

Sensitivity of capital and MFP measurement to asset depreciation patterns and initial capital stock estimates

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Abstract

This paper discusses the sensitivity of capital and MFP measurement to asset depreciation patterns and initial capital stock estimates. Applying the same depreciation rates in the US as in other G7 countries would reduce the US net investment rate and net capital stock by up to one third and increase US GDP by up to 0.5%. Capital and MFP growth would be less affected. Estimating initial capital stocks often involves assuming constant investment growth, but this leads to unreliable results. Relying on average K/Y ratios across countries works well for the US, but this might not be the case for other countries due to the international dispersion in K/Y ratios. Two main recommendations for statistical agencies emerge from this analysis. First, they should regularly review asset depreciation patterns to ensure that measured differences across countries are well justified. Second, they should backcast investment series as much as possible before relying on stationarity assumptions to estimate initial capital stocks.

Keywords: National accounts, Asset depreciation, Capital stock, Capital services, Multifactor Productivity (MFP).

JEL classification: E01, E22, E23, O47.

Résumé

Cet article étudie la sensibilité de la mesure du capital et de la PGF aux profils de dépréciation des actifs et à l'estimation des stocks de capital à une date initiale. L'utilisation des mêmes taux de dépréciation aux États-Unis que dans les autres pays du G7 pourrait réduire d'un tiers le taux d'investissement net et le stock de capital net des États-Unis, et le PIB de ce pays de 0,5%. Les taux de croissance du capital et de la PGF seraient moins affectés. L'estimation des stocks de capital à une date initiale repose souvent sur une hypothèse de taux de croissance constant de l'investissement, mais cela conduit à des résultats peu fiables. L'utilisation de coefficients de capital K/Y moyens conduit à de bons résultats pour les États-Unis, mais cela pourrait ne pas être le cas pour d'autres pays à cause de la dispersion des coefficients de capital entre pays. Deux recommandations principales pour les instituts de statistique ressortent de cette analyse. Tout d'abord, ils devraient revoir régulièrement les profils de dépréciation des actifs pour s'assurer que les différences de mesure entre pays sont bien justifiées. Par ailleurs, ils devraient rétroscander les séries d'investissement le plus possible avant de recourir à des hypothèses de stationnarité pour estimer les stocks de capital à une date initiale.

Mots-clés : Comptes nationaux, Dépréciation des actifs, Stock de capital, Services du capital, Productivité globale des facteurs (PGF).

Classification JEL : E01, E22, E23, O47.

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1. Introduction

1. Capital measurement plays a fundamental role in national accounts, both to assess the economic wealth and the state of infrastructure in a given country, as well as to better understand the sources of economic and productivity growth. Nevertheless, measuring capital is challenging because capital stocks are usually unobserved and need to be estimated by making assumptions on initial capital stocks and cumulating past investment flows while accounting for the depreciation and the retirement of assets, a statistical process known as the Perpetual Inventory Method (PIM).
2. Statistical agencies in different countries tend to use very different assumptions regarding the depreciation and retirement of assets. While such differences may be justified by country-specific factors such as climate, construction techniques (e.g. for buildings and structures) and government investment policies (OECD, 2009), they may also simply reflect differences in statistical assumptions as depreciation and retirement patterns tend to be based on thin empirical evidence or old research (Bennett et al., 2020).
3. Unexplained differences in depreciation and retirement patterns across countries may harm the cross-country comparability of capital stocks and macroeconomic indicators that rely on consumption of fixed capital (CFC). This is obviously the case of economic aggregates that are measured net of depreciation, such as net investment (the difference between gross investment and CFC) and net domestic product (the difference between GDP and CFC). Nevertheless, since CFC also enters the calculation of the output and value added of non-market activities, uncertainty around CFC estimates also potentially affects prominent gross indicators such as GDP.
4. Another practical issue that statistical agencies face when estimating capital stocks and CFC is the estimation of initial capital stocks at a given date in the past in order to initialise the PIM. This issue is particularly important when only short time series of investment are available.
5. This paper discusses the impact on the measurement of capital and multifactor productivity (MFP) of using different asset depreciation and retirement patterns and different assumptions to estimate initial capital stocks.
6. By using the distribution of *cohort* depreciation rates² for a given asset type across countries as a measure of uncertainty, this paper implicitly assumes that all available estimates measure the same unobserved cohort depreciation rate, and that all differences across countries may be related to measurement errors. Therefore, it ultimately provides an

² To avoid any ambiguity, in this paper the term depreciation (without any further qualification) is reserved to describe how the value (i.e. the market price) of a single productive asset declines over time due to the shortening of its remaining service life as time goes by. Depreciation is reflected in the age-price profile of a single asset. Nevertheless, the depreciation process does not take into account that assets belonging to the same cohort (i.e. purchased at the same time) may be retired from the productive capital stock at a different age. *Cohort* depreciation corresponds to the combined effect of (single-asset) depreciation and retirement. It determines how the value of a stock of assets declines over time if depreciation and retirements are not compensated by investment (GFCF) or other positive changes in volume. A synonymous for cohort depreciation rate could be combined depreciation/retirement rate.

upper bound of the implied uncertainty on capital and MFP measurement.³ By highlighting this uncertainty, it aims at encouraging statistical agencies to review asset depreciation and retirement patterns more regularly, including for assets that have long been capitalised in national accounts (e.g. buildings, structures, machinery and equipment). The intention is not to promote a complete standardisation of asset depreciation and retirement patterns across countries, but to ensure that differences are well justified. Similarly, the analysis of the impact of usual assumptions to estimate initial capital stocks on capital and MFP measurement should encourage statistical agencies to use national sources to extend their investment series to the maximum extent before relying on such assumptions.

7. The national accounts produced by the US Bureau of Economic Analysis (BEA) are used as a laboratory to analyse the sensitivity of capital and MFP measurement in this paper. The reason is that the BEA produces the longest and most detailed investment time series in OECD countries, which allows applying the assumptions of other countries and test their impact on US capital and MFP measurement. The sensitivity of capital and MFP measurement to alternative depreciation patterns and alternative methods to estimate initial capital stocks may depend to some extent on the investment share of each asset in each economy. Nevertheless, the composition of investment across OECD countries looks sufficiently similar to consider that the sensitivity of capital and MFP measurement in the United States is a good indication of what would be obtained with the data of other advanced countries (OECD, 2021).

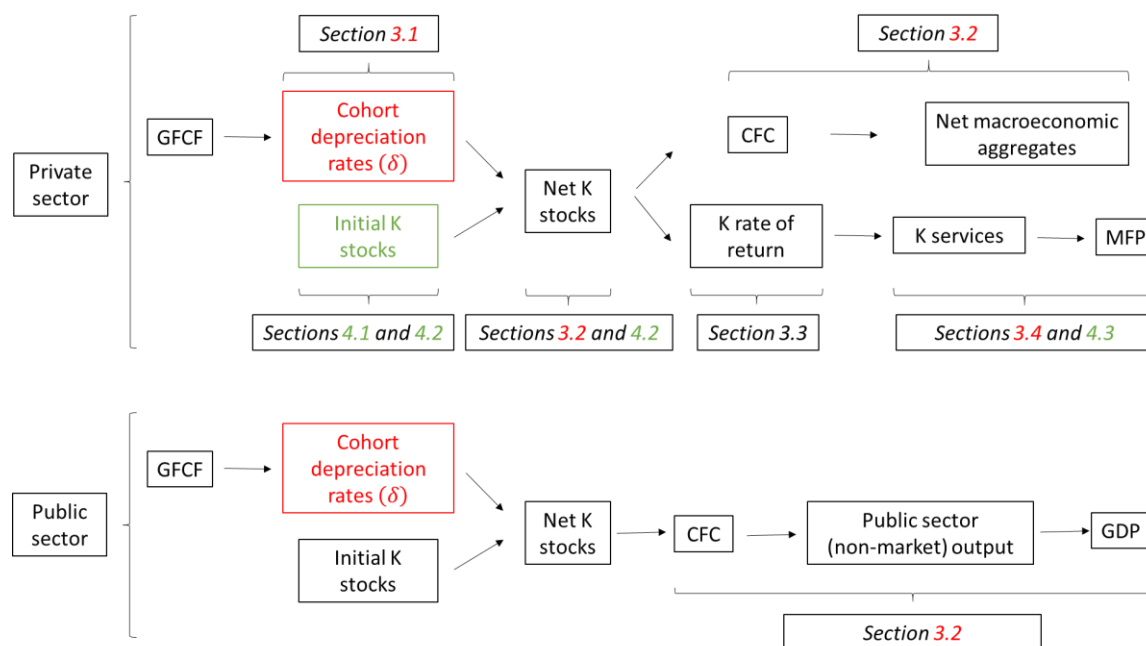
8. This paper extends a previous sensitivity analysis by Inklaar (2010), who focused on the sensitivity of capital services to the type of assets considered and to the measurement of capital user costs. First, the present paper analyses the effect of changing depreciation/retirement patterns and/or initial capital stocks, which Inklaar (2010) did not consider but acknowledged as potentially important factors. Second, it discusses the sensitivity not only of capital services, but also of net capital stocks,⁴ CFC and MFP. Third, it assesses the reliability of different methods to estimate initial capital stocks. Fourth, it compares cohort depreciation rates in Canada, France, Germany, Italy, the United Kingdom and the United States, and therefore extends a recent sensitivity analysis by Giandrea et al. (2021) which focused on Canada and the United States.

9. The rest of this paper is organised as follows. Section 2 discusses the replication of the PIM implemented by the BEA in order to produce benchmark estimates for this sensitivity analysis. Section 3 describes a synthetic way to compare combined asset depreciation and retirement patterns across countries, and the sensitivity of capital and MFP measurement to such patterns. Section 4 discusses two leading methods to estimate initial capital stocks, relying on stationarity assumptions on either investment growth or capital-stock-to-output ratios, and assesses their impact on capital and MFP measurement. Section 5 concludes. Figure 1.1 summarises the organisation of the sensitivity analysis and the paper.

³ Alternatively, a pure Monte Carlo analysis could be considered. Nevertheless, there is no obvious statistical distribution from which to draw cohort depreciation rates. Therefore, this paper relies on the cohort depreciation rates used in different countries as a measure of uncertainty.

⁴ In this paper, the term “net capital stock” is used as synonymous for “net *wealth* capital stock”. The latter is only used when there is a need to distinguish *net wealth* and *productive* capital stocks.

