In the example unit *Dinner Preparation* (see Box 4.1), the problem solver is asked to use an interactive map to accomplish a set of pre-defined goals. The initially static situation becomes dynamic through obstacles that present a change in the presented problem and the available solutions.

Box 4.1 shows two example items for *Dinner Preparation*. The unit starts with a static planning task. In the first item, the problem solver needs to use an interactive map to find the fastest route to accomplish three goals, keeping a set of time constraints in mind. The problem solver needs to: take a child to school by a designated time, purchase ingredients for dinner, and return home by a designated time. This could be considered a standard problem-solving task, in which a solution needs to be found given some constraints that need to be satisfied. In the second item, the situation becomes dynamic as the problem solver has to deal with new circumstances that interfere with the initial problem solution. Impasses must be overcome and additional constraints need to be taken into consideration when adapting the initial problem solution.

Box 4.1. Example unit "Dinner Preparation"

General description of the problem background:

Planning and coordinating different, sometimes contradicting, goals are elementary parts of our everyday lives. This ranges from activities that involve single and multiple goals that have to be planned daily, to long-term goals, and they can arise in a variety of contexts, be it personal, work, or social. However, plans are also repeatedly thwarted by unforeseeable events, or changes in the initial situation. Successfully dealing with such dynamically changing situations, in which the solution is often not directly available requires everybody to engage into APS. More specifically, the emerging problem situation needs to be defined, information about how to approach the situation has to be considered, and the (new) solution has to be applied.

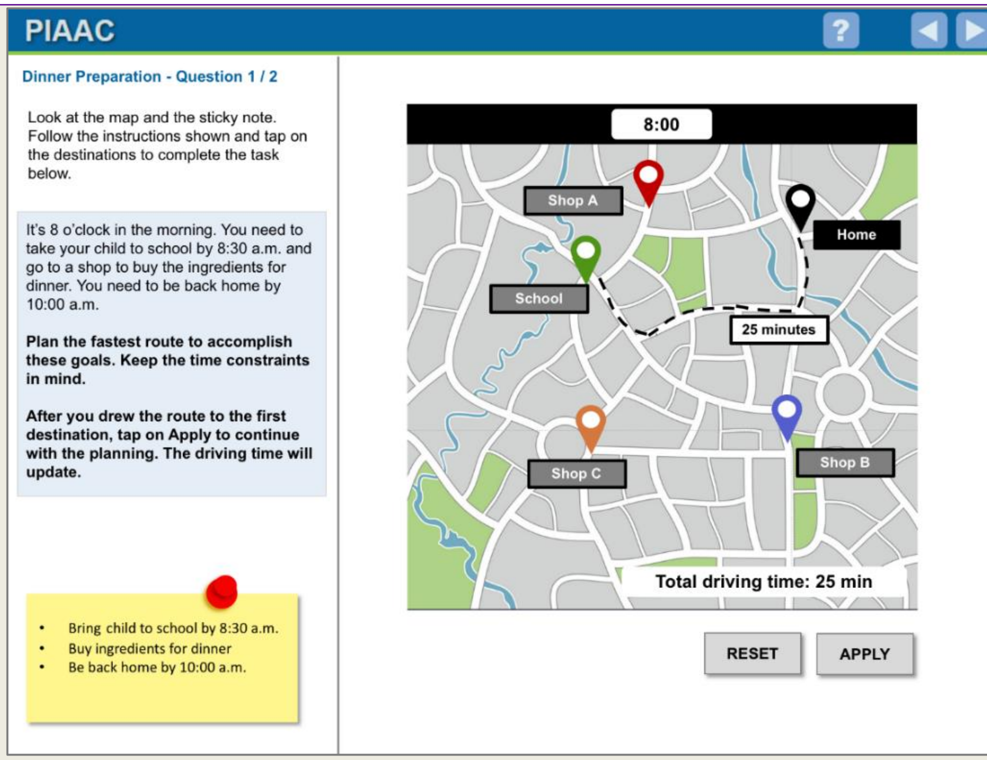
How the unit unfolds:

Imagine that you need to accomplish one single or even multiple goals over the course of the day, such as picking up the child from school, and getting the groceries for dinner. In order to accomplish both goals, you would plan the best route for the car trip, look up the driving times, and make a shopping list. At first, the situation seems to be manageable and quite predictable.

Example Item 1

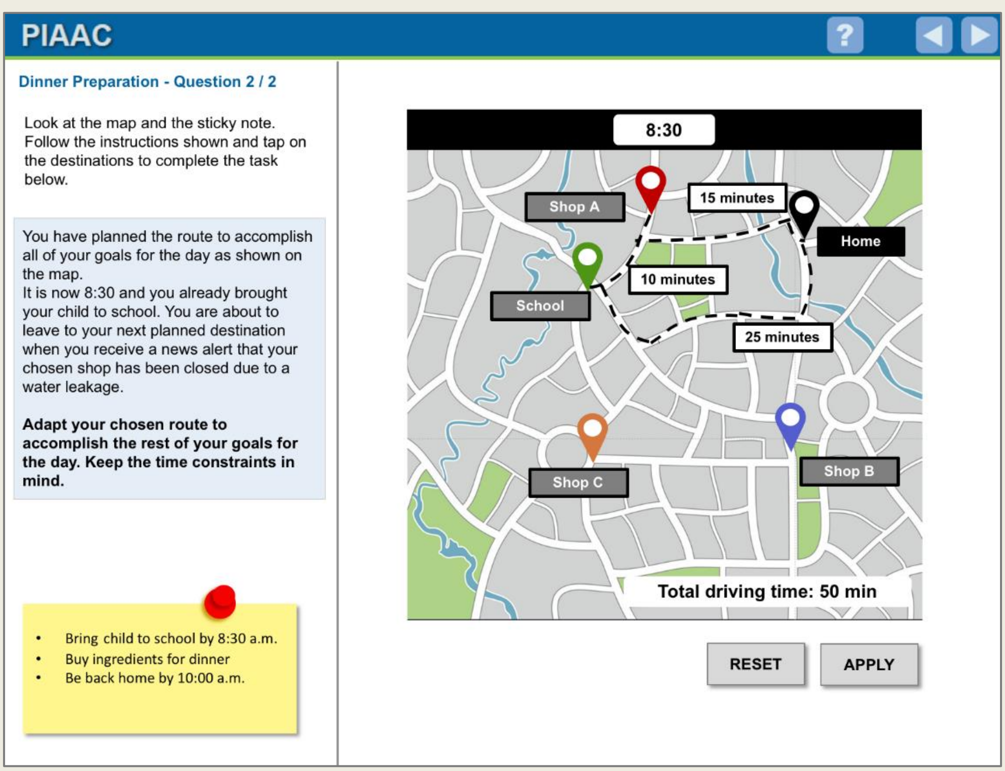
Problem solvers are provided with a map that shows different locations and a sticky note that summarises the goals to be accomplished and the time constraints to be met. A clock shows the time of the day, information on the driving time can be viewed by clicking on the locations. In this first item, problem solvers need to navigate through the map by drawing lines in order to find the fastest way to a) take the child to school by 8.30 and to b) get to a market to buy the ingredients for dinner.

However, just as in real life, while on your way, you suddenly find that one of the local shops is closed and you need to come up with a different plan – you could for example go to a different store, call someone to get the missing ingredient, or change the dinner plans.



Example Item 2

When the problem solvers have planned their route, they get informed that their chosen market got closed due to a water leakage. Problem solvers need to adjust their route while keeping in mind the time constraints.



Example task: "Stock Market"

In the example unit *Stock Market* (see Box 4.2*)*, problem solvers are provided with a stock market simulation, in which they begin with initial stock holdings in five companies, and a small disposable sum of cash that they can invest. They can sell stock for cash, or buy new stock with cash. Stock prices vary on a day-by-day basis. The situation describes a "continuous drip" problem, i.e., the problem is not turn-based, and does not progress to a new stage only after the problem solver commits to an action. It rather evolves in real-time, even if the problem solver does not perform an action – in this case, a new "day" comes on screen every 60 seconds. By judging the history of each company, problem solvers have to make a decision regarding the investment solution that will most likely yield a profit in the future. They then need to sell the undesirable investments that they hold in their portfolio and buy stock in the more promising companies, in order to maximise the value of their portfolio.

While the unit architecture may appear to be quite specialised (i.e., stock market, financial operations), the problem is, in fact, a knowledge-lean task. It does not contain any references to actual companies or industries, and the solution does not depend on specialised knowledge.

Box 4.2 shows two example items for *Stock Market*. In the first item, the problem solver needs to optimise an investment portfolio, while considering the current status and the performance of the five companies over a defined period of time. In the second item, the situation becomes complicated, as the previous pattern of performance for the five companies changes. An impasse is generated by having the two companies with a previous positive evolution turning to negative; this interferes with the initial problem solution and requires problem solvers to rethink their problem-solving strategy.

Box 4.2. Example unit "Stock Market"

General description of the problem background:

Most financially complex situations have a few characteristics in common: a limited number of options are assessed on the go, as part of a dynamically changing situation, in which the optimal state of the system, i.e., when to commit to a decision, is uncertain. Interestingly, financial transactions are typical in a large number of contexts, and are not limited to work, social, or community contexts. Complex financial transactions are now part of everyday life in virtually every culture and are consonant with the demands of the modern world. Throughout their lives, most people will have to solve problems having a complex financial component.

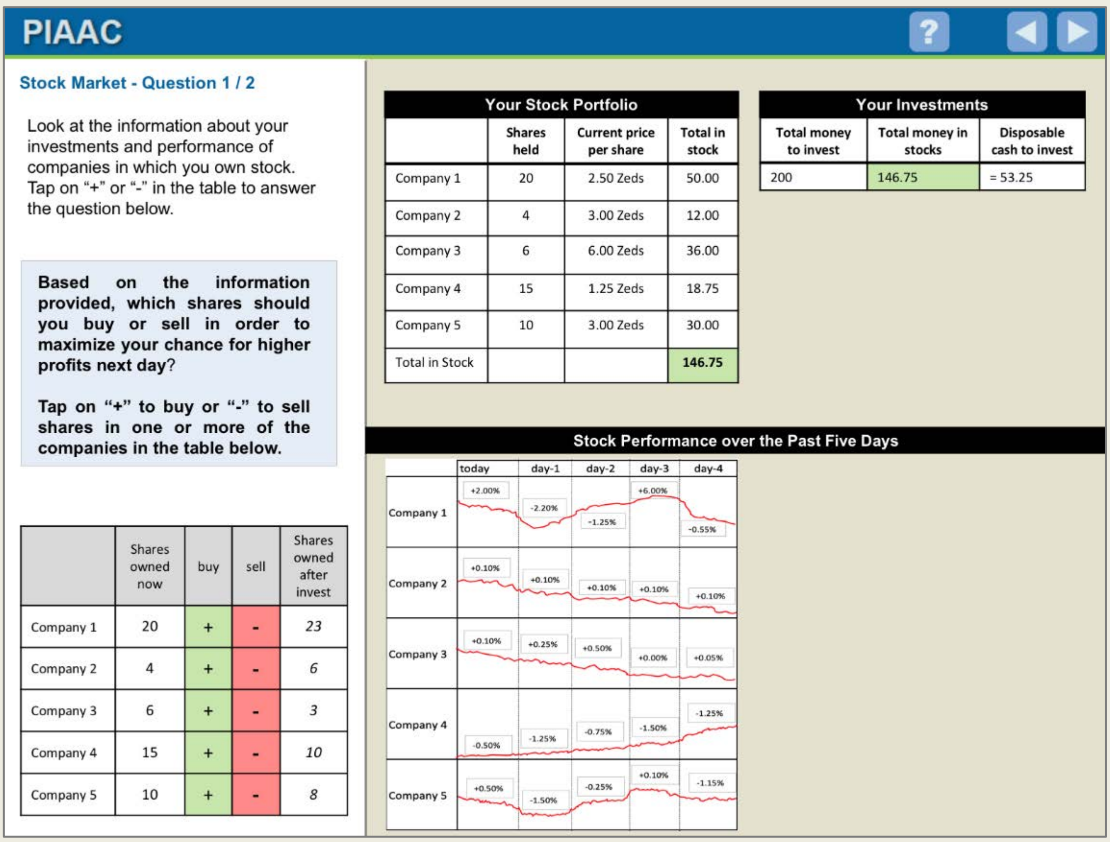
How the unit unfolds:

Imagine that you have to make a number of financial decisions over the course of a week or month, decisions that involve selling uncompetitive assets and buying more competitive ones. In order to accomplish the goal of maximising your money, you will have to consider the evolution of each of your assets each day and decide which ones have become less desirable and should to be sold, and which ones have become more attractive and should be bought to benefit you. The situation is complex from the start, and the problem unfolds day by day – not reacting in a meaningful way may already diminish your investments, as the worth of each share changes day by day.