Figure 1.1. Organisation of the sensitivity analysis and the paper



Note: The red colour indicates a discussion related to cohort depreciation rates, and the green colour a discussion related to initial capital stocks.

2. Replication of the Perpetual Inventory Method used by the BEA

2.1. US private sector

10. In order to build benchmark estimates of net capital stocks for this sensitivity analysis, the PIM implemented by the BEA is first replicated.⁵ Annual investment (i.e. Gross Fixed Capital Formation, or GFCF) time series compiled by the BEA for the US private sector,⁶ broken down into 86 residential and non-residential assets and 63 economic activities over the period 1901-2019 are used.⁷ These series are the longest and most detailed publicly available GFCF series across OECD countries, which allows testing different scenarios for the estimation of capital and MFP.

⁵ See BEA Fixed Asset Accounts: <https://apps.bea.gov/national/FA2004/Details/Index.htm>, extracted in October 2020.

⁶ The US private sector is defined as industries 11 to 81 in the [NAICS 2017](#) classification, and thus excludes federal, state and local government activities.

⁷ Autos, computer and peripheral equipment, and nuclear fuel are excluded because the BEA applies a non-geometric combined retirement/depreciation profile for these assets (BEA, 2003). In 2019, these assets accounted for 2.1% of total GFCF and 0.6% of the net capital stock of the US private sector.

11. For each asset type and industry, *benchmark* net capital stocks and CFC are computed using the geometric cohort depreciation rates and the PIM applied by the BEA.^{8 9} The BEA estimates the net capital stock of a given asset i at the end of period t as follows:¹⁰

$$K_{a,t} = K_{a,t-1} * (1 - \delta_a) + I_{a,t} * (1 - \delta_a/2)$$

where:

$K_{a,t}$ is the net capital stock of asset type a at the end of period t ;

δ_a is the geometric cohort depreciation rate of asset type a (see Section 3.1);¹¹

$I_{a,t}$ is the volume of GFCF in asset type a during period t .¹²

12. All variables above are volumes and expressed in constant prices of a base period. The last term on the right-hand side implies that investment happens at mid-year, or that it is evenly spread out over the year. A volume measure of consumption of fixed capital (CFC) associated to asset a in period t is then derived as:

$$CFC_{a,t} = K_{a,t-1} - K_{a,t} + I_{a,t} = K_{a,t-1}\delta_a + I_{a,t}\delta_a/2$$

Measures of net capital stock and CFC in current prices are finally constructed by multiplying the volume measures with the asset price index between the base period and the current period.

⁸ The list of depreciation rates currently used by the BEA is available at https://apps.bea.gov/national/pdf/BEA_depreciation_rates.pdf. This information is largely based on Fraumeni (1997), with very few differences related to the capitalisation of R&D in national accounts in the meantime and updated depreciation rates for a few assets (e.g. trucks, buses and truck trailers).

⁹ In the United States, two different statistical agencies produce capital stock estimates. The BEA estimates net *wealth* capital stocks, using geometric cohort depreciation rates. These net capital stocks are reported in US national accounts balance sheets. The Bureau of Labor Statistics (BLS) estimates *productive* capital stocks for the business sector, assuming hyperbolic age-efficiency profiles. These productive capital stocks are then used in the estimation of capital services and MFP. The difference between these two capital stock concepts is explained in detail in OECD (2009). Note that the BEA net wealth capital stocks and the BLS productive capital stocks are obtained independently from each other. In the present paper, capital services and MFP are derived from net wealth capital stocks, using asset- and industry-specific capital user costs. Therefore, they differ from the capital services and MFP estimates produced by the BLS, but they are consistent with the net capital stocks produced by the BEA.

¹⁰ Although the industry index is omitted, this formula applies to each asset and each industry. Depreciation rates for a given asset may vary across industries.

¹¹ For very few assets, the BEA depreciation rates changed at some point. In the present paper, the latest depreciation rates are used over the entire period of analysis.

¹² According to Giandrea et al. (2021), the BEA also considers in this formula other changes in the volume of the assets. However, in the absence of specific information on other changes in volume, they are not considered for the estimation of benchmark net capital stocks in the present paper, which does not impair the accuracy of the replication of official US net capital stocks (Table 2.1).

13. Table 2.1 compares the official net capital stocks estimated by the BEA with the benchmark estimates resulting from the replication of the US PIM. These estimates are very close to each other. In the subsequent sections, these benchmark estimates will be used as a basis to analyse the impact on capital and MFP measurement of changing cohort depreciation rates and using different approaches to estimate initial capital stocks.

Table 2.1. Replication of the BEA Perpetual Inventory Method: US private sector

Net capital stock to GDP ratios, current prices, 2019

Assets	BEA official estimates	OECD benchmark estimates
Dwellings	1.1	1.1
Other buildings and structures	0.7	0.7
Transport equipment	0.1	0.1
Other machinery and equipment	0.2	0.2
IT equipment and IPP assets excluding R&D	0.1	0.1
R&D	0.1	0.1
TOTAL	2.3	2.3

Note: Autos, computer and peripheral equipment, and nuclear fuel are excluded from this comparison.
Source: BEA (Fixed Assets Accounts, October 2020), Authors' calculations.

2.2. US government sector

14. In order to assess the impact of changing cohort depreciation rates on non-market CFC and, in turn, non-market output and GDP (Section 3.2), the PIM used by the BEA for the US government sector is also replicated. The GFCF series released by the BEA for this sector are much more aggregated than for the private sector. In particular, the available breakdown by asset does not allow matching government GFCF with depreciation rates as accurately as for the private sector. While the BEA kindly provided some unpublished information for this study, a number of depreciation rates used by the BEA had to be averaged across detailed asset types in order to match the available level of detail of GFCF. Nevertheless, the available information allows reproducing the CFC estimates of the BEA relatively accurately for broad categories of assets (Table 2.2).

Table 2.2. Replication of the BEA Perpetual Inventory Method: US government sector

Consumption of fixed capital, USD billions, current prices, 2019

Assets	BEA official estimates	OECD benchmark estimates
Dwellings	6.4	8.6
Other buildings and structures	234.7	239.7
Machinery and equipment	133.4	128.6
Software and databases	55.6	55.2
R&D	152.0	154.3
TOTAL	582.1	586.4

Source: BEA (Fixed Assets Accounts, August 2022), Authors' calculations.

3. Impact of changing asset depreciation and retirement patterns on capital and MFP measurement

3.1. Comparison of combined asset depreciation and retirement patterns across countries

15. Net capital stocks result from successive vintages of investment in productive assets and the combined effect of their depreciation and retirement over time. The depreciation pattern describes how the value of a single asset declines over time as the asset ages. The retirement pattern takes into account that not all assets purchased at the same time (i.e. belonging to the same cohort) are removed from the capital stock at the same age. For this purpose, non-degenerated probability distributions around average asset service lives are usually considered by statistical agencies.

16. Hulten and Wykoff (1981a) showed how the combination of depreciation and retirement gives rise to convex age-price profiles for cohorts of assets, which can usually be approximated by geometric patterns.¹³ The main advantage of geometric patterns is that they are characterised by a single and constant parameter (the geometric cohort depreciation rate). This simplicity led several statistical agencies such as the BEA and Statistics Canada to rely on geometric patterns to estimate CFC for their national accounts (Fraumeni, 1997; Baldwin et al., 2015).

17. However, not all countries rely on geometric patterns to summarise the combined effect of depreciation and retirement and estimate net capital stocks. For example, France relies on linear depreciation profiles for single assets and combines them with log-normal retirement patterns. Alternatively, the Netherlands and the United Kingdom estimate net *wealth* capital stocks using the combined depreciation and retirement patterns that they derive from hyperbolic age-efficiency profiles combined with Weibull (for the Netherlands) or truncated normal (for the United Kingdom) retirement functions (Office for National Statistics, 2019; Statistics Netherlands, 2019).¹⁴

18. In order to consider countries that rely on any asset depreciation and retirement patterns, this sensitivity analysis follows Cabannes et al. (2013) who estimate geometric approximations of combined depreciation and retirement patterns for France. This method consists in combining depreciation and retirement patterns analytically and estimating the geometric function that provides the best fit to the combined pattern in a least square sense. Annex B discusses how these geometric approximations are obtained for France, Germany, Italy and the United Kingdom.

19. Table 3.1 provides average ratios of Canadian, French, German, Italian and UK cohort depreciation rates to the corresponding US parameters for aggregate asset categories. In most cases, the cohort depreciation rates used in Canada, France, Germany and the United Kingdom are higher, or much higher, than those used in the United States.

¹³ Hulten (2008) later summarised this as follows: “The more assets are grouped together, the more the group experience tends to be a geometric-like pattern, regardless of the actual patterns of the individual assets in the group. If the individual patterns are themselves nearly geometric, the group effect is reinforced, but this is not a necessary condition.”

¹⁴ The United Kingdom’s Office for National Statistics applies this method to all assets except research and development, for which they combine a Weibull retirement distribution with a geometric age-efficiency function. See Annex B for additional information on the asset depreciation and retirement functions used in G7 countries.

This is especially true for dwellings and non-residential buildings, as well as other (civil engineering) structures in Canada.¹⁵ The Italian depreciation rates are closer to the US ones.

Table 3.1. Ratios of cohort depreciation rates in Canada, France, Germany, Italy and the United Kingdom, relative to the United States

Asset label	Canada	France	Germany	Italy	United Kingdom
Dwellings	2.0	5.0	2.4	1.6	2.5
Buildings other than dwellings	3.0	2.8	2.1	1.4	3.1
Other structures	2.7	1.1	1.4	1.6	1.7
Transport equipment	1.5	1.5	1.4	1.1	1.3
Computer hardware	1.3	1.2	0.8	1.4	1.2
Telecom. equipment	2.1	1.4	1.6	2.8	1.2
Other machinery and equipment	1.8	1.1	1.5	1.4	1.1
R&D	1.8	1.0	1.0	1.3	1.8
Software & databases	1.0	0.7	0.9	0.9	0.7
Originals	6.3	2.6	2.7	1.4	1.5

Note: Ratios higher than 1.5 are highlighted in orange, and ratios higher than 2.0 are highlighted in red.

Source: The geometric cohort depreciation rates for Canada and the United States are sourced from Statistics Canada and Giandrea et al. (2021). Geometric approximations are used for France, Germany, Italy and the United Kingdom (see Cabannes et al., 2013 and Annex B). Ratios are first calculated for detailed assets and then aggregated to the upper level of the asset classification using 2019 net capital stock shares in the US private sector as weights.