Example Item 1

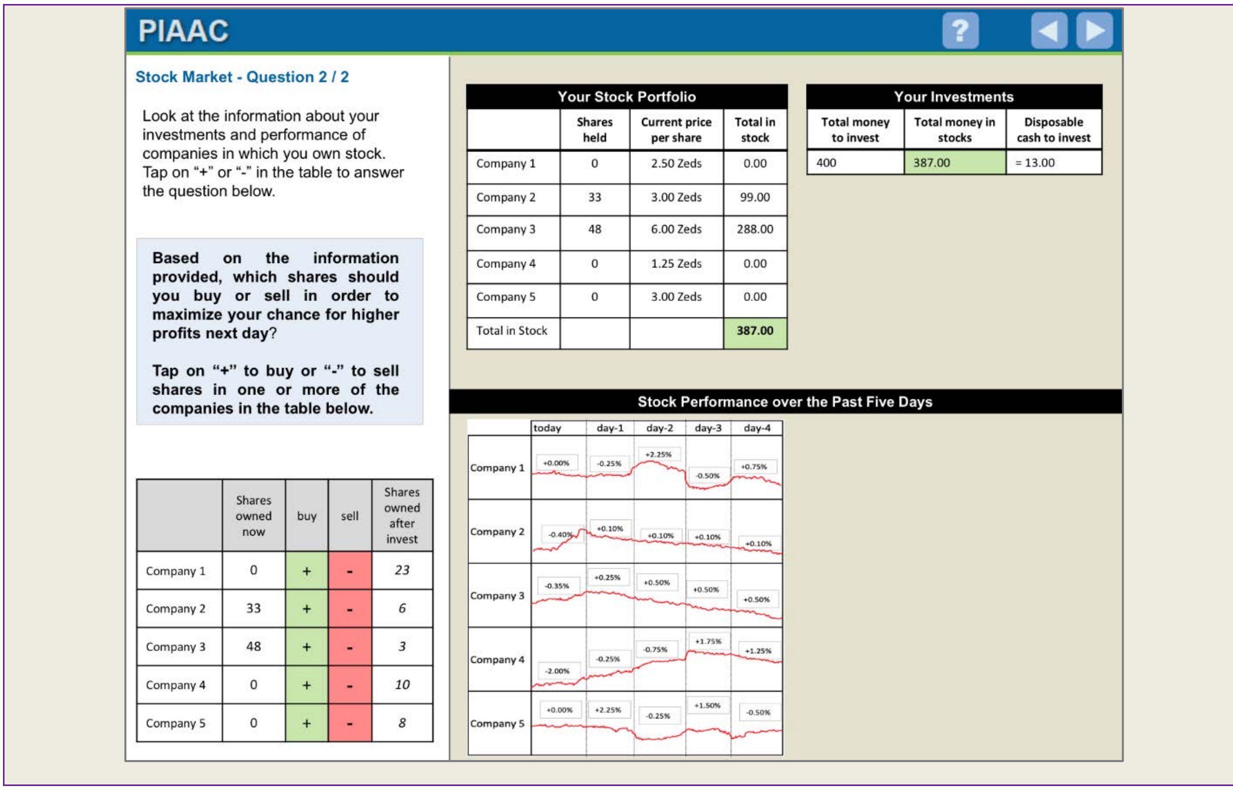
Problem solvers are provided with a stock market simulation, in which they begin with initial stock in five companies, and a small disposable sum of cash that they can invest. They can sell stock for cash, or buy new stock with cash. Stock prices vary on a day-by-day basis. A new “day” comes on screen every 60 seconds, with new information about the evolution of the five companies. A short history, i.e., the last few days in each company’s evolution are displayed on the screen. The pattern of change for some of the companies is transparent, i.e., future change is predictable.

In this first item, problem solvers need to decide, based on the past evolution history of each company, where to invest their money. They need to sell the stock they do not need, and buy stock in the more promising companies, in order to maximise the value of their portfolio.



Example Item 2

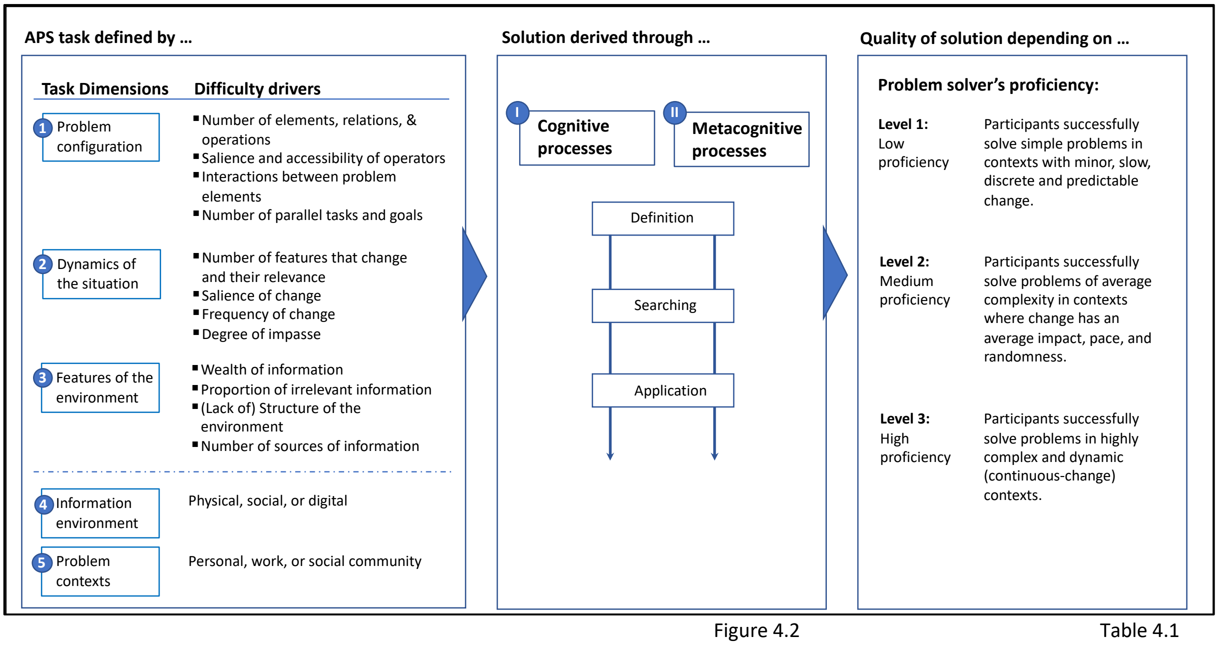
After the problem solvers have committed their portfolio to one or both of the more promising and predictable companies (Companies 2 and 3), the behaviour of these companies changes, and they begin to have negative yield. Problem solvers need to adjust their investment while keeping in mind the ultimate goal to generate as much money as possible.



Core dimensions of the APS domain

So far, we have outlined the theoretical underpinnings of APS. This following section will now focus on the core dimensions that will provide the foundation for the APS assessment. Figure 4.1 illustrates the components of each of the core dimensions. The first panel shows the five task dimensions that define an APS task and their associated difficulty drivers. These are described in more detail below. As shown in the middle panel, and discussed in the next section, a second set of core components are the cognitive and metacognitive processes (i.e., defining the problem, searching for information, and applying a solution) that are crucial for the problem-solving process in greater detail. The third panel presents an overview of the features that define the quality of a solution, as associated with three levels of proficiency in adaptive problem solving. We will then outline the assumed proficiency levels of APS that will form the basis for analysis.

Figure 4.1. The nexus of task dimensions, metacognitive and cognitive processes, and proficiency levels



Task dimensions

To really understand what forms an adaptive problem, it is crucial to identify specific characteristics that make a problem adaptive, and to ask whether there are any qualitative and/or quantitative differences between various adaptive problems. When decomposing a problem, it becomes apparent that each adaptive problem can be described by five problem characteristics, or "task dimensions": (1) the problem configuration, (2) the dynamics of the situation, (3) the features of the environment, (4) the information environment, and (5) the problem context (see left panel of Figure 4.1). These five task dimensions are descriptive of any adaptive problem (see Box 4.3) and will guide the development of the APS assessment in the second cycle of PIAAC.

The first three of these five task dimensions permit changes in *quantity*, and thus can drive the difficulty of the problem. Each of these three task dimensions has four even more specific difficulty drivers and by tweaking these, a problem can become easier or more difficult, requiring different abilities from problem solvers. More specifically, these three dimensions, along with the respective difficulty drivers, can be characterised as follows:

1) **Problem configuration:**

This task dimension refers to the initial problem setup and the goal state(s) including the problem elements, the relations, and the resources/operators. A problem may have more or fewer elements, and these elements may interact with each other or be relatively independent. The different elements may be accessible with ease or with difficulty, and may be more or less salient. The various elements may interact with each other or be relatively independent. And the problem requirement may include the accomplishment of only one or of several goals. All these characteristics of the initial problem configuration drive difficulty in adaptive problems.

The four difficulty drivers that are typical for this task dimension, therefore, are:

(1a) the number of elements, relations, and operations
(1b) the salience and accessibility of operators
(1c) the interactions between problem elements
(1d) the number of parallel tasks and goals

2) **Dynamics of the situation:**

This task dimension refers to change (or absence of change) within the problem situation and the problem constraints across time, and how this affects the problem configuration.

For example, change may happen in one or more features of the problem, these features that change may be more or less relevant for attaining the goal, change may be more or less frequent, and change may generate a difficulty and impasse (or not). All these characteristics of the "dynamism" of the problem drive the difficulty of adaptive problems.

The four difficulty drivers that are typical for this task dimension therefore are:

(2a) the number of features that change and their relevance
(2b) the salience of change
(2c) the frequency of change
(2d) the degree of impasse

3) **Features of the environment:**

This task dimension refers to various features that are characteristic of the environment and the information and resources available from it. For example, the environment in which the problem is set and unravels may be rich in information, and that information may be more or less relevant to solving the problem, and may be more or less structured. These characteristics of the environment have a direct impact on the difficulty of the adaptive problem.

The four difficulty drivers that are typical for this task dimensions therefore are:

(3a) the wealth of information
(3b) the proportion of irrelevant information
(3c) the (lack of) structure of the environment
(3d) the number of sources of information

Task dimensions (1) to (3) and their respective difficulty drivers are the building blocks through which a purposeful construction of the units and items of the test is able to elicit the relevant cognitive and metacognitive processes in problem solvers. It is indispensable to understand their structure and role in the architecture of adaptive problems. It is also important to mention that we do not consider these difficulty drivers to be exhaustive in any way. The ones used here reflect important aspects of APS and can be manipulated with relative ease when constructing the test items. We have therefore settled on them, while explicitly acknowledging the possibility to also describe the problem configuration, the dynamics of the situation, and the features of the environment under other, different parameters. Annex 4.A. more specifically defines the respective difficulty drivers and relate them to how simple and difficult problems would look like.

The last two task dimensions only permit changes in the *quality* of the context in which the problem is set and therefore these two task dimensions do not drive the difficulty of the problem. Task dimensions (4), i.e., information environment, and (5), i.e., problem contexts, give context to the problems featured in the items. Contextualisation is important for any problem-solving effort: no actual problem that people encounter in their lives is free of context. Any problem occurs (and is solved in) an environment with its specific information that may not be directly part of the problem, but that may shape both, the "flavour" of the problem, and the resources that are available for a meaningful solution. More specifically, any problem occurs in a context that is related to people's lives: some problems are personal, other occur in work settings, or in community and social contexts. The goal in specifying these two dimensions is to ensure that the item pool reflects a range of information environments and contexts.

4) **Information environment:**

This task dimension refers to the sources for the resources that are available for solving the problem. The nature of the information environment can be physical, social, or digital. Of course, all these resources appear more or less simultaneously in a digital problem-solving effort, but the problem imposes the need to handle (at least mentally) a specific kind of resource. These resources will be simulated in the assessment tasks.

(4a) Physical resources are those that require hands-on handling: driving a car, operating a machine by pressing buttons and pulling levers, connecting pipes, and others.
(4b) Social resources are those that require the problem solver to engage in interpersonal and social interactions with other people, such as leading a group, planning an activity with friends or family, or presenting a speech to an audience.
(4c) Digital resources are those that require the problem solver to interact with digital features or devices and make use of digital knowledge and skills, such as sorting a table, sending an e-mail, searching the web, formatting a text and others.

5) **Problem contexts:**

This task dimension refers to the situational embedding of the problem, whereby people encounter problems in their personal life, at work or in social and community contexts.

(5a) Contexts that are personal may refer to one's home, family, career, education, hobbies, or financial investments; these problems will therefore require problem solvers to solve a problem that occurs in the context of their personal life.
(5b) Contexts that are work-related may require problem solvers to solve a work-related task, or place them in a work-related context, in which they work under supervision or with co-workers.
(5c) Contexts that are social and community related may refer to interaction with other people in leisure activities (e.g., going to a party or hiking in the mountains) or with community resources (e.g., police, firefighters, or administrative institutions).

Box 4.3. Task dimensions in the example units

The *Dinner Preparation* unit has a specific problem configuration: it asks test-takers to accomplish two goals at the same time, the problem elements are accessible and salient and presented in a visually ordered fashion. The information environment of this example is not rich, and not much information, relevant or irrelevant, is provided beyond the problem itself. The dynamic of the situation is average: when change is induced, test-takers are prompted to the change, and the specifics are explained; still, changes can produce an impasse. The problem is placed in a personal problem context and a mixed digital and physical information environment.

The *Stock Market* example also has a specific mix of these characteristics. The problem configuration requires solving of only one goal, and is based on a high number of elements, that are salient and easily accessible to problem solvers. The problem environment is not very rich and does not offer much information, relevant or irrelevant, beyond the problem itself. The dynamic of the situation is high, with frequent but salient change, that does not create an explicit impasse. The information environment is digital, and the problem context is personal.

The various task dimensions are critical in the description of any given adaptive problem, and the difficulty drivers are the operational building blocks through which task dimensions are implemented in the units and items of the test (right panel of Figure 4.1). However, the task dimensions only reflect the *adaptive problem*, and they do not directly describe in any relevant manner the cognitive and metacognitive processes underlying adaptive problem *solving*.

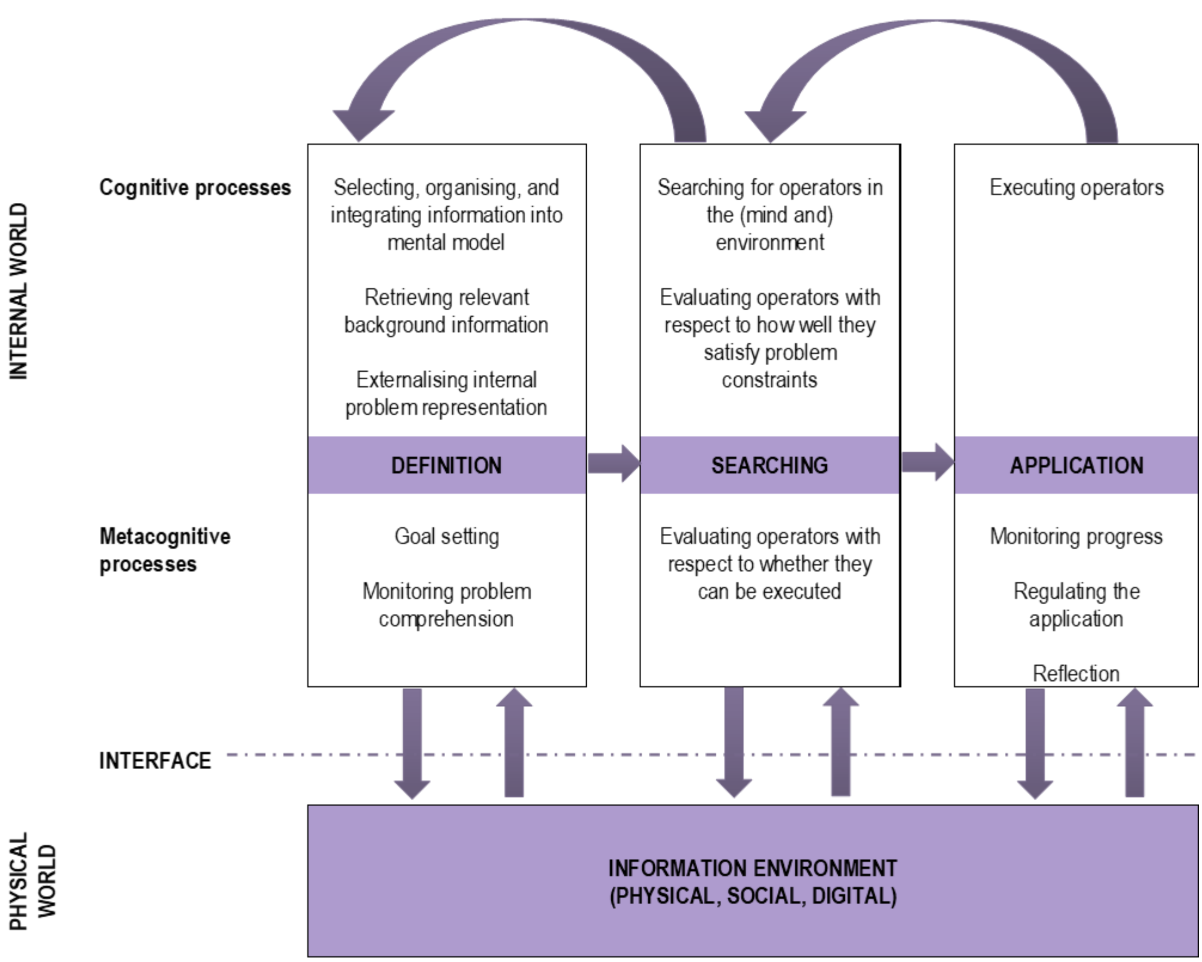
For the cognitive and metacognitive processes, it is assumed that three distinctive cognitive and metacognitive stages, i.e., definition of the problem, search for a solution, and apply the solution (second panel of Figure 4.1), are involved to differing degrees in the process of solving the respective problem tasks. These cognitive processes are inherently bound to the problem-solving process.

Purposeful construction of the units and items of the test uses the task dimensions and their respective difficulty drivers as building blocks with which to elicit the relevant cognitive and metacognitive processes in problem solvers. The next section will focus directly on these important, and often intertwined processes that are key to any APS task.

Cognitive and metacognitive processes in adaptive problem solving

As stated in the definition of APS, there are multiple cognitive and metacognitive processes a problem solver has to accomplish in order to arrive at a problem's solution. These processes can be organised with respect to three stages of problem solving, namely, *defining the problem*, *searching for information* relevant to its solution, and *applying a solution*. Figure 4.2 illustrates how APS is conceptualised according to these stages (shown as boxes organised from left to right to reflect the overall process of adaptive problem solving) and the processes embedded within each stage.

Figure 4.2. Adaptive problem solving



Source: Adapted from Greiff et al. (2017, p. 19[6]).

In the following, we will define the cognitive and metacognitive processes within each stage of APS from an assessment perspective and illustrate them by referring back to the example tasks provided in Boxes 4.1 and 4.2. For each process, we will make connections to the previous section on task dimensions to exemplify how they elicit cognitive and metacognitive processes and render them more or less challenging for problem solvers. Only few references will be made to task dimensions (4) and (5) since they refer to the contextual embedding of the problem and its solution-relevant information only; it is assumed that these task dimensions have no systematic influence on the quality of the cognitive and metacognitive processes that need to be conducted to solve a problem (e.g., constructing a mental model of the problem is not inherently different for problems embedded in either a physical or digital information environment nor do personal problems require different processes than social ones).

The present section ends with some general remarks regarding the relationship between the conceptual framework of APS (Greiff et al., 2017[6]) and the way the cognitive and metacognitive processes are considered when conceptualising them from an assessment perspective. While the present description is grounded in the conceptual framework (Greiff et al., 2017[6]), some amendments are necessary to take into account the specific requirements and constraints of the assessment context.

The remainder of this section will start with the definition of cognitive processes (shown in the upper part of Figure 4.2) and then turn to metacognitive processes (shown in the lower part of Figure 4.2). This is done because cognitive processes, which refer to reasoning about the problem and its solution, are involved in any kind of problem solving irrespective of how the task dimensions are implemented in the problem. In easy problems, cognitive processes may be conducted without considerable effort. Especially in more complex problems, however, these cognitive processes may require effortful monitoring and control to ensure that they are correctly executed. For instance, any change of information about the problem (as introduced in example item 2 of the *Dinner Preparation* example) will make it necessary for a problem solver to verify the understanding of what the problem is about and whether the initially derived solution plan still matches the current problem configuration. As a consequence, problem solvers also need to apply metacognitive processes by reasoning about the quality of their own thinking. Box 4.4 illustrates the cognitive and metacognitive processes necessary for the two example units.

In general, more complex problems are more likely to require metacognitive processes in order to be solved effectively. That is, the more (interacting) elements and relations are involved in the problem configuration (task dimension 1), the more dynamic a problem is (task dimension 2), and the richer, the more unstructured and less salient the information environment is (task dimension 3), the higher the likelihood that metacognitive processes will be involved. Of all these task dimensions with their respective difficulty drivers, task dimension 2 (dynamics of the situation) is likely to contribute most strongly to metacognitive requirements in APS, since any change in the problem configuration or the information environment always requires monitoring whether one's reasoning is still aligned with the newly evolving situation and possibly modifying one's cognitive structures (i.e., the mental model of the problem and/or the solution plan).

Box 4.4. Cognitive and metacognitive processes in the example units

First of all, the problem would need to be defined on a cognitive and a metacognitive level. From a cognitive point of view, the *Dinner Preparation* example requires problem solvers to search for the relevant information about the goals by browsing the map, the problem requirements and by selecting, organising and integrating the information to plan the fastest route. The *Stock Market* example requires problem solvers to mentally organise and integrate the information about the companies and their histories in order to plan the most promising investment strategy. From a metacognitive point of view, the *Dinner Preparation* example requires problem solvers to set subgoals – for example, to first drive to school, then to the store. Both problems require problem solvers to monitor their problem comprehension.

On a cognitive level, the second stage of the adaptive problem-solving process, searching for solution, would involve the search for relevant information on the map and the sticky note in the *Dinner Preparation* example. For the *Stock Market* example, it would involve a continuous search of changes in the problem statement and the environment, and an analysis of these continuous changes. On a metacognitive level, problem solvers would need to evaluate different alternatives to accomplish both goals in time in the *Dinner Preparation* example. In the *Stock Market* example, problem solvers would need to constantly look at the most promising investment alternatives continuously opening up as a function of the "daily" changes in company prices.

In the apply the solution stage, in both of the examples, the plans would then be applied to solve the problem on a cognitive level, while, on a metacognitive level the progress would be monitored.

Cognitive processes

In the following we will describe the different cognitive processes as specified in Figure 4.2.

Problem definition: Mental model construction

In order to define a problem, a person needs to construct a mental model of the state of affairs described in the problem (Mayer and Wittrock, 2006[10]; Nathan, Kintsch and Young, 1992[11]). This mental model comprises information on the initial state (i.e., the problem configuration, cf. task dimensions), the goal state to be achieved, the legal operators, and the set of intervening states that are required in order to move from the initial state to the goal state; together these various states make up the problem space (Klahr, 2002[12]; Klahr and Dunbar, 1988[13]; Newell and Simon, 1972[14]; Vollmeyer, Burns and Holyoak, 1996[15]). Accordingly, items assessing mental model construction need to provide an account of the accuracy and comprehensiveness regarding the problem solver's understanding of what the problem is about. Three cognitive sub-processes were identified in the conceptual framework (Greiff et al., 2017[6]) as contributing to mental model construction (cf. lower left corner of Figure 4.2). In the following, these will be re-introduced and discussed from an assessment perspective.

1) Selecting, organising, and integrating problem information into mental model

To define the problem, one first needs to *select relevant information* about the initial problem state. This means that a problem solver will need to decide for every piece of available information whether it is necessary in order to understand the current problem configuration. The exploration of information will be rather broad and involve the use and evaluation of multiple sources of information as resources with respect to their reliability, relevance, adequacy, and comprehensibility. The selected information will then need to be *organised and integrated into a coherent mental representation* that comprises all information that is known about the problem configuration.

The more (interacting) elements and relations a problem contains, the less salient the problem information is (e.g., because problem-irrelevant information is also included in the problem statement, task dimension 1), and the more the problem information is subject to change over time (task dimension 2), the more difficult will it be for a problem solver to select, organise, and integrate problem information into an accurate mental model. Accordingly, items can be varied along these dimensions to make this cognitive process more or less challenging for problem solvers. Items assessing mental model construction need to reflect whether a problem solver considered all relevant information for defining the problem, while ignoring irrelevant information also embedded in the storyline.

Box 4.5. Selecting, organising, and integrating problem information into mental model in the example units

For instance, in the *Dinner Preparation* example, an example item could consist of a list of options describing which information is available for solving the problem (e.g., driving times to reach a grocery store, its opening hours, availability of organic food). The problem solver is then asked to tick all information categories that s/he wants more details on. In example item 1, only driving time matters for the problem definition; hence, none of the other options should be ticked. Such an item provides information on a problem solver's accuracy in solving the problem, while at the same time delivering information on the underlying cause of problem-solving failure, namely, a problem solver's inability to construct an adequate mental model of the problem.

2) Retrieving relevant background information

In real-world problem solving, relevant background knowledge will help an individual to distinguish between relevant and irrelevant information as well as building a coherent mental model. Memories from past problem-solving activities are one important source of background knowledge. Thus, a problem solver has to activate these memories from past problem-solving activities, which has been shown to be difficult for many problem solvers who fail to recall these past activities and do not recognise that they possess potentially helpful past experiences (Ross, 1989[16]). Moreover, many problem solvers will fail to distinguish between a problem's structural features, which will affect how the problem can be solved and superficial or contextual features that are irrelevant to its solution (Braithwaite and Goldstone, 2015[17]; Ross, 1989[18]). Therefore, they will activate memories of past problems that are only superficially similar to the problem at hand or construct a situation model that is heavily based on irrelevant information, which will misguide the subsequent problem-solving steps.

Accordingly, a problem solver's ability to make effective use of his or her past experiences and knowledge is likely to have a profound impact on performance in real-world problem solving. However, assessing this sub-process in the second cycle of PIAAC is problematic for various reasons. It is not known what kind of prior or expert knowledge problem solvers already possess nor can it be comprehensively assessed; moreover, expert knowledge is likely to vary between individuals and countries. The goal in the assessment is to include problems that are accessible to most people, thereby also not confounding availability of expert knowledge with a person's ability to solve problems. Accordingly, while problems cannot be totally free of background knowledge, problems in which expert knowledge is required or where those with expert knowledge will find that the scenario conflicts with what they know should be avoided.

3) Externalising internal problem representation

Even though problem solving itself is mostly an internal process (Mayer and Wittrock, 2006[10]), it can largely benefit from externalising one's thoughts. With respect to the construction of a situation model, problem solving will benefit from *forming an external representation of a problem's main features* [e.g., in a drawing or table; (Ainsworth, Prain and Tytler, 2011[19]; Fischer, Greiff and Funke, 2012[20]; Zhang, 1997[21])].

From an assessment perspective, these externalisations can provide important insights into the way a person conceptualises a problem and into his or her misconceptions or gaps in the mental model (Lee, Jonassen and Teo, 2011[22]). Hence, it is suggested to include externalising tasks in the assessment that ask problem solvers to make a drawing or create a table, where they would need to include all the relevant features and show the relationships among those features. Because problem solvers are explicitly instructed to create externalisations, such tasks do not assess spontaneous use and hence the cognitive process underlying it. Rather, such tasks are recommended because they are instrumental to the assessment of yet another, albeit pivotal process contributing to mental model construction, namely, selecting organising, and integrating problem-relevant information in a specific format. In consequence, the same task dimensions as for selecting, organising, and integrating problem information affect the difficulty by which a mental problem representation can be externalised.