20. It is worth noting that the above method is better than relying on Declining Balance Rates (DBRs) to plug the depreciation and retirement patterns of other countries into the PIM used by the BEA. DBRs were first introduced by Hulten and Wykoff (1981b) to provide a simple inverse proportional relationship between geometric cohort depreciation rates (δ) and average asset services lives (T):

$$\delta \equiv \frac{DBR}{T}$$

21. Nevertheless, DBRs do not have an obvious economic meaning. Annex A shows that they are not universal constants as they depend on the shape of the underlying depreciation and retirement functions used by national statistical agencies. Therefore, DBRs are country specific, and estimating geometric cohort depreciation rates for France, Germany, Italy and the United Kingdom based on the asset service lives of these countries and the DBRs of the United States would be misleading. By contrast, the geometric approximations to asset depreciation and retirement in this paper are only based on national assumptions and summarise all aspects of asset depreciation and retirement in each country.

¹⁵ The results for Canada and the United States are in line with Giandrea et al. (2021). The present paper extends the comparison to France, Germany, Italy and the United Kingdom.

3.2. Sensitivity of CFC and net capital stocks to changes in cohort depreciation rates

3.2.1. US private sector

22. This section analyses the sensitivity of capital measurement to changes in cohort depreciation rates. In order to explore the range of possible depreciation patterns, the geometric cohort depreciation rates used by Canada, France, Germany, Italy and the United Kingdom are successively introduced into the US PIM along with the original US GFCF time series to recalculate the CFC, net investment rates and net capital stocks for all assets of the US private sector.¹⁶

23. Consistently with the evidence provided in Table 3.1, Figure 3.1 shows that the US ratio of CFC to gross value added (GVA) would be significantly higher if the BEA relied on the same cohort depreciation rates as Canada, France, Germany and the United Kingdom (15.9%, 15.5%, 15.2% and 15.2% against 14.2%, respectively). It would be only slightly higher if the BEA relied on the same cohort depreciation rates as Italy (14.6% against 14.2%). The main difference with the official US accounts relates to the CFC of residential and non-residential buildings.

24. Accordingly, Figure 3.2 and Figure 3.3 show that the US net investment and net capital stocks would be significantly lower, by up to one third, if the BEA relied on the same cohort depreciation rates as Canada, France, Germany and the United Kingdom, and only slightly lower if it relied on the Italian cohort depreciation rates. Here again, differences are mainly related to residential and non-residential buildings.

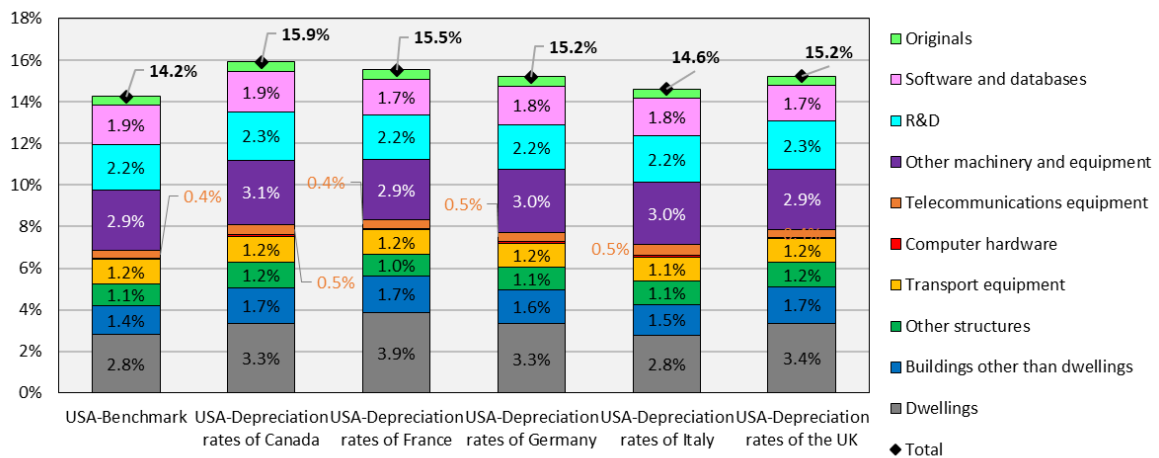
25. Nevertheless, the impact of switching to other countries' cohort depreciation rates is more limited on the growth rate of the US net capital stock (at constant prices) than on its level (at current prices). This is because an increase in the depreciation rate of an asset has two opposite effects on the growth rate of its net capital stock. Rewriting the generic capital accumulation equation $K_t = I_t + (1 - \delta)K_{t-1}$ in terms of capital growth rate $\frac{\Delta K_t}{K_{t-1}} = \frac{I_t}{K_{t-1}} - \delta$ shows that an increase in δ has a direct negative effect as well as an indirect positive effect on $\frac{\Delta K_t}{K_{t-1}}$ because it reduces K_{t-1} . This latter effect is more muted in a period of low investment ($I_t \rightarrow 0$). In this case, an increase in δ is more likely to reduce the growth rate of the net capital stock.

26. As expected, Figure 3.4 shows that the impact of changing cohort depreciation rates on the growth rate of the US net capital stock has the largest impact in the period corresponding to the Great Recession and the immediately following years, which is a period of low investment. Nevertheless, on average between 1998 and 2019, the annual growth rate of the US net capital stock only changes from 1.9% to 1.7%-1.8% when using Canadian, French or German cohort depreciation rates, and it is unaffected when using Italian or UK cohort depreciation rates.

¹⁶ For France, this paper relies on the geometric approximations provided by Cabannes et al. (2013). For Germany, Italy and the United Kingdom, it is based on geometric approximations of the combined age-price/retirement profiles in each country. The asset classifications used in the five countries are mapped together using information from Cabannes et al. (2013), Giandrea et al. (2021) and the replies by Statistic Canada, ISTAT and the ONS to the 2019 Eurostat-OECD Questionnaire on the Methodology underlying Capital Stocks (Annex C).

Figure 3.1. Sensitivity of consumption of fixed capital to changes in cohort depreciation rates

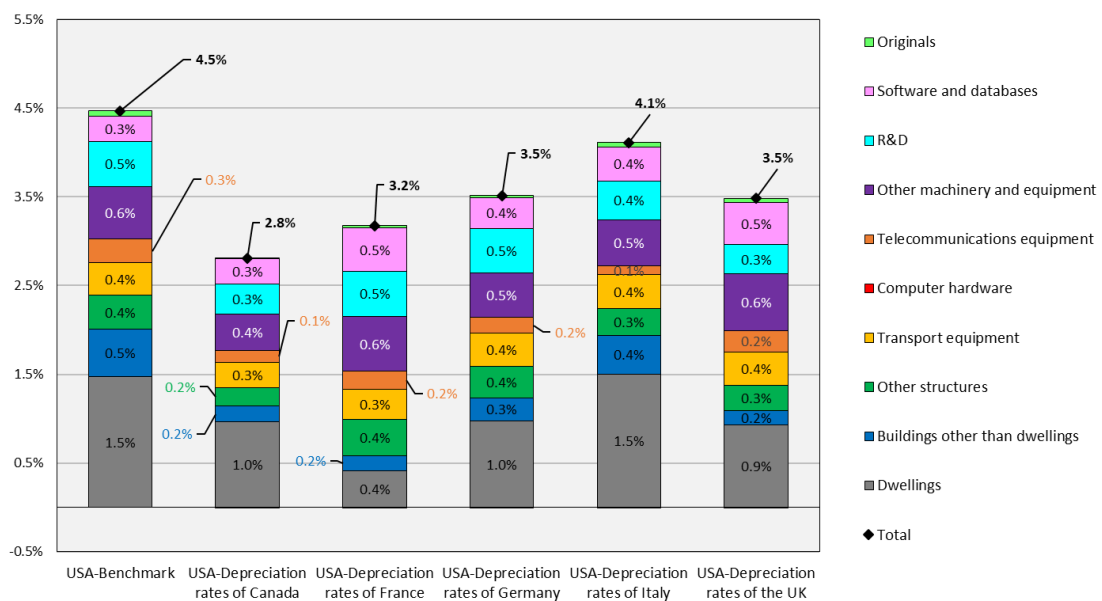
Ratio of consumption of fixed capital to gross value added, US private sector, 2019



Source: Authors' calculations, based on BEA depreciation rates, Cabannes et al. (2013), Giandrea et al. (2021), information shared by Statistics Canada, DESTATIS (Germany), ISTAT (Italy) and the ONS (United Kingdom).

Figure 3.2. Sensitivity of net investment to changes in cohort depreciation rates

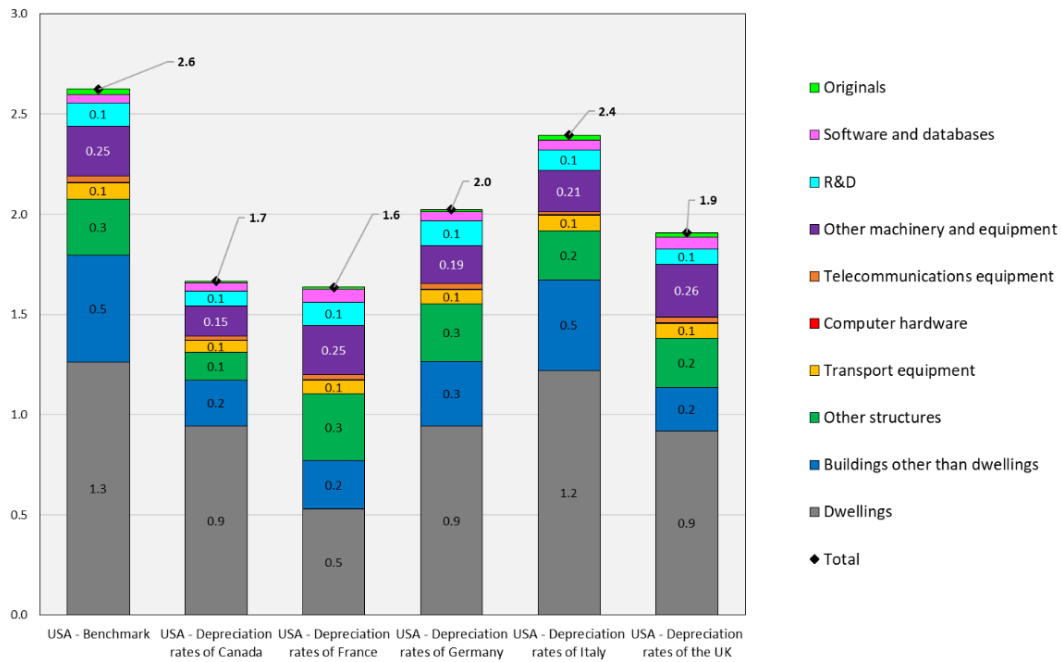
Ratio of net investment to gross value added, US private sector, 2019



Source: Authors' calculations, based on BEA depreciation rates, Cabannes et al. (2013), Giandrea et al. (2021), information shared by Statistics Canada, DESTATIS (Germany), ISTAT (Italy) and the ONS (United Kingdom).