Search solution: Identifying effective operators

This second stage heavily relies on the mental model that was built when defining the problem (cf. middle box in Figure 4.2). The solution of the problem can be described as the sequence of steps necessary to get from the initial state of the problem to the goal state. The process of searching for a solution marks the distinction between a task and a problem. A task is present if a solution can be directly retrieved from memory and applied to the situation at hand effortlessly and without modification. A problem, on the other hand, requires that a person breaks down a problem into parts, searches for a solution among different alternatives, plans a sequence of actions, and possibly tries out different ways of reaching the goal state (Gick, 1986[23]). The search for a solution thus requires cognitive strategy knowledge on different solution methods and the metacognitive skills to handle this knowledge (Fischer, Greiff and Funke, 2012[20]; Mayer and Wittrock, 2006[10]).

Two cognitive sub-processes were identified in the conceptual framework (Greiff et al., 2017[6]) as contributing to solution search. In the following, these will be re-introduced and discussed from an assessment perspective.

1) Searching for operators in the (mind and) environment

Whereas information search aimed at defining the problem is tailored towards understanding the problem with the goal of acquiring as much knowledge as possible about the problem, the search during this stage aims at identifying possible operators that will help to make the transition from the initial state to the goal state [cf. dual space theory (Klahr and Dunbar, 1988[13]); see also (Greiff, Wüstenberg and Avvisati, 2015[24])]. Operators may be in the mind of the problem solver (i.e., cognitive actions such as adding two numbers) or they may be located in the information environment. In general, the more complex the problem configuration and the features of the information environment (task dimension 3) are, the more difficult searching for operators will become.

Box 4.6. Searching for operators in the example units (1)

For instance, in the *Dinner Preparation* example there is one overarching operator that refers to taking the car to go grocery shopping and that has different instantiations in that the stores are differentially suited to fulfill the problem's constraints given the driving times to them. In the *Stock Market* example, on the other hand, there are two operators (i.e., buying and selling stocks) with various stock options, making this problem harder than the *Dinner Preparation* example (cf. task dimension 1). As for the complexity of the problem configuration, the map used in the *Dinner Preparation* example might not be as clean as the one presented above, but might be very cluttered with unnecessary information and occlude the relevant information on driving times, in which case searching for operators would be far more difficult.

Sequences of operators that are determined prior to executing solution steps make up problem-solving plans. In the remainder, we will always talk about operators only, even though in a specific problem they might be composed into a problem-solving plan.

Searching for operators involves using appropriate devices, tools or information as well as communicating and coordinating one's activities with other parties [cf. collaborative problem solving (OECD, 2017[8])]. Resources for locating operators may hence be located in the social, physical, or digital environment. Due to the digital assessment to be implemented in the second cycle of PIAAC, access to resources is always embedded in a digital interface for the sake of representing the problem, but this does not mean that the resources would be necessarily digital in the real world as well.

Box 4.7. Searching for operators in the example units (2)

For instance, in the *Dinner Preparation* example, the map to read off driving times to different grocery stores might as well be a physical map; on the other hand, the diagrams illustrating the dynamics of the stock market are likely to be digital even in the real world since they need to be updated in real time.

Because the situations in which 21st century citizens solve problems often undergo change over time (cf. task dimension 2), APS requires that they constantly update their knowledge about operators.

Box 4.8. Searching for operators in the example units (3)

For instance, in the second, dynamic example item of the *Dinner Preparation* problem the problem solver receives a message while being on the road that there has been a water leak in the designated grocery store, thereby requiring a change of plans. Similarly, in the second example item of the *Stock Market* problem there is constant change in the performance of the different companies that needs to be considered when buying or selling stocks.

2) Evaluating operators with respect to how well they satisfy problem constraints

There may be many operators that come up during the aforementioned search for operators, but not all of them may be legal. That is, they may fail to satisfy the constraints as expressed in the problem configuration.

Box 4.9. Evaluating operators in the example units (1)

For instance, while grocery store A and B may both offer the required food choices, store A may have opening hours that conflict with the requirement of being home before 10 a.m. Hence, for every potential operator it has to be determined whether it is effective in principle (i.e., enables the transition from initial to goal state) and whether it satisfies all constraints.

Evaluation of operators becomes harder for problem solvers, if there are many potential operators and many constraints to be considered (cf. task dimension 1) as well as if information on these operators is embedded in a rich and unstructured environment (cf. task dimension 3). Moreover, whereas in static problems a problem solver can rely on the operators' (un-) suitability for problem solving once it has been evaluated, in dynamic problems, a problem solver has to continuously re-evaluate whether either the operators or the constraints have changed, thereby affecting the effectiveness of the solution.

Box 4.10. Evaluating operators in the example units (2)

For instance, a grocery store is no longer available due to a water leakage or a formerly well-performing company does no longer make any profit, which is why its stocks should potentially be sold rather than bought (cf. the dynamic example items of the two sample problems).

In real-world problem solving, the sub-process of evaluating operators typically includes two aspects: evaluating whether the operator is in line with the options that have been provided (e.g., is store A better suited than store B?) and evaluating whether the problem solver is capable of using the operator. The prior evaluation refers to a cognitive process since it requires reasoning about the problem. The latter requires problem solvers to consider their own or the fictitious problem solver's resources that they could invest into applying the solution, thereby addressing metacognitive aspects. From an assessment perspective, these two aspects are difficult to disentangle in an artificial problem-solving context. For this reason, it is recommended that items in this category might be coded on both dimensions for analysis purposes (see section on assessing APS below).

Apply solution: Applying plans and executing operators

During this third stage, a problem solver *applies plans to solve a problem* and *executes the specified operators* (cf. right box in Figure 4.2). This stage relies on having procedural knowledge available (Mayer and Wittrock, 2006[10]). The nature of this procedural knowledge will depend on the requirements of the problem and may, for instance, comprise algebra skills to solve equations, logical reasoning skills or other domain-specific operators. In the context of simulating problem solving for the purpose of assessing problem-solving skills, this process must be confined to selecting an operator, as problem solvers do not actually perform any actions (i.e., they do not actually go grocery shopping).

Note that the conceptual framework (Greiff et al., 2017[6]) mentioned 'predicting the environment' as yet another cognitive sub-process relevant to applying a solution. However, the expert group agreed that this aspect was not well defined and could not be measured so it will be dropped as a process to be included in the assessment.

A summary of the cognitive processes of APS together with a brief definition is provided in Box 4.11.

Box 4.11. Cognitive processes in adaptive problem solving in a nutshell

Defining

(1) *Selecting, organising, and integrating information into mental model*: Constructing a mental representation of the problem space (initial state, goal state, legal operators).

(2) *Retrieving relevant background information*: Accessing memory to retrieve background knowledge (note: assessment tasks should be designed to avoid necessity of this process).

(3) *Externalising internal problem representation*: Creating an external representation (e.g., drawing, table) that illustrates the problem solver's mental model of the problem.

Searching

(1) *Searching for operators in the mind and environment*: Locating information about available action options that might be suited to solve the problem.

(2) *Evaluating operators with respect to how well they satisfy problem constraints*: Determining which of the action options will be best to reach the goal while considering all possible constraints.

Application

(1) *Applying plans and executing operators*: Implementing the selected operator(s) to solve the problem.

Metacognitive processes

As already mentioned, metacognitive processes are also inherently bound to the process of problem solving. However, metacognitive processes become more important to the extent that problems are more complex and difficult to comprehend, that the problems change, and that progress towards the solution becomes more difficult.

Problem definition: Setting goals and monitoring problem comprehension

Problem-solving situations in real life may differ in whether the goal (i.e., what is to be achieved) is clear and whether only the way to get there is not yet known. In particular, there may be ambiguous problems where the goal and hence the direction to take in order to solve the problem needs to be figured out first. Moreover, especially in complex problems, that is, problems whose solutions are composed of multiple steps (cf. task dimension 1) or that require adaptation to changing circumstances due to their dynamic nature (cf. task dimension 2), the problem solver has to constantly evaluate whether the current understanding of what the problem is about still matches the current state of affairs. Thus, s/he must monitor the quality of the cognitive processes regarding the definition of the problem. Due to the fact that goal setting and monitoring problem comprehension require thinking about one's own state ('what do I want to achieve?') and mental representations rather than contemplating about the problem, these processes are metacognitive in nature.

Accordingly, the conceptual framework mentioned 'goal setting' and 'monitoring problem comprehension' as two important metacognitive sub-processes (Greiff et al., 2017[6]), which are shown in the lower left part of Figure 4.2. For reasons mentioned below, the assessment framework will consider only the latter process.

1) Goal setting

Goal setting refers to defining dimensions of the problem that require a change and identifying features that characterise the state one wants to achieve. Different from the initial problem state, the definition of the goal state crucially depends on the problem solver, his/her motives, and the resources that s/he has available, and also willing to invest these, for a favourable outcome. Hence, setting goals requires reflection about one's own cognition and motivation, thereby making it a metacognitive process.

In real life, goal setting is an important metacognitive process when solving a problem for one's own purpose, since it gives direction and is the motivational driving force behind many actions taken towards solving the problem. However, from an assessment perspective, letting problem solvers chose among different goals would impose immense challenges in terms of scoring their performance, since problem solvers would differ in their goals, which in turn determine which solution steps would be appropriate. Hence, every goal would require its own scoring rules; moreover, problem solvers might even set goals whose achievement is not supported by the information environment made available in the assessment. For these reasons, goal setting will not be assessed in the APS tasks as the goals will be given to the problem solver in the description of the units.

Box 4.12. Goal setting in the example units

For instance, in a real-world situation, a problem solver faced with the *Dinner Preparation* problem might actually decide to give up the initial goal of preparing a healthy dinner and get take-out food instead; in the *Stock Market* problem s/he might contemplate between making a quick, but potentially risky bargain versus optimising profit in the long run at a medium level, but with less risk involved.

2) Monitoring problem comprehension

An accurate understanding of the problems' initial and goal state (i.e., "where am I and where do I need to be?") is crucial for all subsequent problem-solving steps. Hence, problem solvers need to monitor whether their understanding of the problem is sufficient in order to find a solution to it. An accurate comprehension monitoring is especially important, since it will determine whether the process of defining the problem is adequately regulated (Nelson and Narens, 1990[25]). For instance, overconfidence in one's understanding of the problem may lead to a premature termination of the search for problem-relevant information, whereas underconfidence may yield an inefficient construction process, where information search is continued even after all relevant information has been identified. Research on metacognitive judgements has shown that many people, especially those with little prior knowledge, make rather inaccurate judgements of their level of comprehension and rely on invalid cues when making these judgements (Bjork, Dunlosky and Kornell, 2013[26]). Notably, monitoring becomes more difficult the more information needs to be considered when constructing a mental model of the problem (task dimension 1). Moreover, dynamic problems require constant monitoring of problem comprehension, since the problem configuration may be affected by the dynamics (task dimension 2).

In contrast to some of the other metacognitive processes, monitoring problem comprehension can be assessed relatively easily by administering items in which problem solvers have to indicate whether they would require additional information on the problem before they can start solving it.

Box 4.13. Monitoring problem comprehension in the example units

For instance, in the *Dinner Preparation* example, only upon taking an action (e.g., activating an additional display option by clicking on it) would the map display not only the locations of the grocery stores but also the problem solver's location, which is necessary to infer the driving distances. Problem solvers who take this action are aware of the fact that their understanding of the problem's initial state is incomplete and that further information is necessary. Similarly, items could ask problem solvers whether they have understood the problem and relate their answers to their actual comprehension performance. Ideally, corresponding questions should be asked by an agent or problem-solving partner, thereby embedding the assessment into the story line and making the assessment of metacognition less evident. In the *Dinner Preparation* example, for instance, a problem solver may respond to a friend's question that s/he has looked up the opening hours to grocery store A, so that s/he is ready to go – thereby not accounting for the fact that driving there would take far too much time in order to be back home at 10 a.m.

Search for solution: Evaluating operators with respect to whether they can be executed

Operators need to be eventually selected based on an integrated evaluation of their effectiveness and their ability to satisfy problem constraints as well as internal constraints such as the problem solver's ability to apply an operator (cf. middle box in Figure 4.2). Because these two evaluation criteria are difficult to disentangle in an artificial problem-solving context, it is suggested to code items in this category as reflecting both, cognitive and metacognitive processes for analysis purposes. Accordingly, metacognitive evaluation is affected by the same task dimensions, in that it becomes more difficult if there are many potential operators and many constraints (cf. task dimension 1) as well as if relevant information is embedded in a rich and unstructured environment (cf. task dimension 3). Moreover, the need to constantly update the evaluation process makes dynamic problems more challenging (task dimension 2).

Box 4.14. Search for solution in the example units

For instance, to assess metacognitive evaluation processes, in the *Stock Market* example, the problem solver could be involved in a discussion with another broker who suggests two (or more) different plans that fulfill the problem constraints to different degrees. The problem solver could be asked to continue the discussion by making a decision regarding the suggested options and also providing a reason for this decision (e.g., possible answer options: 'both options sound good to me. I will decide spontaneously which stocks to buy'; 'I will go for option A, because …[right/wrong reason]'; 'I do not think that either option will work, because …[right/wrong reason]'). Such a task requires that the problem solver reflects upon the adequacy of the cognition (solution plan) rather than about the problem, which is why such a task is assumed to mainly trigger metacognitive reasoning processes. Again, an agent is introduced to not make the need for metacognitive evaluation less evident and to not trigger processes that, in the real world, would have to be carried out spontaneously.

Apply solution: Monitoring progress and regulating the problem-solving process

When applying a solution, problem solvers need to evaluate whether they are making progress towards the goal and/or take actions if this is not the case (cf. right box in Figure 4.2). Especially in dynamic problems (task dimension 2) there may be changes in the problem configuration or obstacles that may affect the availability of operators, thereby making it necessary to regulate the problem-solving process and to modify existing plans in order to steer towards goal achievement.