Figure 3.3. Sensitivity of net capital stock to changes in cohort depreciation rates

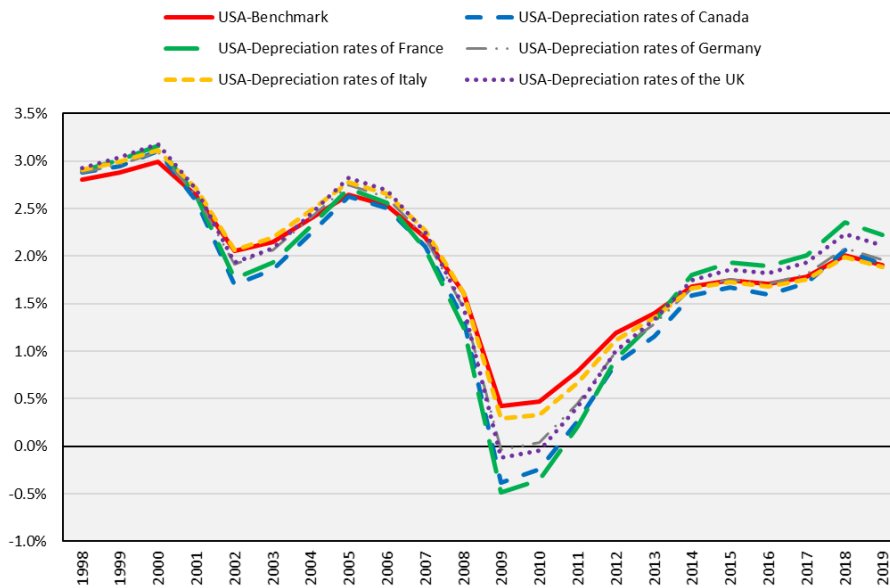
Ratio of net capital stock to gross value added, US private sector, 2019



Source: Authors' calculations, based on BEA depreciation rates, Cabannes et al. (2013), Giandrea et al. (2021), information shared by Statistics Canada, DESTATIS (Germany), ISTAT (Italy) and the ONS (United Kingdom).

Figure 3.4. Sensitivity of net capital stock growth to changes in cohort depreciation rates

Constant prices, US private sector, 1998-2019



Source: Authors' calculations, based on BEA depreciation rates, Cabannes et al. (2013), Giandrea et al. (2021), information shared by Statistics Canada, DESTATIS (Germany), ISTAT (Italy) and the ONS (United Kingdom).

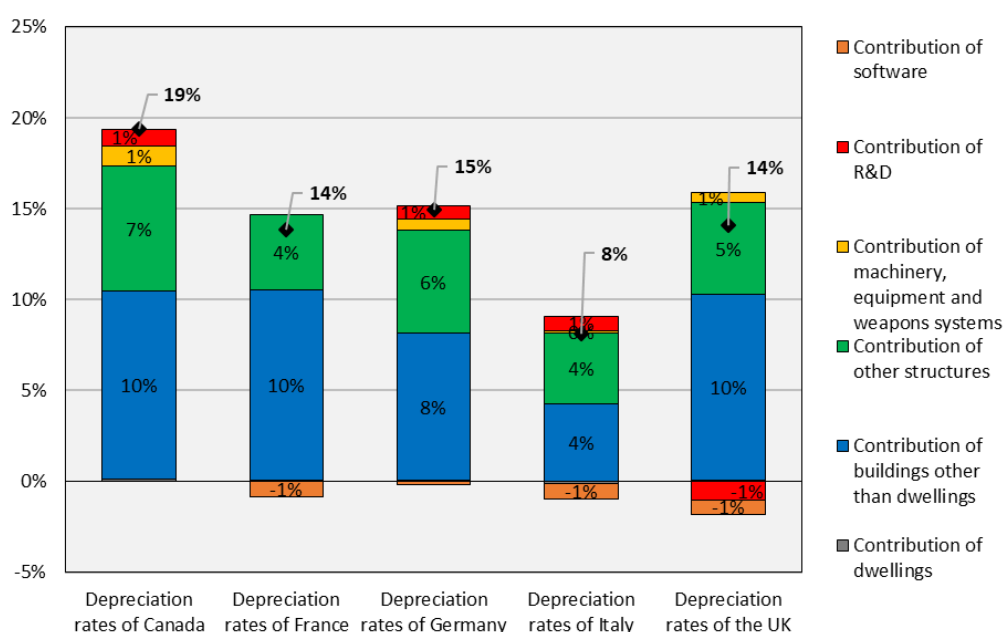
3.2.2. US government sector

27. This section extends the analysis of the previous section to the US government sector. While the lack of detailed GFCF series does not allow assessing how changes in cohort depreciation patterns affect non-market CFC at a detailed asset level,¹⁷ it is possible to assess how they affect overall non-market CFC. Since the gross output of the government sector is calculated as the sum of purchases of intermediate goods and services, compensation of employees and CFC (BEA, 2021), any change to CFC affects the gross output and the value added of the government sector and, in turn, nominal GDP.

28. The CFC of the US government sector in 2019 would increase by up to 19% if the BEA relied on the same cohort depreciation rates as Statistics Canada (Figure 3.5). Accordingly, the US GDP in 2019 would be revised upwards by up to 0.5% (Table 3.2).

Figure 3.5. Sensitivity of consumption of fixed capital to changes in cohort depreciation rates

Percentage increase in CFC and contribution of underlying assets, US government sector, 2019



Note: The CFC of the US government sector would increase by 19% if the BEA relied on the same depreciation rates as Statistics Canada. Buildings other than dwellings would contribute to this increase by 10 percentage points.

Source: Authors' calculations, based on BEA depreciation rates, Cabannes et al. (2013), Giandrea et al. (2021), information shared by Statistics Canada, DESTATIS (Germany), ISTAT (Italy) and the ONS (United Kingdom).

¹⁷ Detailed GFCF series matching the granularity of depreciation rates used by the BEA would be required for this purpose. Note that, with this information, it would also be possible to assess how changes in cohort depreciation rates affect the stock, average age and remaining service life of infrastructure assets, a large part of which (e.g. roads, schools, hospitals) is being owned by the government sector (Bennett et al., 2020).

Table 3.2. Sensitivity of government sector value added and GDP to changes in cohort depreciation rates

Increase in government sector value added and GDP, 2019

	Depreciation rates of Canada	Depreciation rates of France	Depreciation rates of Germany	Depreciation rates of Italy	Depreciation rates of the United Kingdom
Government sector value added	+4.7%	+3.4%	+3.6%	+2.0%	+3.4%
GDP	+0.5%	+0.4%	+0.4%	+0.2%	+0.4%

Source: Authors' calculations, based on BEA depreciation rates, Cabannes et al. (2013), Giandrea et al. (2021), information shared by Statistics Canada, DESTATIS (Germany), ISTAT (Italy) and the ONS (United Kingdom).

3.3. Selecting a rate of return for the measurement of capital services

29. The paper now focuses again on the US private sector for the analysis of capital services. Indeed, the estimation of capital user costs that underlies the measurement of capital services is inconsistent with the fact that non-market output is defined using a sum of costs approach excluding any return to capital (OECD, 2009).

30. There are two main approaches to calculate the rate of return (rr_{jt}) in the user cost formula for the measurement of capital services:

$$u_{ajt} = p_{ajt-1}(d_{aj} - \Delta p_{ajt} + d_{aj}\Delta p_{ajt} + rr_{jt})$$

where:

u_{ajt} is the user cost of capital of asset type a in industry j and period t ;

d_{aj} is the depreciation rate of asset type a in industry j ;

p_{ajt} is the price of asset type a in industry j and period t ; and

Δp_{ajt} is the 5-year centred moving average of changes in the price of asset type a in industry j and period t . The main justification for using a moving average is to capture inflation expectations rather than current inflation.

31. The preferred approach in this paper is to rely on exogenous and time-varying (annual) rates of return, estimated based on financial market information. Two different estimation methods are tested:

- a weighted average cost of capital (WACC) taking into account the cost of debt and equity financing. This rate is estimated using the same method and data sources as Inklaar (2010) and his time series are extended from 2005 to 2019.¹⁸

¹⁸ Following Inklaar (2010), the weighted average cost of capital is calculated as follows: $WACC_t = s_t^E C_t^E + (1 - s_t^E)(1 - \tau_t)C_t^D$. s_t^E is the share of equity in total funding, constructed as the ratio of the stock of equity and investment fund shares in the total liabilities of US non-financial corporations, sourced from the OECD National Accounts database (Table 720). C_t^E , the cost of equity, is computed as the sum of the earning and dividend yields of the S&P500. C_t^D is the cost of debt, estimated as Moody's BAA corporate bond yield for the United States. It is multiplied by one minus τ_t , the marginal corporate tax rate, in order to reflect the tax-deductibility of interest payments. τ_t is proxied by the Statutory Corporate Tax Rate sourced from the [Tax Foundation](#).

- an exogenous nominal rate of return (ENRR) obtained by combining a constant real long-term interest rate and a smoothed inflation rate. In practice, the real long-term interest rate r^* is estimated as the long-term average of the AAA and BAA corporate bond yields for the United States produced by Moody's, adjusted for CPI inflation. This leads to a value of 4.2% for r^* . Denoting the 5-year centred moving average of the CPI inflation rate as π_t , the ENRR is finally estimated as:

$$ENRR_t = (1 + r^*)(1 + \pi_t) - 1$$

A similar method is advocated by Diewert (2001) and Schreyer (2010).

32. An alternative approach is to estimate the capital rate of return endogenously, in such a way that the value of capital services exactly exhausts the gross operating surplus (GOS) and the capital component of gross mixed income (Jorgenson, 1963). The estimation of an endogenous rate of return has several advantages (in particular, it relies on available national accounts data only and leaves no unexplained residual income). Nevertheless, it is only consistent with a fully competitive economy and production processes with constant returns to scale. Moreover, it assumes that all assets contributing to the production process are taken into account, which may not be very plausible in light of the increasing importance of certain unmeasured intangibles and natural assets that enter the production process (Schreyer, 2010).

33. An endogenous rate of return is estimated for each aggregate industry by equating the value of capital services to a measure of the industry's capital income, measured residually as all income produced in the industry that is not accruing to labour. In practice, this residual income ($KInc$) is calculated as the sum of GOS, the capital component of mixed-income, and taxes less subsidies on production (see Annex D for details). Equating the industry's residual income with the total value of capital services leads to:

$$KInc_{it} = \sum_{j=1}^{J_i} \sum_{a=1}^A u_{ajt} K_{ajt}$$

where:

$KInc_{it}$ is the residual income in industry i at date t ;

u_{ajt} is the user cost of capital of asset type a in (sub-)industry j at date t ;

K_{ajt} is the net capital stock of asset a in (sub-)industry j at date t ;¹⁹

A is the number of asset types; and

J_i is the number of sub-industries within industry i .

¹⁹ The volume of capital services of a given asset in a given industry is assumed to be proportional to the volume of its net capital stock. Time-variation in the factor of proportionality relating the two measures (e.g. due to changes in the capacity utilisation rate of capital) is neglected here.

34. The user cost formula takes into account that asset depreciation rates and the revaluation of asset prices can vary across sub-industries (j), but endogenous rates of return are estimated for only 13 aggregate industries (i) belonging to the US private sector. Indeed, data quality is probably lower at the level of sub-industries, and estimating endogenous rates of return at a higher level limits the number of extreme values likely reflecting measurement errors (Annex D). Introducing the user cost formula in the previous expression leads to:

$$KInc_{it} = \sum_{j=1}^{J_i} \sum_{a=1}^A p_{ajt-1} (d_{aj} - \Delta p_{ajt} + d_{aj} \Delta p_{ajt} + irr_{it}) K_{ajt}$$

The endogenous, or internal, rate of return (irr_{it}) can then be calculated as follows:

$$irr_{it} = \frac{KInc_{it} - \sum_{j=1}^{J_i} \sum_{a=1}^A p_{ajt-1} (d_{aj} - \Delta p_{ajt} + d_{aj} \Delta p_{ajt}) K_{ajt}}{\sum_{j=1}^{J_i} \sum_{a=1}^A p_{ajt-1} K_{ajt}}$$

35. Figure 3.6 compares the WACC, the ENRR and the average internal rate of return across industries over the period 1998-2019. While the WACC and the ENRR are close to each other, the average internal rate of return is higher and shows larger fluctuations. Increases in the internal rate of return may partly capture increases in overall mark-ups (Calligaris et al., 2018; Basu, 2019; Schreyer and Zinni, 2020).

36. Table 3.3 shows the sensitivity of capital services growth in the US private sector over 1998-2019, based on the three different rates of return.²⁰ The endogenous (or internal) rate of return results in significantly lower growth rates of capital services over 1998-2019 and all sub-periods. However, using one exogenous rate of return or the other leads to very similar growth rates of capital services. The fact that using an endogenous rate of return leads to significantly lower capital services growth in the US private sector after the mid-1990s was also noticed by Inklaar (2010), but his estimates stopped in 2005. According to the evidence presented in Table 3.3, the same applies over the following 15 years.

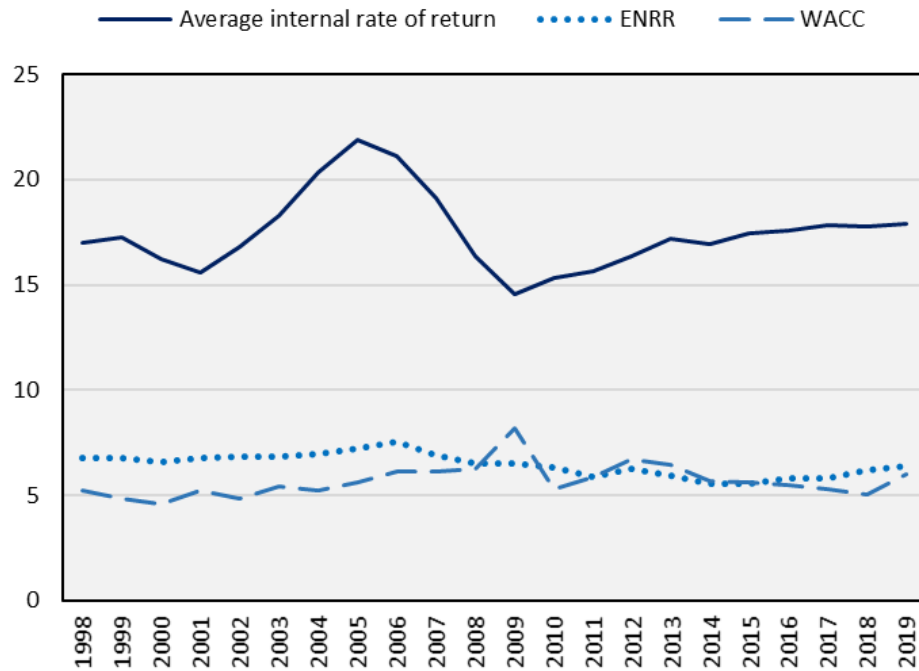
37. Similarly, Table 3.4 shows the sensitivity of MFP growth in the US private sector over 1998-2019, based on the three different rates of return. Here again, using an endogenous rate of return leads to lower MFP growth, and the two exogenous rates of return lead to similar results. Perhaps surprisingly, lower capital services growth with an endogenous rate of return does not translate into higher, but lower, MFP growth. Indeed, a higher endogenous rate of return increases the weight received by capital services in the growth accounts, thus overcompensating their lower growth and reducing growth in MFP.

38. Given the strong assumptions that are required to justify the use of endogenous rates of return, capital user costs entering the calculation of capital services and MFP will be based on the ENRR in the rest of this paper.

²⁰ The calculation of capital services follows the same methodology as in the OECD Productivity Database. See *OECD Productivity Statistics - Methodological Notes*, www.oecd.org/sdd/productivity-stats/OECD-Productivity-Statistics-Methodological-note.pdf.

Figure 3.6. Endogenous and exogenous rates of return

Percentage points, US private sector, 1998-2019



Note: The average internal rate of return corresponds to the weighted average of the internal rates of return estimated for 13 aggregate industries in the US private sector, where industry shares in gross value added are used as weights.

Source: Authors' calculations.

Table 3.3. Sensitivity of capital services growth to the use of different rates of return

Average annual percentage changes, US private sector, 1998-2019

Period	Rates of return		
	ENRR	WACC	Internal rate of return
1998-2019	2.8	2.9	2.4
1998-2006	3.6	3.9	3.2
2006-2012	1.8	1.9	1.5
2012-2019	2.7	2.8	2.1

Source: Authors' calculations.

Table 3.4. Sensitivity of MFP growth to the use of different rates of return

Average annual percentage changes, US private sector, 1998-2019

Period	Rates of return		
	ENRR	WACC	Internal rate of return
1998-2019	0.6	0.6	0.5
1998-2006	0.7	0.8	0.4
2006-2012	1.5	1.5	1.4
2012-2019	-0.3	-0.3	-0.2

Source: Authors' calculations.

3.4. Sensitivity of capital services and MFP growth to changes in cohort depreciation rates

39. Similarly to what is observed for the evolution of net capital stocks, the average evolution of capital services between 1998 and 2019 is not significantly affected by changes in cohort depreciation rates (Table 3.5).

40. The impact of changing cohort depreciation rates is more significant during the Great Recession and the immediately following years. Over 2006-2012, the average growth rate of capital services is 1.8% per year with US and Italian depreciation rates, 1.6% with German depreciation rates, 1.5% with Canadian and UK depreciation rates, and it is further reduced to 1.2% with French depreciation rates (Figure 3.7). Dwellings and non-residential buildings are the main contributors to these differences, as expected given that cross-country differences in depreciation patterns are larger for these assets.

41. An increase in the depreciation rate of a given asset impacts the growth rate of its capital services via three different channels: it increases the user cost (i.e. the rental price) of this asset, decreases the level of its net capital stock, and modifies the growth rate of its net capital stock. The first two channels have opposite effects on each asset's weight in aggregate capital services. Indeed, this weight is the share of each asset's capital services value (defined as the product of a user cost and a capital stock volume) in the total value of capital services. As already discussed above (Section 3.2), an increase in the depreciation rate of an asset also has an ambiguous effect on the growth rate of its net capital stock. Nevertheless, a depreciation rate increase has a more negative impact on capital accumulation in a period of low investment. This is why the impact on capital services growth of switching to the higher depreciation rates of Canada, France, Germany and the United Kingdom is more visible in the low investment years following the Great Recession.