1. Monitoring progress

When executing a problem-solving strategy, a problem solver needs to constantly monitor the degree to which progress towards solving the goal has been made. To do so, it is important that the goal has been defined in a way that clear criteria for goal achievement exist against which the current problem state can be evaluated. In the case the goal state has been achieved, the problem-solving process can be terminated. However, monitoring will often lead to the detection and interpretation of unexpected events, impasses, or breakdowns. If there is no or too little progress towards the goal state, problem solvers will need to identify possible reasons for this in order to regulate their future efforts accordingly (see below). Importantly, again test items need to be designed in a way that they do not trigger monitoring.

Box 4.15. Monitoring progress in the example units

For instance, a variant of the *Dinner Preparation* example could involve a more complex task where the problem-solving process is interrupted at a point where two subgoals (e.g., doing part A of the grocery shopping and picking up the child) have already been achieved. The problem solver could be asked what next options would be. If s/he decides to drive home to prepare dinner – thereby forgetting that part B of the shopping in a different store has not yet been accomplished – this suggests poor progress monitoring. Similarly, in the *Stock Market* problem the goal could be to buy and sell stocks so that at a given point in time the custody account is of a certain value. If a problem solver stops interacting with the simulation prior to having reached this value, this would denote poor progress monitoring.

2. Regulating the application of operators

The process of regulating the application of operators heavily depends on progress monitoring (Bjork, Dunlosky and Kornell, 2013[26]; Nelson and Narens, 1990[25]). When progress monitoring implies that the goal has been reached, the application process can be terminated. When a problem-solving failure due to an inadequate plan has occurred, the problem solver needs to devise a modified or entirely novel plan, thereby backtracking to earlier stages of the problem-solving process. Alternatively, the plan may have been adequate, but a problem solver may have failed to carry out the involved operators, because s/he lacked the procedural knowledge. In this case, the formerly devised plan can still be used to solve the problem, but its execution needs to be optimised. Finally, modifications might be necessary because of changes in the problem configuration and its constraints (cf. task dimension 2), which would be noticed by a problem solver if s/he was good at monitoring problem comprehension.

Box 4.16. Regulating the application of operators in the example units

For instance, in a variant of the *Dinner Preparation* example impasses may occur during execution of the plan such as that the problem solver notices that store A actually ran out of fish, which is, however, a necessary ingredient for dinner. In contrast, there could also be other things on the shopping list that are not available at this moment as well, but that are not necessary for dinner on that day. Items can assess whether problem solvers in the first scenario will plan to go to a different store instead to fetch the missing ingredient there (correct option) or drive home instead; for problem solvers in the second scenario driving home without making a detour to a second store is the correct option. In the *Stock Market* problem, the change from example item 1 to 2 where suddenly formerly well-performing companies now show a dip in their performance requires that the problem solver notices that these companies should no longer be considering in buying stocks.

As can be seen, regulation also requires the comparison of different solutions, which is why the latter process that had been mentioned separately in the conceptual framework is subsumed here.

3. Reflection

People who are good at solving problems have been shown to reflect upon their problem-solving experiences and abstract strategy knowledge from it that can be put to use in future problem-solving situations. Thus, problem solving is assumed to leave memory traces, which can be used in the future. This sub-process involves the development of a principal or set of principals related to general problem solving. While being an important aspect for the development of problem-solving expertise, it is unlikely that this can be assessed in the context of a large-scale assessment.

A summary of the metacognitive processes in APS together with a brief definition is provided in Box 4.17.

Box 4.17. Metacognitive processes in adaptive problem solving in a nutshell

Defining

(1) *Goal setting*: Deciding upon what the to-be-achieved state is about (cannot be considered in large-scale assessments because allowing problem solvers to set their own goals would yield too many degrees of freedom).

(2) *Monitoring problem comprehension*: Supervising whether one's mental model of the problem matches the current state of affairs.

Searching

(1) *Evaluating operators with respect to whether they can be executed*: Determining which of the action options will be best to reach the goal while considering all possible constraints.

Application

(1) *Monitoring progress*: Determining whether executing operators achieves the desired outcome.

(2) *Regulating application of operators*: Modifying selection of operators in case the problem configuration has changed (cf. monitoring problem comprehension) or impasses have been noted (cf. monitoring progress).

(3) *Reflection*: Deliberating about one's own capabilities to solve problems with the goal of abstracting knowledge from it that can be applied in the future (cannot be considered in a large-scale assessment context because it requires repeated confrontation with similar problem-solving instances).

Conclusions

In the previous section we have attempted to illustrate the cognitive and metacognitive processes that constitute APS referring back to the example items provided in Boxes 4.1 and 4.2, to describe how they are affected by the different problem characteristics, i.e. task dimensions, described previously, and commented on their relevance and how well they can be assessed in a large-scale context. General principles regarding the design and scoring of items for the assessment of APS will be addressed in the next section; however, here we would like to point out some important issues that arise when attempting to consider cognitive and metacognitive processes underlying APS in a large-scale assessment such as PIAAC.

(1) Not all processes are equally important to APS. For instance, once a comprehensive mental model of a problem has been constructed and the correct operators identified, applying operators from a cognitive perspective may just be a technicality. On the other hand, metacognitive processes during the latter stage can play a major role for problem-solving success, especially if the problem solver faces impasses or the problem configuration changes. Hence, it is unlikely that processes will be equally distributed across problem-solving assessment scenarios without distorting their naturally occurring distribution in real-world problem solving.

(2) Not all processes can be considered in a large-scale assessment context. Some processes such as setting a problem-solving goal and managing this goal during problem solving (i.e., making sure it is maintained and shielded against distractions) are highly relevant from a metacognitive perspective in that they can provide substantial barriers for problem solvers; however, the test-taking situation requires that the goal is already pre-defined so that its accomplishment can be unambiguously scored as correct or

incorrect. As a consequence, some processes, albeit important from a conceptual perspective are not considered in the assessment framework discussed here.

(3) Not all processes can be unambiguously disentangled in a large-scale assessment context. Some processes are difficult to tease apart in an assessment situation where no “real action” is required. For instance, selection of a set of problem-solving operators and its application appear to be the same in a test, where, for example, a problem solver does not actually need to drive the route to get to a shop. As a consequence, in some cases it is suggested to merge processes into one, where no separation in an assessment context seems possible. Moreover, in real life, cognitive and metacognitive processes can usually not directly be observed and they are tightly intertwined with each other. For this reason, in some cases it is suggested to devise items that can be scored both ways, as being evidence for cognitive and metacognitive processes.

(4) An explicit assessment of processes is likely to alter their occurrence. Especially metacognitive processes may often be implicit only. Thus, they may often be better reflected in the ease of problem solving (e.g., in response times, choices NOT made, or feelings of confidence in one’s decisions) than in a ratable response to an explicit question. Moreover, explicit questions tailored towards metacognitive processes may serve as trigger for these processes, which would otherwise not have been conducted spontaneously by the problem solver. For instance, explicitly asking a problem solver whether s/he has fully comprehended the problem will most likely make him or her monitor comprehension in that situation; however, the response will not be a good indicator of spontaneous monitoring. This problem pervades research on metacognition and a lot of effort is invested into identifying more implicit measures of metacognition. For the assessment context, it is suggested to embed tasks targeting the problem solver’s metacognition as much as possible into the storyline of the problem, so that their true purpose remains concealed.

Reporting proficiency in adaptive problem solving

So far, we have described the different task dimensions that define an APS task and specified the various cognitive and metacognitive processes that form the basis of the problem-solving process. We also outlined how these processes translate into the actual assessment of APS. In a next step we describe the way in which the quality of the solution of an adaptive problem depends on the problem solver’s proficiency to deal with the various demands. These demands are inherent in the *quantitative* task dimensions (1) to (3) and their respective difficulty drivers (see right panel of Figure 4.1 and previous section). Task dimensions (4) and (5) however, are only of *qualitative* nature and do not contribute to the actual process of problem solving.

More specifically, whether a problem solver scores high or low in APS will depend on how s/he deals with different problem configurations (task dimension 1), the dynamics of the situation (task dimension 2), and features of the environment (task dimension 3), whose respective difficulty is determined by the assumed difficulty drivers (see Annex 4.A. for a detailed description of the difficulty drivers and how they shape the difficulty of a problem). In the following, we differentiate high from low scorers in the three relevant task dimensions to build the ground for the specification of the assumed APS proficiency levels (see right panel of Figure 4.1).

Problem solvers may score low or high when confronted with different problem configurations (cf. task dimension 1). Low and high scorers will exhibit different levels of cognitive and metacognitive processes. In any possible adaptive problem,

A low scorer:

- integrates in his/her mental model only a small number of elements, relations and operations;
- accesses only that extra information that is readily available and that does not require the problem solver to take extra steps (such as pushing a button in the interface);

- understands only simple, clear, direct and straightforward effects and understands incompletely or incorrectly those problems that contain indirect effects, or effects generated by interactions between various elements;
- identifies operators that are not salient, i.e., resources that are not readily available and identifiable as such;
- handles only one task at a time, has difficulties in handling several tasks in parallel;
- considers only one of several goals (end states) at the same time for a problem; only focuses on a single goal at a time; if several goals are given for the problem, needs to accomplish them one after the other (consecutively).

A high scorer:

- mentally manipulates and integrates in his/her mental model a large number of elements and the relations between them;
- accesses information that is not immediately and readily available by taking the extra steps needed;
- understands complicated effects based on non-linear relationships, and on interaction effects between operators;
- identifies resources and relationships that are not salient, i.e., are not straightforwardly defined as such, but are “hidden” in the context;
- handles multiple tasks at the same time, such as controlling multiple effects towards an end goal; considers several goals at the same time, as end states of the problem-solving process, and works towards their accomplishment in parallel (not consecutively).

Box 4.18. Task dimension 1 low and high scorers in the example units

For instance, low scorers in the *Dinner Preparation* example will have difficulty in keeping in mind the various elements of the problem, and will need to continuously check on the routes and on the sticky note. They will try to only handle one task at a time and will have difficulties in handling potentially competing goals. They will use the resources that are on screen, but in case the problem will permit invoking a calculator to aid in planning the route, they may not press the button that is needed in order to make use of this resource. In the same example, high scorers will handle various goals at the same time, will use the resources available on screen while also identifying those resources that are not readily available (such as the calculator), and will keep in mind all the various elements of the problem.

Problem solvers may also score low or high when confronted with different dynamics in a situation (cf. task dimension 2). Low and high scorers will have different abilities to cope with dynamic changes during the problem-solving process. In any possible adaptive problem,

A low scorer:

- identifies only some of the features that change;
- identifies only the most salient features, and may miss those that are less salient;
- reacts only to change that is transparent, for example when s/he is prompted that something changed;
- is based in reasoning on the current situation, has difficulties in predicting future change based on past changes (or prior information);
- builds incomplete or incorrect mental models of the change process (to understand how and why "things" change);

- adjusts the mental model to change incompletely or incorrectly (e.g., has difficulties in making adequate changes to resolution strategy).

A high scorer:

- identifies all relevant features that change, irrespective of their number, salience, transparency;
- predicts likely future changes based on past changes (prior information);
- constructs a mental model of the actual change (not only of the problem) (i.e., understands how and why things change);
- adjusts the mental model to changes (e.g., changes resolution strategy if needed).

Box 4.19. Task dimension 2 low and high scorers in the example units

For instance, low scorers in the *Stock Market* example might not identify that the prices for all stocks have changed. They will have difficulty predicting future changes in any of the stocks, and may only be able to predict how stocks will vary in the case of those that have a very transparent and univocal past evolution. They may build incomplete or incorrect mental models of the problem and its dynamics. In the same example, high scorers will quickly identify that change takes place in all the stocks, on a "daily" basis, will correctly predict future changes based on prior evolutions of these stocks, will build a correct mental model of the problem and its dynamics. Based on these abilities to constantly monitor the problem solution and to react to changes, they will easily adjust this mental model to any supplementary change, if induced, i.e., they will adapt to the new circumstances.

Finally, problem solvers may score low or high when confronted with different features of the environment (cf. task dimension 3). Low and high scorers will have different abilities. In any possible adaptive problem,

A low scorer:

- works with only one or a small number of variables about the state of the environment;
- integrates only one or a small number of variables from the environment in the conceptualisation of the problem;
- filters out distractors with difficulty and incompletely; is distracted by irrelevant information; continuously manipulates variables that have no effect on anything;
- is distracted by background material; does not recognise distractors; continues to consider all material, even if not relevant (e.g., reads through all the update notes);
- interacts with structured environments, but interacts in an inefficient (and sometimes not meaningful) way with environments that are not structured.

A high scorer:

- mentally manipulates and integrates in mental models a large number of variables about/from the environment;
- integrates "the environment" (and its variables) in the conceptualisation of the problem;
- filters out distractors (irrelevant information);
- focuses on relevant variables from the environment, is not distracted by stimuli that are external to the task or are irrelevant for the task;
- recognises the distracting background material;
- interacts efficiently with unstructured environments (i.e., structures environment, constructs mental model of environment).

Box 4.20. Task dimension 3 low and high scorers in the example units

For instance, low scorers in the *Dinner Preparation* example will only integrate a small amount of the available information in their conceptualisation of the problem. They will be distracted by irrelevant background information and will operate the map in an inefficient way. In the same example, high scorers will integrate a large number of only relevant information into their mental model of the problem. They will recognise changes in the environment and will interact with the map efficiently even if the map would be cluttered with irrelevant information.

The described core task characteristics and their difficulty drivers form the basis upon which the high and low scorers of APS can be described. However, the final score of problem solvers is not directly interpretable, unless related to their proficiency level. Using the task characteristics and difficulty drivers identified in the framework, the expert group will define levels of proficiency and explain what each level means. In other words, what are the specific components of APS that can be performed with proficiency by a high scorer, but cannot be performed by an average scorer, and what are those components that are performed by average scorers and cannot be performed by low scorers? Further, what are the specific components that are expected to be performed even by low scorers?

The proficiency levels will define the scale and will provide a useful way to understand the progression of APS skills. These proficiency levels are associated with the competency of problem solvers, but are also associated with the complexity of items, i.e., the specific components of APS skills that are required by each progressively more difficult items. In Table 4.1, we present a preliminary proposal for APS skills, divided into three proficiency levels. This proposal is based on theoretical considerations about how proficiency may be distributed in the population with the task dimensions as well as the cognitive and metacognitive processes outlined in this framework in mind. This proposal is not based on actual data, and analysis of the main study data will require changes in the number of levels as well as the specific descriptions of those levels of the proficiency scale. The table contains four descriptions for each proficiency level:

a) a general statement of that proficiency level, that can help readers to quickly understand each level;
b) a description of how problem solvers at that specific proficiency level deal with (i.e., adapt to) dynamically changing problems – which is, after all, the basis of adaptive problem solving;
c) a description of the various cognitive processes that are typical for that proficiency level;
d) a description of the various metacognitive processes that are typical for that proficiency level.

Table 4.1. Descriptions of the three APS proficiency levels proposed

	General statement	Dealing with dynamics	Cognitive processes	Metacognitive processes
1	At Level 1, problem solvers successfully solve simple problems in contexts with minor, slow, discrete, and predictable change. They may also be able to solve static (and not dynamic) problems, or only tasks that are part of a static or dynamic problem.	• Problem solvers at Level 1 deal well with infrequent, discrete, or slow changes. They also deal well with changes to which they have been prompted, if these are slow, explicit, discrete, and predictable. • They may perceive THAT changes in the problem environment have occurred, but may need to be prompted towards HOW specifically these changes occurred. • They integrate relevant changes into their problem-solving approach, if prompted to them.	• They define problems with low complexity and low dynamics, especially if prompted towards them, and later identify the relevant changes in the problem statement or the problem environment. They integrate them in a mental model. • They devise partial or complete solutions to static problems and react to changes that are presented in small and visible increments. They adapt their approach in order to retrieve goal-relevant information when they are prompted to them. • They adapt their resolution strategies to changes in the problem statement and the environment, if these changes are of small complexity, and especially if the changes are visible or if they are prompted towards the relevant changes.	• They may successfully evaluate their comprehension of the problem for simple problems, especially when prompted to do so. • They may be able to monitor their progress towards simple goals. • If asked to, they may be able to set subgoals for their progress, and evaluate simple alternatives in order to choose among them. • They may be able to search for solutions to the problem, yet without evaluating alternative solutions.
2	At Level 2, problem solvers successfully solve problems of average complexity in contexts where change has an average impact, pace, and randomness.	• Problem solvers at Level 2 deal well with changes of average frequency and pace. • They usually have good awareness for change, that is, they identify both THAT something has changed and HOW specifically it has changed, but may need to be prompted to specific aspects of the change. • They discriminate between changes that are relevant or trivial to the problem situation. • They predict correctly the general future behaviour of a system based on information that they have about its past behaviour.	• They successfully define problems with average complexity and dynamics (i.e., average pace or frequency) and can later identify the relevant changes in the problem statement or environment. They integrate them in a working mental model. • They devise solutions to a given problem and react to changes that are presented in visible increments. They adapt their approach in order to retrieve goal-relevant information, i.e., information that they consider relevant. • They adapt their resolution strategies to changes in the problem statement and the environment, if these changes are of small or average complexity.	• They monitor their progress towards a goal. • They search for solutions by evaluating alternative solutions to the problem. • They reflect on their solution strategy only when an impasse occurs and when forced to adapt.

	General statement	Dealing with dynamics	Cognitive processes	Metacognitive processes
3	At Level 3, problem solvers successfully solve problems in highly complex and dynamic (continuous-change) problem contexts. They solve complex problems with multiple constraints in the problem configuration and with complex features of the problem environment, and adapt their problem-solving process well to highly dynamic changes in these problems.	• Problem solvers at Level 3 deal well with frequent and even continuous changes. • They have a good awareness for change, that is, they are successful in identifying both THAT changes in the problem environment occurred and HOW these changes occurred. • They discriminate well between changes that are relevant and less relevant or even trivial to the problem situation. • They predict correctly the future behaviour of a system based on information that they have about its past behaviour. They adapt their behaviour according to the expected change.	• They can successfully define highly dynamic problems by selecting relevant information about both the problem and the change. They generate a corresponding mental model that adequately describes the problem situation. • They actively search for solutions by continuously evaluating the information provided by the environment. They adapt their approach in order to continuously retrieve goal-relevant information. • They continuously adapt their solution strategies to changes in the problem statement and the environment; this adaptation is also proactive, as they predict likely changes in their environment.	• They successfully monitor their comprehension of the problem and the changes, as well as of their progress towards their goal. • They search for solutions by setting subgoals and evaluating alternative solutions to the problem. • They continuously reflect on their approach to solving the problem and can successfully get over an impasse by revising their strategy. • They cope well with frequent and unpredictable change and adapt their solution strategy in order to advance their goals.

Assessing adaptive problem solving

The previous section presented the domain of APS and outlined the task dimensions, difficulty drivers, the cognitive and metacognitive processes involved in APS, and the proposed proficiency levels. These elements define the overall, conceptual framework of APS and form the basis for the development of test units and their corresponding items. Ensuring a sufficient match between the conceptual framework and what the APS units and items assess is critical to the crafting of a validity argument. Hence, achieving the greatest possible coverage of the task dimensions and APS processes is the key goal for the test development. The assessment of APS in the second cycle of PIAAC will emphasise the dynamic nature of problem-solving situations as defined earlier and will present problem solvers with newly developed test units that will be suited in information-rich environments.

This section provides an overview of the anchoring of the APS units in the task dimensions outlined in the previous section (see also Figure 4.1), describes overarching test design principles, and explains the scoring and capturing of data beyond item responses that will form the basis of the different proficiency levels.

Anchoring the APS assessment in the task dimensions

The APS units will represent tasks that are comprised of multiple items (i.e., questions). In this sense, an APS unit contains the following key elements: a task stimulus (e.g., introduction to the task, description of functionalities of interactive elements) and multiple items that require the problem solver to adapt to changing situations. The design of the items within a unit will be guided by (1) the task dimensions, and (2) the cognitive and metacognitive processes, as described in previous sections.

Concerning (1), the following five task dimensions formed the development of APS items: problem configuration (i.e., the initial problem setup and goal states), dynamics of the problem situation (i.e., the degree to which the problem situations and its constraints change over time), the features of the environment (i.e., construct-relevant features of information and resources), the types of information sources (i.e., physical, social, and digital), and the contexts (i.e., personal, social community, and work; as defined in the first PIAAC cycle (OECD, 2012[4]). Each and every unit will be mapped onto these five dimensions. However, as we assume information environments and problem contexts in real life to be not equally distributed (cf. section defining APS), we propose to target slightly different proportions of all the problems to be placed in the various environments and contexts as displayed in Table 4.2.

Table 4.2. Proposed distribution of the information environments and problem contexts

Task dimension 4: Information environment	Task dimension 5: Problem context
• Physical: 30% • Social: 35% • Digital: 35%	• Personal: 30% • Work: 30% • Social community: 40%

Concerning (2), all items within the APS units are located within the framework of cognitive and metacognitive processes. These processes comprise defining the problem, searching for a solution, and applying the solution (see section on cognitive and metacognitive processes in APS and Figure 4.2). For a specific item, these three processes may be required, both on the cognitive and metacognitive side. Given that the cognitive and metacognitive processes are intertwined, a clear separation of these processes – for instance, in the form of empirically distinct indicators or scores – is hardly possible. As a consequence, the APS items may require problem solvers to engage in multiple processes rather than a single process within the APS framework. Besides, to successfully solve a problem that is subject to change over time, problem solvers have to understand the problem situation and develop a mental model about it (Ericsson and Pool, 2016[27]). Ultimately, the processes of understanding the problem form the basis for all subsequent processes of search and applying a solution. This dependence between the three processes of APS results in the anchoring of the APS items in multiple cognitive or metacognitive processes. However, for a given item, some processes may be more pronounced than others and these items will be assigned to the respective, dominant processes.

The proposed distribution of the three main processes in the APS item pool is shown in Table 4.3.

Table 4.3. Proposed distribution of the three main cognitive and metacognitive processes

Processes	Cognition	Metacognition
(1) Defining the problem	Constructing a mental model (30-40%)	Monitoring the comprehension of a problem (30-40%)
(2) Searching for a solution	Searching for operators in the problem environment (40-50%)	Evaluating operators/plans (40-50%)
(3) Applying the solution	Applying plan and executing operators (20-30%)	Monitoring/regulating progress (20-30%)

As stated earlier, for reasons of test fairness and validity, reference to expert knowledge should be avoided from an assessment perspective. Accordingly, items should be designed so that information on operators should be provided through them. In this regard, the *Stock Market* example is potentially a borderline case, since experience with buying and selling stocks may be very limited in some populations. To make this scenario accessible to problem solvers, it has to be simplified compared with its real-world counterpart.

Test design

Test administration

The APS units will be administered on tablets and allow problem solvers to interact with the problem and information environments directly. The technology-based test administration further enables the implementation of problem situations that change over time or make new sources of information available to the problem solver during the problem-solving process. Moreover, in selected items and units, log-file data of specified actions may be used to inform the development of the described APS proficiency levels.

For the main study, the APS assessment will be administered together with the assessments of numeracy and literacy. Participants will be randomly assigned to two of the three domains. For these assessments, an adaptive test design is anticipated so that each participant does not work on all items within the respective domains. The adaptive testing procedure will be based on units, depending on the dependencies between items within a task. At the beginning of the assessment, participants will be assigned to one of three pathways based on their initial performance on a locator test of their literacy and numeracy skills. This design combines adaptive testing with multi-stage testing and is aimed at maximising the information about the participants gained from the assessments (OECD, 2013[28]).

Design elements

The design of the APS units and items contains several elements that facilitate the assessment of *adaptive* problem solving and ensure the fairness of the test:

a) *Explicitness of change*: In some APS tasks, change in the problem situation is not made explicit so that problem solvers will have to recognise these changes. This design element is construct-relevant as it stimulates metacognitive processes of reflecting on the problem situation and initial mental models given the changes in the environment. This element, however, increases the difficulty of the items and is thus used sparsely. In fact, most APS items make explicit the changes in the problem environment.

b) *Rescue elements*: The design of APS units as a sequence of items that gradually introduce changes to the problem environment may create dependencies between items. In other words, if a problem solver does not succeed in one item, s/he may have a disadvantage in solving subsequent items. To circumvent this problem and to ensure the comparability of items among problem solvers, the APS units will contain rescue elements. These elements represent a certain decision or problem solution to the problem solver that are based on a previous item. However, these elements do not evaluate the problem solvers actual responses on previous items but are entirely independent from the correctness of these responses. In this sense, all problem solvers receive the items with these rescue elements to ensure test fairness.

c) *Gradual introduction of changes*: At the outset of an APS task, problem solvers will be presented with a static problem. The subsequent items will gradually unfold and introduce the dynamics of the problem situation. These changes are mostly made explicit (see above) and may be of discrete or continuous nature. The initial, static tasks will ensure that a measure can be established that forms the baseline for problem solvers' performance on subsequent items.

Demands on literacy and ICT skills

The APS units and items will be designed in a way that the level of literacy required to successfully solve the problem is kept minimal [see Greiff et al. (2017[6])]. To accomplish this, the stimulus material and item statements will be formulated briefly and as clearly as possible, except when the complexity of the materials is construct-relevant (e.g., amount of distracting information for information-rich problems). Furthermore, APS units will not present problem solvers solely with written text but will also provide information in tables, schemes, diagrams, and interactive simulations to reduce the reading load and exploit the advantages of multiple representations of testing material. At the same time, a certain level of literacy will be required to successfully solve the problems, especially in order to understand the problem situation and the information material. How APS distinguishes from other core abilities, namely literacy, numeracy and ICT, will be described in detail in the following section.

Along similar lines, the technology-based administration of the APS assessment in the second cycle of PIAAC will require basic skills to deal with ICT. Whether problem solvers are likely to have these skills will be determined in the tablet training. It must be noted that the required level of ICT skills will be kept low, and APS units will mainly demand the navigation through items, switching between two to three information pages, selecting response options, inserting short responses into text boxes, and manipulating well-defined variables by operating a small number of buttons or sliders. In fact, participants will only need to tap on a selection with a stylus or finger, use drag and drop, and highlight (underline) text. To further assist problem solvers in maneuvering through the APS units, a tablet tutorial will be provided at the outset of the PIAAC test administration. This tutorial supports participants in familiarising themselves with the tools to navigate through the tests. Moreover, PIAAC Cycle 2 chose to administer the performance tests on tablets to facilitate an intuitive handling of the test environment (OECD, 2018[29]).

Drivers of item difficulty

The main purpose of the APS assessment is to assess problem solvers' capacity to successfully solve dynamic problems. To capture the broad variation of proficiency in the PIAAC population of 16- to 65-years old participants, APS units and items will need to vary with respect to their item difficulty. To achieve this, the items will be distributed along the difficulty drivers as described in detail earlier in this chapter (see also Table 4.A.1. in the Annex).

As the second cycle of PIAAC focuses on the adaptive component of problem solving, the manipulation of the dynamics of the problem situation is key to the item development. At the same time, the elements a problem situation is comprised of (i.e., its configuration and the characteristics of the information sources) also play an important role in driving item difficulty. Furthermore, in some instances, the instructions to solve a problem are not fully provided, for instance, when problem solvers interact with a simulation and thereby acquire knowledge about its functionalities. This design feature is relevant to the measurement of APS, as it presents problem solvers with an actual problem situation and triggers metacognitive processes to develop and refine a mental model about the problem situation (i.e., in this case, the functionalities of the simulation).