Table 3.5. Sensitivity of capital services growth to changes in cohort depreciation rates

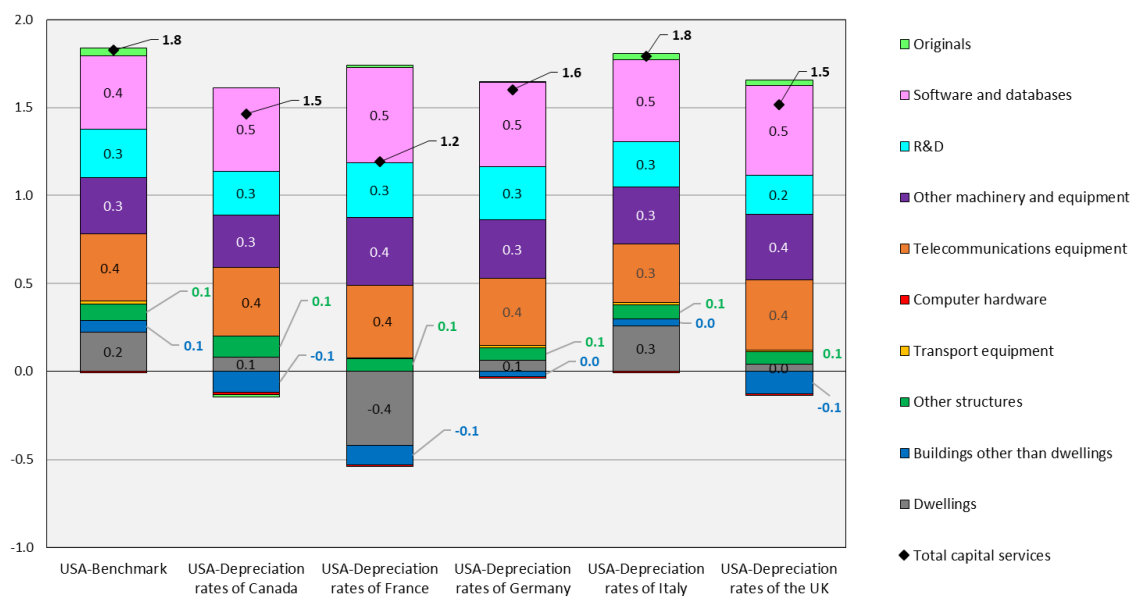
Average annual percentage changes, US private sector, 1998-2019

	USA - Benchmark	USA – Depreciation rates of Canada	USA – Depreciation rates of France	USA – Depreciation rates of Germany	USA – Depreciation rates of Italy	USA – Depreciation rates of the United Kingdom
1998-2019	2.8	2.7	2.7	2.8	2.9	2.8
1998-2006	3.6	3.4	3.9	3.8	3.7	3.7
2006-2012	1.8	1.5	1.2	1.6	1.8	1.5
2012-2019	2.7	2.8	2.7	2.8	2.8	2.8

Source: Authors' calculations.

Figure 3.7. Sensitivity of capital services growth to changes in cohort depreciation rates

Average annual percentage changes, US private sector, 2006-2012



Source: Authors' calculations.

42. Consistently with the results obtained for capital services, US MFP growth rates are only marginally affected by changes in depreciation patterns (Table 3.6).

Table 3.6. Sensitivity of MFP growth to changes in cohort depreciation rates

Average annual percentage changes, US private sector, 1998-2019

	USA - Benchmark	USA – Depreciation rates of Canada	USA – Depreciation rates of France	USA – Depreciation rates of Germany	USA – Depreciation rates of Italy	USA – Depreciation rates of the United Kingdom
1998-2019	0.6	0.7	0.7	0.6	0.6	0.6
1998-2006	0.7	0.8	0.6	0.7	0.7	0.7
2006-2012	1.5	1.7	1.8	1.6	1.6	1.7
2012-2019	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3

Source: Authors' calculations.

4. Impact of initial capital stock estimates on capital and MFP measurement

4.1. Options for estimating initial capital stocks in the absence of long investment time series

43. In addition to specific assumptions on the depreciation and the retirement of assets, the estimation of capital stocks using the PIM requires investment time series and initial capital stocks to initiate the estimation process. Initial capital stocks matter all the more if the available investment time series are shorter and the corresponding assets have longer service lives.

44. Unlike the United States, several European countries, mostly in Central and Eastern Europe, only dispose of investment series going back to the mid-1990s. In such cases, there are two main avenues for the estimation of initial capital stocks. The first possibility is to estimate initial capital stocks from national sources such as population censuses (giving information on the number of dwellings owned by households) and company accounts (giving information on the fixed assets owned by firms). Note that company accounts usually value assets at their book value (i.e. at their historical purchase price) and need to be supplemented with specific assumptions on the depreciation and information on the date of purchase of all assets to be able to value them at the price of a given year using national accounts' deflators. The second possibility is to rely on stationarity assumptions to backcast investment time series, or estimate initial capital stocks directly. In the absence of long investment time series, both avenues to estimate initial capital stocks require strong assumptions. Since the use of national sources to estimate initial capital stocks is country-specific and the lessons one may draw for the United States would be difficult to generalise to other countries, the present paper focuses on the second possibility (stationarity assumptions).

45. When the available investment time series are shorter than the desired length of the capital stock series plus the maximum service life of an asset, researchers and statistical agencies usually rely on stationarity assumptions to estimate initial capital stocks.

These assumptions may concern the growth rate of investment, in which case they are used to backcast investment time series, or capital stock-to-output ratios, in which case initial capital stocks are derived from the value of output (GDP) at the initial date.

4.1.1. Stationarity assumption on investment growth rates

46. A standard procedure to estimate initial capital stocks is to assume that investment for each asset type grows at a constant rate, usually taken equal to the average growth rate observed over the period where data are available (OECD, 2009). In this case, if the average growth rate of investment in asset type i is equal to θ_i and its geometric cohort depreciation rate is equal to δ_i , the initial capital stock of asset i at the end of period t can be calculated as follows:

$$K_{i,t} = \sum_{j=0}^N (1 - \delta_i)^j I_{i,t-j} = \sum_{j=0}^N \left(\frac{1 - \delta_i}{1 + \theta_i} \right)^j I_{i,t}$$

Provided that $\left| \frac{1 - \delta_i}{1 + \theta_i} \right| < 1$ and letting N tend to infinity, the previous formula simplifies to:

$$K_{i,t} = \frac{1 + \theta_i}{\theta_i + \delta_i} I_{i,t}$$

In this case, the initial capital stock at date t can be estimated based on investment at date t , as well as θ_i and δ_i .

4.1.2. Stationarity assumption on capital stock-to-output ratios

47. Alternatively, it can be assumed that the capital stock-to-output ratio is constant over time. This assumption is based on the Solow (1957) growth model where, on a balanced growth path, capital and output grow at the same rate. Initial capital stocks in the Penn World Tables are estimated in this way (Inklaar and Timmer, 2013; Feenstra et al., 2015).

4.2. Accuracy of initial capital stock estimates and impact on net capital stocks at later dates

48. In order to assess the accuracy of initial capital stock estimates and their impact on net capital stocks at later dates, it is assumed that the US investment time series start in 1950, 1980 or 1995, instead of 1901 as in the BEA national accounts.²¹ The above-described stationarity assumptions on investment growth rates and capital stock-to-output ratios for specific assets are then used in turn.

²¹ These cut-off dates are representative of the typical length of publicly available investment time series across OECD countries. While according to the 2019 Eurostat-OECD Questionnaire on the Methodology underlying Capital Stocks, many OECD countries rely on unpublished historical investment series to implement their PIM, this is apparently not the case for Central and Eastern European countries, for which investment time series do not seem to be available before 1995.

49. In the first case, average investment growth rates are estimated for each aggregate asset and industry²² over the first 20 years for which investment series are available.²³ These average growth rates are then used to backcast investment series for each underlying asset and industry.

50. In the second case, the asset-specific capital stock-to-output ratios calculated by Inklaar and Timmer (2013) are used. They are reported in Table 4.1. These are average capital stock-to-output ratios estimated on a sample of 142 countries with investment series going back at least to 1970.²⁴ Output corresponds to GDP and both capital and GDP are measured at current national prices.

Table 4.1. Stationarity assumptions on capital stock-to-output ratios to estimate initial capital stocks

Asset category	Capital stock-to-output ratio (total economy)
Structures (residential and non-residential)	2.2
Transport equipment	0.1
Other machinery and equipment	0.3
All other assets (i.e. IT equipment, Software, and Originals)	0

Note: Inklaar and Timmer (2013) did not cover R&D which, at the time, was considered intermediate consumption (not investment) in the System of National Accounts (SNA).

Source: Inklaar and Timmer (2013, Table 4).

51. For the purpose of this sensitivity analysis focusing on the US private sector, the three capital stock-to-output ratios given by Inklaar and Timmer (2013) are multiplied by a factor 0.8, corresponding to the ratio between the capital stocks in the US private sector and the US economy as a whole.²⁵ Initial capital stocks are then further broken down into assets and industries based on their respective investment shares over the first 20 years for which investment series are available. Finally, these initial capital stocks are used as

²² More precisely, average investment growth rates are estimated for Dwellings, Buildings other than dwellings, Other structures, Transport equipment, Computer hardware, Telecommunication equipment, Other machinery and equipment, R&D, Software and Originals, in each aggregate industry shown in Table D.1 of Annex D.

²³ For example, for the scenario where investment series are assumed to start in 1950, average investment growth rates are estimated over the period 1950-1969 for each aggregate asset/industry.

²⁴ Note that the adjustment advocated by Inklaar et al. (2019) to account for the slight increase in global capital stock-to-output ratios over time is not implemented in the present paper. Since the US ratios in the BEA accounts do not show any time trend (see Figures 4.3, 4.4 and 4.5), this adjustment would not improve the accuracy of initial capital stocks estimates for the United States. Similarly, their method to account for the fact that there is a large dispersion in capital stock-to-output ratios across countries is not implemented here. Since the United States is close to the cross-country average, this method would not improve the accuracy of initial capital stock estimates for the United States either.

²⁵ This ratio is taken from the actual BEA accounts. Nevertheless, this operation does not bias the results in favour of this method because the actual capital-stock-to-output ratio for the US economy as a whole (2.75) is close to the cross-country average (2.6) calculated by Inklaar and Timmer (2013), which is the key reason why this method works well for the United States. The multiplication by 0.8 simply allows focusing on the US private sector rather than the US economy as a whole.

starting points to apply the PIM and estimate net capital stocks at the same level of detail as the BEA (see Section 2).

52. Table 4.2 shows the accuracy of both methods to estimate initial capital stocks by comparing their results with the official capital stocks published by the BEA.²⁶ As expected, initial capital stocks have a long-lasting influence on future capital stocks for Structures and, to a lesser extent, for Transport equipment and Other machinery and equipment. For example, out of the initial capital stocks of structures estimated in 1950, 1980 and 1995, 25%, 52% and 76%, respectively, remain in use in 2005.²⁷ It is especially for long-lived assets that the accuracy of the methods to estimate initial capital stocks should be assessed.

53. The first conclusion that can be drawn from Table 4.2 is that the stationarity assumption on investment growth rates to estimate initial capital stocks can be very misleading, especially in the case of structures for which estimated capital stocks with investment series starting in 1995 are 16 times higher than in the official BEA accounts in 2005. This reflects the fact that the growth rate used to backcast investment series before 1995 is far below the actual average growth rate over the past, which leads to far too large estimates of past investment, especially for buildings other than dwellings.

²⁶ The BEA capital stock series start in 1947, or even 1925 for some assets, but these estimates are based on unpublished historical investment time series. Based on publicly available investment series starting in 1901, capital stocks for the longest-lived assets (residential buildings) cannot be recalculated before 1981. Therefore, the published BEA capital stock series, rather than the recalculated ones, are used in Table 4.2.