Assessing metacognitive processes

As noted earlier, metacognition plays an important role in the APS processes, especially as problem solvers monitor their comprehension of the problem, evaluate operators and solution plans, and monitor their progress towards the goal. As these metacognitive processes interact directly with the cognitive processes during problem solving, disentangling them from the measurements of cognition poses a challenge. For example, evaluating one's personal resources and capabilities is an aspect of metacognition that cannot be addressed in a survey such as PIAAC that does not report individual results. Moreover, test questions that are aimed at making problem solver's understanding of a problem explicit by asking them "How well do you think you understood the problem?" seem artificial (and may lack face validity) and could prompt problem solvers' responses in following items or even units.

To obtain some measures of metacognition, the APS assessment provides implicit and explicit indicators that can be derived from item scores or log-file data. For instance, in some APS items, log-file data can provide information whether a problem solver accessed certain information sources (i.e., navigation behaviour). This information may serve as an indicator of metacognitive processes to evaluate certain information sources during "searching for a solution" – in some instances, it may also indicate whether problem solvers reconsider certain pieces of information during the "applying the solution" stage. In general, the navigation behaviour may indicate certain metacognitive strategies to solving the problem.

Next to these implicit measures, some APS items explicitly assess metacognition. For instance, at the end of a problem-solving process, problem solvers may be asked to evaluate a given solution to the problem according to pre-defined criteria. Additionally, problem solvers may be asked to evaluate certain problem-solving strategies according to their efficiency and applicability. Mastering the latter is indicative of problem solvers' metacognitive strategy knowledge [e.g., (Antonietti, Ignazi and Perego, 2000[30]; Efklides and Vlachopoulos, 2012[31])]. Overall, the APS assessment will contain both explicit and implicit measures of metacognition. However, given the nature of metacognitive processes and the challenges inherent in their assessment, metacognitive processes, albeit essential to APS, will not form the major focus of the assessment itself.

For the two example units, metacognitive processes could be traced using several measurement approaches. These approaches are described below (Box 4.21).

Box 4.21. Assessment of metacognition in the example units

Metacognition in the *Dinner Preparation* example is implicitly assessed in item 2 only. It can be assessed whether problem solvers adapt their initial solution according to the new information. Metacognition in the *Stock Market* example is not assessed explicitly in this unit, but implicitly. Item 2 requires the problem solvers to understand that the previously employed and efficient solution is not working any more, due to changes in the environment. They will need to detect the impasse, to understand the reason, and to adapt decisions accordingly.

Item scoring and data capturing

General scoring principles

Each APS item will be scored according to criteria that define the correctness of the responses. For most items, the answers provided by problem solvers (e.g., by selecting a response among given response options, or by selecting certain sets of values for a set of variables) are scored dichotomously as either correct (code: 1) or incorrect (code: 0). Missing responses are also coded (code: 9). For some items, the solution must fulfill multiple criteria so that partial credits may be given. Nevertheless, the item scoring is aimed at providing scores that allow the application of parsimonious item response models – hence, a dichotomous scoring is preferred.

To exemplify the item scoring, Box 4.22 describes how problem solvers' responses are scored in the two sample units.

While the preferred scoring method is to dichotomise problem solvers' performance in items (correct vs. incorrect), in some instances, the scoring may allow for partial credits. Partial credits will be used only if the different scores represent qualitatively different responses or processes. Field trial data will be used to evaluate the appropriateness of partial credit scoring for the main study. The key criterion for considering partial credit scores is therefore their construct-relevance.

As noted earlier, the cognitive and metacognitive processes stimulated by the APS items are intertwined, and, in most APS units, their indicators cannot be separated clearly. As a consequence, the scaling of problem solvers' APS performance will not result in two distinct APS dimensions representing the two types of processes. Along the same lines, the APS assessment in the second cycle of PIAAC does not aim for distinguishing the three processes, define the problem, search for a solution, and apply the solution empirically into three correlated APS dimensions. The reporting of the APS performance scale will therefore most likely not be along these processes, and will most likely result in a single APS scale.

Given the variation of APS items and units across the task dimensions, a possible distinction between dimensions may be based on the dynamics of the situation (e.g., static vs. dynamic items) or the inclusion of metacognitive processes (e.g., items requiring metacognition vs. items not requiring metacognition to a substantial degree). These possible dimensions will, however, not be made psychometrically explicit, for instance, in the form of separate APS scores - they may be used to craft a validity argument for the APS assessment.

Box 4.22. Scoring in the example units

Dinner Preparation

Item 1: "Plan the fastest route to accomplish these goals. Keep the time constraints in mind"
Code 1: Route from Home to School to Shop A selected
Code 0: *Other responses*
Code 9: *Missing*

Item 2: "Adapt your chosen route to accomplish the rest of the goals for the day. Keep the time constraints in mind"
Code 1: Route correctly adapted School to Shop A to Home OR School to Shop C to Home
Code 0: *Other responses*
Code 9: *Missing*

Stock Market

Item 1: "Based on the information provided, which shares should you buy or sell in order to maximise your chance for higher profits next day"
Code 1: The problem solver uses the correct investment pattern to maximise profit
Code 0: *Other responses*
Code 9: *Missing*

Item 2: "Based on the information provided, which shares should you buy or sell in order to maximise your chance for higher profits next day"
Code 1: The problem solver uses the correct investment pattern to maximise profit
Code 0: *Other responses*
Code 9: *Missing*

Log-file data

Next to the scoring of problem solvers' item responses that they submitted directly after completing an item, log-file data are used to retrieve and evaluate certain behaviours while solving a problem. These data may include the sequence of actions, whether or not certain elements in the problem environment were selected or accessed, and the time spent on the tasks. Whereas the latter may be useful to identify test-taking effort or aberrant responses (Goldhammer, Martens and Lüdtke, 2017[32]; Marianti et al., 2014[33]), the former can provide insights into metacognition. Some of these behaviours may even be scored.

For instance, whether or not a problem solver makes use of a certain information source (e.g., a hyperlink to a text that contains relevant information) may be an indicator of both cognitive and metacognitive processes of search for information and understanding the problem. If, indeed, a problem solver does not access this information, the problem-solving success may only be limited due to missing information or a resultant solution that does not fully meet all criteria. For instance, considering the information about time restrictions in the *Dinner Preparation* example is essential to the APS performance. In this sense, log-file data aid the analysis or the description of problem-solving performance within the task. Overall, log-file data may provide data beyond the mere correctness of an item response to indicate test-taking behaviour and, in some cases, metacognitive processes.

Adaptive problem solving in the nexus of related constructs and implications for PIAAC Cycle 2

Up on this point we have described the importance of APS in today's changing world, defined and explained what is meant by APS and have introduced the core dimensions that form an adaptive problem before concretising how APS can be assessed. It is, however, also crucial to theoretically describe what differentiates APS from other core competencies, since APS addresses a set of higher-order cognitive skills that are related to other domains, such as literacy, numeracy, or digital competencies. For example, APS often relies on verbal and pictorial representations that the person has to be able to parse in order to acquire information that is needed to solve the problem. The *Dinner Preparation* example presented in Box 4.1 involves written instructions, a map and a sticky note; and the *Stock Market* example (Box 4.2) has a set of tables and graphs. Regardless of their ability to adaptively solve the problem, problem solvers need to be able to parse and make sense of the information in these representations, which is arguably related to their literacy skills.

In the present section we discuss the status of APS in relation to some of these overlapping domains. We review the similarities and differences between the domains and we list a number of distinctive features that differentiate APS as a construct. We also explain how the design of APS task intends to reduce the potential influence of these related domains.

Adaptive problem solving and literacy

The word literacy is sometimes used in the restricted sense of "knowing to read and write". However, over the past 20 years, the definition has been expanded to reflect abilities related to the functional use of documents, which reflects the growing pervasiveness of reading and writing in post-industrial societies (Rouet and Britt, 2017[34]). In turn, the functional use of a document often entails forms of reasoning that amount to problem solving (for instance, making a decision about which product to purchase based on two descriptions of competing products). Therefore, it is important to clarify the boundaries between APS and literacy.

Literacy is bound to overlap with most areas of assessment because most assessment procedures rely on natural language communication. Put in a concrete way, whatever the testing domain, participants always have to read and comprehend written instructions, questions, and stimuli in order to demonstrate their ability in the respective domains. Completing APS tasks is no exception to this rule as a minimum level of literacy is required to solve an adaptive problem. However, several dimensions contribute to making APS a distinct domain. Some of the main dimensions are the types of representations used in the testing materials, the level of problem specification, and the dynamics of the environment (Table 4.4).

Table 4.4. PIAAC Cycle 2 APS and literacy assessments

	PIAAC Cycle 2 APS assessment	Reading literacy assessment
Types of representations	Materials include verbal and non-verbal representations, including interactive graphs and simulated devices	Materials include texts possibly together with static graphs
Task definition	Tasks may be well defined or ill defined	Tasks are generally well defined
Characteristics of the task environment	Environment may change with time as a function of problem solvers actions or other factors (i.e., a dynamic environment)	Environment is static

Note: Other dimensions that are specific to literacy are not represented here.

In a reading literacy assessment, materials include by definition written texts sometimes with other, adjunct representations such as a graph or a picture. Materials included in the APS assessment will encompass a range of stimuli, some of them almost entirely non-verbal. In addition, reading literacy tasks are meant to be well defined, whereas some problem-solving tasks are intentionally left partially implicit. Finally, a reading literacy environment involves one or several passages of text that are provided at the onset and remain the same throughout the task. APS environments may change with time as a function of a range of factors including the problem solvers actions.

In order to maximise the specificity of APS assessment, care will be taken to develop tasks that do not pose significant challenges from a reading literacy perspective. For example, for those APS tasks that include written texts, these will be limited to short and simple passages in combination with non-verbal representations. For instance, the *Dinner Preparation* example involves a simple narrative and a short list of things to do. The *Stock Market* example contains no extended text passage. Difficulty in this unit clearly comes from the need to handle multiple dynamic sources of mostly non-verbal information, which arguably makes it distinct from a reading literacy task.

Adaptive problem solving and proficient use of information and communications technology (ICT)

Throughout the second half of the 20th century, digital devices (e.g., mainframes, computers, laptops, iPads and smartphones) have spread rapidly and profoundly in developed societies. People's ability to handle these devices has had an increasingly important impact on their access to employment, civic participation and their personal life in general. Numerous calls have been made for governments and other organisations to assess people's ability to use computers and related devices, under various constructs ranging from "ICT literacy" (Eshet-Alkalai, 2004[35]), to "digital competence" [Ferrari (2013[36]), to cite just a few].

Proficient use of digital devices involves knowing how to perform basic operations such as opening a folder, naming a file or updating a piece of software, but also to perform more complex tasks such as managing a photo or e-mail archive, addressing issues with system or application compatibility, or contacting a customer service in order to obtain information. Surveys and assessments addressing people's use of computers have typically included tasks at various levels of difficulty.

Digital devices are used to perform an ever-increasing range of tasks, including non-routine ones. In addition, these devices are typically dynamic and interactive, offering numerous opportunities for adaptation. Therefore, it is relevant to ask how APS differs from an assessment of digital competence. Table 4.5 highlights two of these dimensions.

Table 4.5. PIAAC Cycle 2 APS and digital competence

	PIAAC Cycle 2 APS	**"Digital competence" 1**
Role of digital devices in task environment	Variable from none to central	Typically large
Status of tasks	Tasks involve non-trivial goals	Range of tasks from routine to complex

1. Here the phrase "Digital competence" subsumes the various constructs and frameworks that have addressed people's knowledge of and proficiency at using digital devices.
Source: Adapted from Greiff et al. (2017[6]).

Firstly, some APS tasks will require the use of digital devices and applications whereas others do not. For instance, the *Dinner Preparation* task uses a static map even though it could be set in the context of embarked information systems such as a GPS editor. The *Stock Market* example also uses simple representations although a spreadsheet application could be of some use to people with a high level of digital competence. Ideally, prerequisites in terms of digital competence should remain minimal in an assessment of APS.

Secondly, APS tasks involve non-trivial goals whereas assessments of digital competence may involve routine as well as non-routine uses. For instance, in the *Stock Market* example, information about two companies changes during the completion of the task, requiring the problem solver to adjust their investment decisions accordingly. The demand on ICT use is minimal, although the complexity in terms of goal management is expected to be moderate to high.

Adaptive problem solving and problem solving in technology-rich environments

The prevalence of problem solving in ICT use has prompted efforts to understand what participants can or cannot do when faced with tasks involving non-routine uses of technology. Therefore, the assessment of traditional competencies, namely literacy and numeracy, was augmented by an assessment of individuals' ability to effectively use information and communications technology to solve problems [i.e. PS-TRE; (OECD, 2012[4])]. The domain was defined as:

> *"using digital technology, communication tools and networks to acquire and evaluate information, communicate with others and perform practical tasks." (OECD, 2012, p. 47[4])*

Since the assessment of APS will also use technology-rich environments in which the problem is embedded, it is important to also compare the APS with the assessment of problem solving in the first cycle of PIAAC.

PS-TRE focused on "non-routine" uses of technology, i.e., those in which individuals have to set up *ad hoc* goals and plans, and to access and use information presented on the computer. Thus, the assessment of PS-TRE in the first cycle of PIAAC was an assessment of problem-solving skills as they apply to technology-rich environments. The stimuli were presented in the context of simulated web browser, e-mail, and spreadsheet environments. The tasks required the participants to access information relevant to their needs by using the tools available in the computer applications(s). Depending on the task, one or several applications were available. For example, a task might require respondents to use a web-based reservation system to manage requests to reserve a meeting room and send e-mails to decline requests if reservations could not be accommodated. The environment typically included more information than was needed to solve the task.