²⁷ These numbers are implied by the BEA geometric cohort depreciation rates. See the note underlying Table 4.2.

Table 4.2. Accuracy of stationarity assumptions to estimate initial capital stocks

Starting date of investment series (D)	Asset	Share of initial capital stock remaining in 2005 (%)	Stationarity assumptions on investment growth rates		Stationarity assumptions on capital stock-to-output ratios	
			Ratio between estimated and official (BEA) capital stocks at initial date (D)	Ratio between estimated and official (BEA) capital stocks in 2005	Ratio between estimated and official (BEA) capital stocks at initial date (D)	Ratio between estimated and official (BEA) capital stocks in 2005
1950	All structures	23.5	2.0	1.0	1.0	1.0
	Of which: Dwellings	20.5	1.5	1.0	1.0	1.0
	Of which: Other buildings and structures	25.0	2.7	1.0	1.0	1.0
	Transport equipment	0.6	1.0	1.0	1.6	1.0
	Other machinery and equipment	0.8	1.1	1.0	1.1	1.0
	IT equipment, Software and Originals	0.1	0.9	1.0	0.0	1.0
	R&D	0.0	0.9	1.0	not estimated	not estimated
	Total		1.8	1.0	1.0	1.0
1980	All structures	48.4	1.3	1.1	1.0	0.9
	Of which: Dwellings	41.3	0.7	0.9	0.9	0.9
	Of which: Other buildings and structures	52.0	2.3	1.3	1.0	1.0
	Transport equipment	5.2	1.8	1.1	1.1	1.0
	Other machinery and equipment	6.5	1.0	1.0	0.8	1.0
	IT equipment, Software and Originals	2.4	1.2	1.0	0.0	1.0
	R&D	1.0	1.0	1.0	not estimated	not estimated
	Total		1.3	1.0	0.9	0.9
1995	All structures	72.5	26.1	15.8	1.2	1.0
	Of which: Dwellings	64.7	3.8	2.7	1.1	1.0
	Of which: Other buildings and structures	76.5	59.0	37.1	1.2	1.1
	Transport equipment	24.6	1.2	1.0	1.5	1.0
	Other machinery and equipment	28.2	1.1	1.0	1.0	1.0
	IT equipment, Software and Originals	15.9	1.2	1.1	0.0	0.9
	R&D	11.3	1.1	1.0	not estimated	not estimated
	Total		20.5	13.0	1.1	1.0

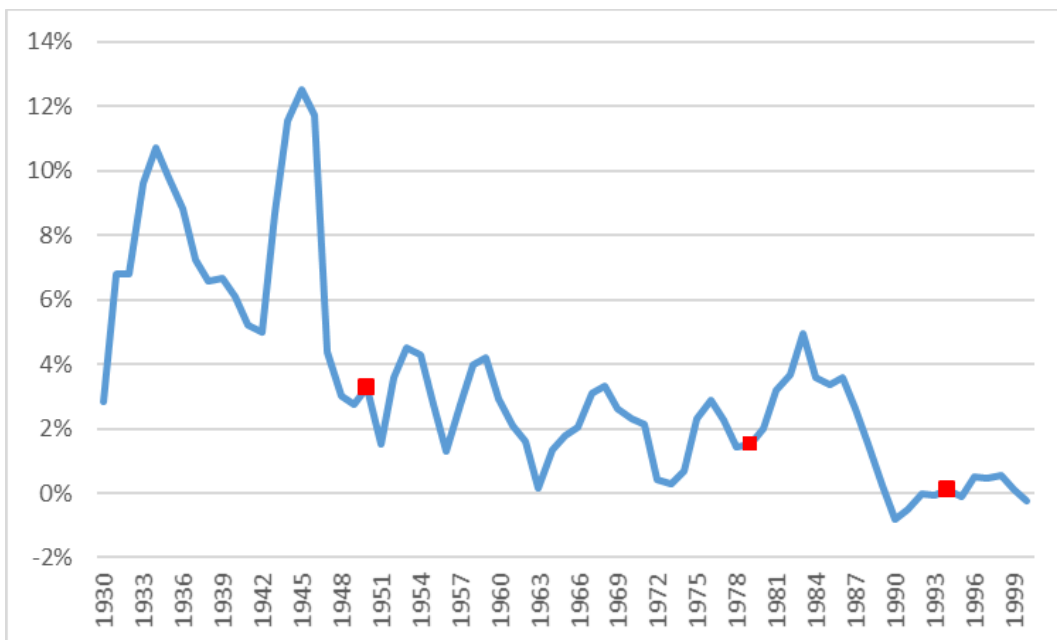
Note: The asset-specific shares of initial capital stock remaining in 2005 are calculated as $(1 - \delta_i)^{2005-D}$, where δ_i is the geometric cohort depreciation of asset i and D the initial starting date of investment series. These shares only depend on geometric cohort depreciation parameters, not on initial capital stocks themselves. In case assets have industry-specific depreciation parameters, or these parameters are set at a low level of the asset classification, an unweighted average of the corresponding shares is reported in Table 4.2. This unweighted average is only reported for quite homogeneous asset categories (e.g. Structures or Transport equipment), but not for the whole economy.

Source: Authors' calculations.

54. As shown by Figure 4.1 and Figure 4.2, the US private sector exhibits large fluctuations and/or long-term trends in investment growth rates for dwellings and buildings other than dwellings, even when these growth rates are averaged over 20 years. Therefore, using investment growth rates that are observed on a specific sample to backcast investment series over long periods in the past may lead to very inaccurate results. This issue is of course magnified if available time series are short, like in the 1995 scenario. Nevertheless, given that more than half of the initial capital stock in structures remains in use after 25 years, a similar issue could have easily happened in the 1980 scenario. Therefore, relying on the stationarity assumption of investment growth rates to estimate initial capital stocks of long-lived assets such as structures should be avoided, except maybe if the PIM is run over several decades before the resulting capital stocks start to be used for economic analysis.

Figure 4.1. Investment growth rate in dwellings

20-year forward moving average, US private sector, 1930-2000

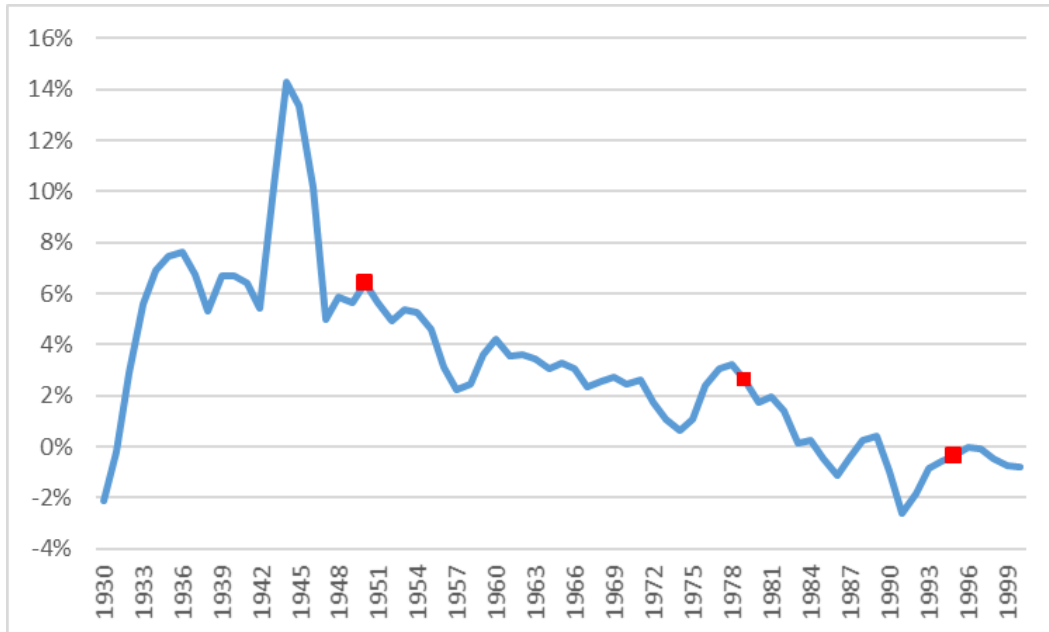


Note: The red dots indicate the 20-year forward moving average investment growth rates that are used to backcast investment time series from 1950, 1980 and 1995 backwards, respectively.

Source: Authors' calculations, BEA Fixed Assets Accounts.

Figure 4.2. Investment growth rate in buildings other than dwellings

20-year forward moving average, US private sector, 1930-2000



Note: The red dots indicate the 20-year forward moving average investment growth rates that are used to backcast investment time series from 1950, 1980 and 1995 backwards, respectively.
Source: Authors' calculations, based on BEA Fixed Assets Accounts.

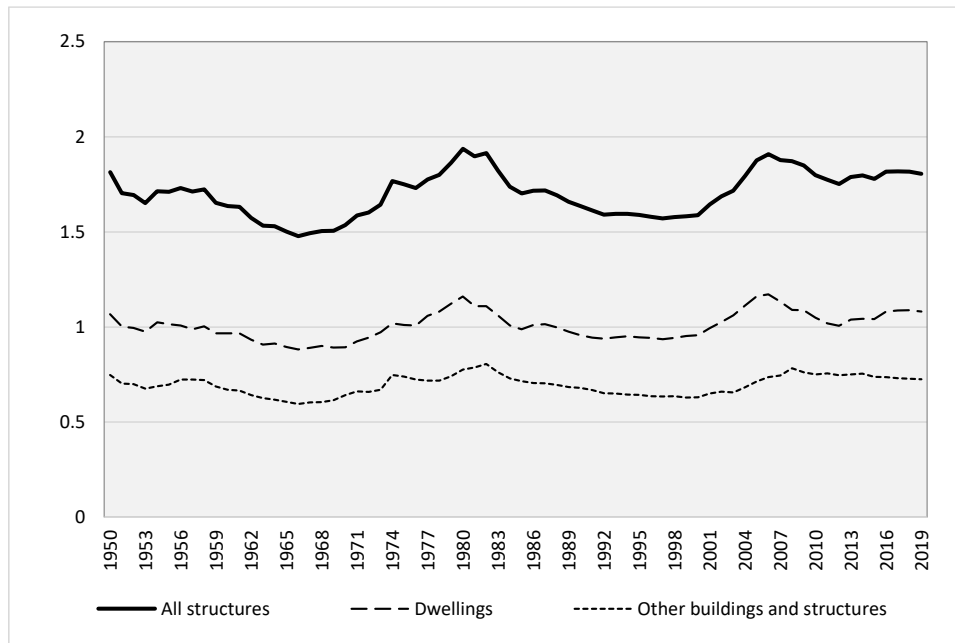
55. By comparison, Figure 4.3, Figure 4.4 and Figure 4.5 show that capital-stock-to-output ratios for the US private sector are much more stable over time than investment growth rates. They are also relatively close to the cross-country averages estimated by Inklaar and Timmer (2013).²⁸ Assuming zero initial net capital stocks for IT equipment, software, and originals as Inklaar and Timmer (2013) looks reasonable given the actual values for these ratios and the relatively short service lives of these assets.