In contrast, the assessment of APS in the second cycle of PIAAC will not systematically assess the proficiency of problem solvers to interact with technology-rich environments. Instead, APS focuses on problem solvers' ability to adapt to changing conditions, such as a change in the problem definition, unexpected difficulties when taking a path towards a solution, or simply a dynamic environment that changes in more or less predictable ways as a function of time (see section defining APS). Proficient problem solvers are expected to be able to detect and manage those changing conditions. This may include giving up an initial path towards a solution, backtracking to previous stages in the problem-solving process, and/or incorporating the new conditions into one's strategy to solve the problem.

In summary (Table 4.6), APS tasks will involve a variable amount of information, and most tasks will implement a constraint to adapt to changing conditions.

Table 4.6. PIAAC Cycle 2 APS and PIAAC Cycle 1 PS-TRE

	PIAAC Cycle 2 APS	PIAAC Cycle 1 PS-TRE
Amount of information presented and/or required to solve the problem	Variable	Typically large
Use of computer applications[1]	Required in some tasks, proficient use not part of the assessment	Required in all tasks
Need to adapt to changing conditions	Required in most tasks	Required in a few tasks

1. Both PIAAC Cycle 1 and 2 use simulations of mainstream computer applications such as a spreadsheet or a web browser. The simulations typically feature a limited set of functions (for instance, a sort function on the spreadsheet), which are presented in standard ways so as to maximise transfer from real-life applications.

Summary and conclusion

In this section we have examined the relationship of APS with three related constructs and domains: literacy, digital competence and PS-TRE. Because of their breadth and the universal use of written language to convey instructions and stimuli, the domains are bound to overlap. However, we have listed a few aspects that make APS distinct from the other domains. One aspect is the diversity of the representations used in the problem-solving environment; another is the non-trivial and sometimes partly implicit nature of tasks. Finally, APS uniquely implements environments that are dynamic and interactive.

The domain of competencies that is implemented in APS reflects current demands on individuals, both at the workplace and in society in general. In particular, it addresses the need for individuals to adjust to conditions that may change at a rapid pace and sometimes in unpredictable ways.

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Annex 4.A. Description of difficulty drivers

Annex Table 4.A.1. Description of difficulty drivers

(1) Problem configuration		
Difficulty drivers		**Problem description**
1a: Number of elements, relations, and operations	How many elements does the problem solver need to consider in the context of the problem. This refers not only to elements that are relevant to solving the problem, but also to "clutter".	A **simple problem** will only have very few elements, and all will be relevant to the task. For example: only one dial, and one readout. A **difficult problem** will have a larger number of elements, with relations among them, and some not relevant for the task. For example, four dials and six readout panels, four of the panels react normally to dials, and two of the panels react to interaction effects between dials. Only one dial and one interaction effect are needed to solve the problem, the rest is irrelevant clutter.
1b: Salience and accessibility of operators	How visible are the resources needed to solve the problem? How accessible are they on screen and more generally in the problem environment?	A **simple problem** will have operators that are readily available from the start, arranged in a visible and logical manner on the interface. In such a problem, the problem solver will have no need to take extra actions in order to access these elements. For example, if needed to solve the problem, an extra window showing progress towards the solutions (in percent's) could show up automatically or be available in a corner of the screen all the time. A **difficult problem** will force the problem solver to take extra steps in order to access information or other resource. Such a problem will not have the resources arranged in a visible manner (they may need to be picked up from a larger number of resources, available in a "basket", or will need to be "invoked" on screen by pressing a button), or the resources will not be readily available at the beginning, but will need to be created during the problem-solving process (e.g., in a chemistry simulation, mixing base substances in order to obtain a higher level element, and some of these higher level elements can be then used to solve the problem).
1c: Interactions between problem elements	Do the manipulable elements of the interface interact in creating an effect?	A **simple problem** will have each button or dial create a clear and unique effect on a readout panel. A **difficult problem** will have the manipulable elements (e.g., buttons, dials, levers) creating effects by interaction. For example, while each of two buttons generates a readout on a dedicated panel, a third readout shows the outcome produced by the interaction of those two dials (e.g., dials for temperature and humidity, with a third readout showing the estimated time to completion of a biological culture). Or, the readout of each of the dials is dependent on the other dial (e.g., when the temperature increases, pressure also increases automatically on the pressure readout, even if the dial is not operated).
1d: Number of parallel tasks and goals	How many goals does the problem prescribe? How many tasks need to be processed in parallel in order to reach these goals?	A **simple problem** may require the problem solver to reach one goal (e.g., set the temperature of an incubator). If several goals are given, the problem solver is not required to solve them in parallel, but one after the other (one at a time, consecutively). For example, it will require the problem solver only to operate one dial in order to observe change in the readout panel. A **difficult problem** may require the problem solver to reach two or more separate goals (e.g., set the temperature and the humidity of an incubator would require the problem solver to push two buttons, or operate two dials at the same time, in order to observe a change in readout), or to reach one or several goals in a maximum number of steps (parsimony on problem solving, i.e., keeping under that threshold of steps, is a goal in itself). The problem solver would also need to work towards these goals at the same time (not one after the other).
The “Dinner Preparation” example is of average-to-high difficulty from this point of view. It asks the test-taker to accomplish two goals at the same time (shop for groceries and take the child to school, respectively pick the child up from school again) – this raises the cognitive and metacognitive demands on the test-taker. But the problem only has a low number of locations to visit, the routes that can be used are very salient and accessible to the problem solver on the interface, as well as are all of the other needed information. The “Stock Market” example is of high difficulty in terms of problem configuration. While it asks the test-taker to accomplish only one goal (reach a certain level of cash), it has a high number of elements in the initial problem statement: the different portfolios each have a history of variation that need to be considered. On the other hand, all these elements are salient and readily available to the test-taker.		

(2) Dynamics of the situation

Difficulty drivers		Problem description
2a: Number of features that change and their relevance	How many features change from one iteration to another? How relevant is change in these features for the problem-solving process? Change may be induced in critical elements or in less critical or even trivial issues.	A **simple problem** may have only one feature that changes from one step to the other. For example, one element of the interface changes position, or one dial changes function, or one parameter (e.g., temperature) changes from one iteration to another. Also, a simple problem has changes induced in trivial aspects of the problem, aspects that are not critical to the problem-solving process. Change is rather a distractor in this case, i.e., the outside temperature has changed, but the outside temperature is not relevant for solving a problem that requires the problem solver to set the luminosity of a lightbulb. A **difficult problem** has a larger number of elements that change. For example, the whole interface is re-arranged, and buttons change position. Or a larger number of buttons (all?) change functionality: they begin to interact now, or their effect on the readouts is no longer linear but exponential etc. Also, a difficult problem changes elements that are critical to the problem being solved and that need to be understood by the problem solver and factored into the problem-solving process in order to be successful. For example, if the problem solvers do not understand the new non-linear effect of a dial they will not be able to solve the problem.
2b: Salience of change (if something changes)	Is the problem solver prompted to the change? Is the change announced or in other way obvious, or is it hidden and needs to be discovered by the problem solver? This refers to the IF of the change (if something has changed). When the problem solver is prompted to change in an element, the particular manner in which it has changed may also be explained (or not). This refers to the HOW of the change (in what way has something changed).	A **simple problem** will announce the change to the problem solver, e.g. state that a change was made. A simple problem will also explain to the problem solver exactly what has changed and in what way. A **difficult problem** will not announce the change - it simply introduces a new element in the problem, that may be visible from the start, but appearance of change is not prompted for the problem solver. Or it may change the functionality of an element of the interface (e.g., button), but the fact that this has changed is not prompted. A difficult problem will also not explain to the problem solver how things have changed. For example, the function of an element of the interface may have changed, and its effect on the readout may no longer be linear, but curvilinear.
2c: Frequency of change	How frequent is the change? It could be iterative, i.e. not very frequent, or "continuous drop" change, i.e., constant.	A **simple problem** may have a low-frequency change: from one item to the other, or even every 2-3 items, there is some change in the problem statement. Throughout a whole problem with 10 items, maybe there are 2-3 changes. There is no change inside the item, but only from one item to another. A **difficult problem** has elements changing constantly, even inside a specific item. For example, temperature fluctuates constantly and the problem solver has to adjust dials while taking account these fluctuations in temperature.
2d: Degree of impasse	Is the change likely to induce an impasse? i.e., does the change actually create another problem that needs to be solved first, or complicates the solving of the initial problem? How likely is it that the induced change will close one avenue of solving the problem that was obvious before the change, i.e., will it require the problem solver to rethink the problem from zero?	A **simple problem** will introduce change that, while bringing with its supplementary information, will not induce impasse - the obvious avenues for solving the problem before the change remain the same after the change. For example, if the problem solver has to regulate the temperature of a room by working a dial, even if the dial no longer has a linear but an exponential effect, the effect remains positive if the dial is turned to the right. A **difficult problem** will induce impasse, i.e., it will throw the problem solver off the course that was obvious for problem solving until the introduction of change. It will either go against how the problem was previously solved (e.g., the same button that the problem solver knew from the previous interaction was doing something, is doing now something else), or interact with how the problem solver thought he/she would solve the problem (e.g., the problem solver works towards the goal in a predictable way with current resources, and some of those resources disappear after the change, so he/she has to rethink the problem).

The “Dinner Preparation” example is of low difficulty from this point of view. The problem configuration does not change at all, and only one element, i.e. one route, is manipulated. More impasse could be engineered in the problem, for example by having one store go out of one ingredient. But change is certainly explicit, transparent and infrequent in this example.

The “Stock Market” example is of average-to-high difficulty in terms of the dynamics of the situation. The change is continuous and frequent, and happens in a large number of elements (in all the stocks the problem solver has investments in). Change is however salient and explicit. Impasse could be engineered into items by changing the pattern with which the various stocks vary from one iteration to another.

(3) Features of the environment		
Difficulty drivers		**Problem description**
3a: Wealth of information	How much information is in the problem statement? This includes both elements that are relevant and those that are not relevant for solving the problem.	A **simple problem** has a very limited set of elements - the barely minimum to define the problem, not much context around it, no extra irrelevant information. For example, a dial is given, a readout, and a basic description of the phenomenon (say, temperature of an oven). A **difficult problem** contains a large number of elements, some of which are needed to define the problem (for example, a larger number of dials and readouts, a description of the entire interface, a description of the context and the motives why the problem needs to be solved, a description of the larger story the problem is set in etc.), the functionality of the interface and the task, some of which are irrelevant to the problem, but enrich the problem environment (e.g., details could be given about how other tasks are performed with the same basic resources, or about the status of other resources that are not needed for the problem at hand).
3b: Proportion of irrelevant information	How much "clutter", i.e. irrelevant information is there in the problem environment?	A **simple problem** does not have irrelevant information: all information given is relevant for solving the problem, every single piece is critical: taking that piece away will make the problem unsolvable. A **difficult problem** has a larger quantity of information that is not relevant for solving the problem. If such a piece of information would be taken away, the problem would be just as easily solvable. Such information does not contribute to solving the problem, but is a distractor and challenges the problem solver to also discern what is relevant and critical from what is not.
3c: (Lack of) Structure of the environment	How structured is the environment?	A **simple problem** is constructed in a well-structured environment. Well-structured environments will have both an intuitive and a simple structure with a small number of categories that are clearly labelled and defined. Data may be presented in clear tables or charts, well grouped and structured. A **difficult problem** is constructed in an unstructured environment. The environment may be "structurable" by the problem solver, i.e. the problem solver could structure the available information in logical categories, but the information is not presented in such a structured manner. Unstructured environments have in principle several categories (e.g. data from several sources, regarding several phenomena) and data from these categories is provided in a narrative form and intercalated with one another, so that no structure is visible on a first glance. Structuring the information is one of the tasks the problem solver would be challenged with in order to solve the problem.
3d: Number of sources of information	How many sources does information come from? These could be the actual problem statement (introduction), the solving process itself, the system through its various buttons, help panels etc.	A **simple problem** has only one source of information: the problem description. No other information is available to the problem solver. A **difficult problem** has a larger number of sources of information. Basic information will come from the problem statement, but a number of other sources of information will be available. These could be extra buttons (e.g., help button, a "read the history" button, a simulated "Google search" of "Wikipedia button" etc.). The problem-solving process itself could provide continuous information and feedback on the task, especially for more complex tasks. A narrator could come up to give extra information, or maybe even several narrators, giving information from other areas.
The “Dinner Preparation” example is of low-to-average difficulty from this point of view. The environment is not extremely wealthy, it does not offer much information beyond what is absolutely necessary to solve the problem (the routes, the shops, the shopping list). No irrelevant information is presented, no separate sources of information are present and the environment, such as it is, is structured. The “Stock Market” example is also of low difficulty in terms of features of the environment: no extra information beyond the actual problem is presented in the environment.		

OECD Skills Studies

The Assessment Frameworks for Cycle 2 of the Programme for the International Assessment of Adult Competencies

The OECD's Programme for the International Assessment of Adult Competencies (PIAAC) represents a comprehensive international comparative assessment of the information processing skills of adults vital for the full participation in social and economic life in the 21st century. PIAAC is now in its second cycle and continues a series of international assessments of adult skills that began in the mid-1990s with the International Adult Literacy Survey (IALS).

The Assessment Frameworks for Cycle 2 of PIAAC provide an essential background for understanding the skills assessed by the PIAAC assessment and for interpreting the results of the study. The Assessment Frameworks define and describe the skills assessed in the study – literacy, numeracy and problem solving – and outline the key features of the assessment of these skills. In addition, the relationship between Cycle 2 of PIAAC and previous assessments of these skills among the adult population is explained and an overview is provided of the changes that have occurred in the conceptualisation of these skills in the different international assessments of adult skills implemented over the last two decades.

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