56. Overall, estimates of net capital stocks in 2005 are in the +10/-10% range around official values reported by the BEA for all main asset categories and under all scenarios (investment series starting in 1950, 1980 or 1995) when capital-stock-to-output ratios are used to estimate initial capital stocks. Nevertheless, given the dispersion around the mean of capital-stock-to-output ratios across countries reported by Inklaar and Timmer (2013, Figure 1), it cannot be excluded that the same method would give less reliable results for other countries than the US. Exploring this issue is left for further research.

²⁸ As explained above, the capital stock-to-output ratios estimated by Inklaar and Timmer (2013) are multiplied by a factor 0.8 in the present paper, in order to focus on the US private sector.

Figure 4.3. Capital-stock-to-output ratios for structures

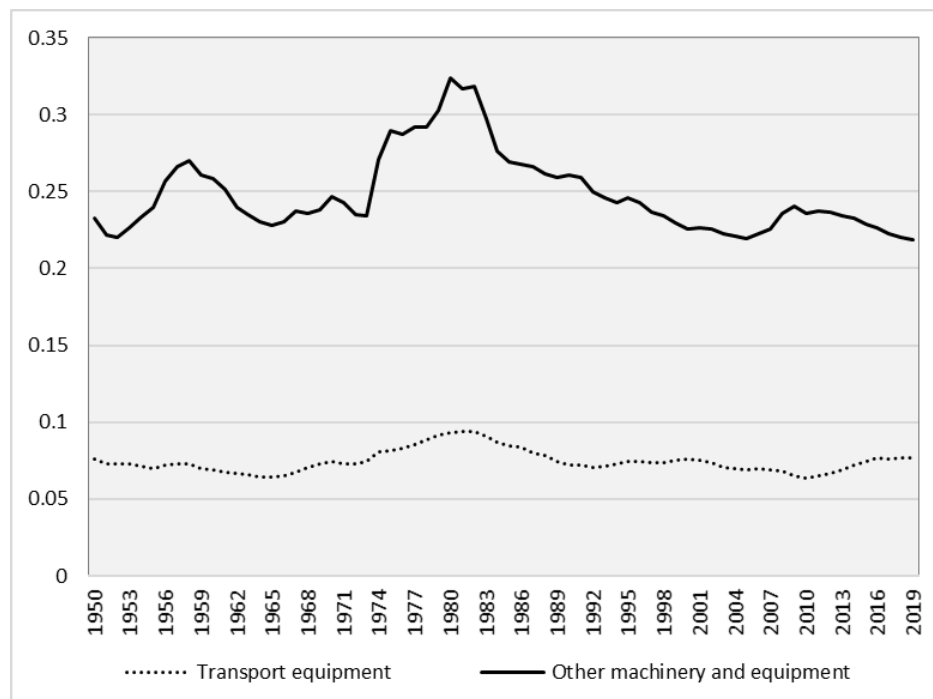
Current prices, US private sector, 1950-2019



Source: Authors' calculations, based on BEA Fixed Assets Accounts.

Figure 4.4. Capital-stock-to-output ratios for transport equipment, and other machinery and equipment

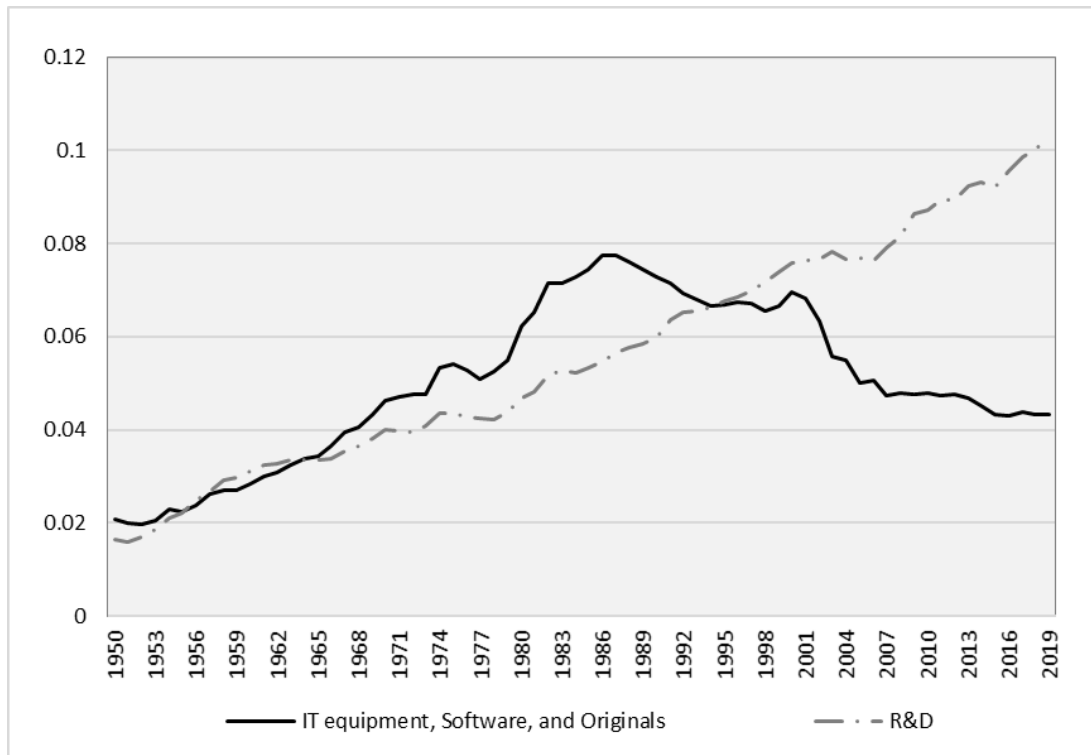
Current prices, US private sector, 1950-2019



Source: Authors' calculations, based on BEA Fixed Assets Accounts.

Figure 4.5. Capital-stock-to-output ratios for IT equipment, software, originals and R&D

Current prices, US private sector, 1950-2019



Source: Authors' calculations, based on BEA Fixed Assets Accounts.

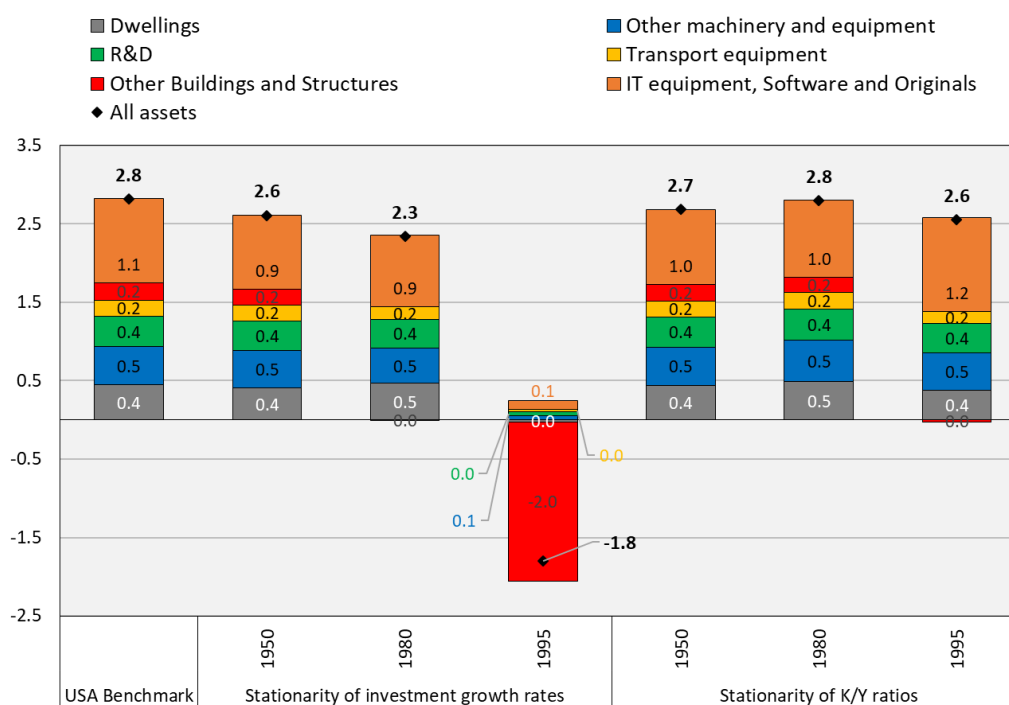
4.3. Sensitivity of capital services and MFP growth to initial capital stock estimates

57. Figure 4.6 shows that the combination of stationarity assumptions on investment growth rates and short investment time series may lead to very inaccurate estimates of capital services growth. This reflects to a large extent the difficulty to estimate initial capital stocks, and hence capital services, for real-estate assets when relying on stationarity assumptions on investment growth. Long investment time series are required to mitigate this problem.

58. By contrast, Figure 4.6 also shows that stationarity assumptions on capital-stock-to-output ratios to estimate initial capital stocks give relatively accurate estimates of US capital services growth, even when short investment time series are available. Nevertheless, the same caveat as for the estimation of net capital stocks holds (Section 4.2). Indeed, the findings in this paper are limited to the United States, for which the average capital-stock-to-output ratios estimated by Inklaar and Timmer (2013) on a large cross-section of countries work reasonably well. Considering the dispersion in capital-stock-to-output ratios across countries, this method may give less reliable results for other countries than the United States.

Figure 4.6. Sensitivity of capital services growth to initial capital stock estimates

Average annual percentage changes, US private sector, 1998-2019



Note: This figure shows the sensitivity of capital services growth to initial capital stock estimates. Two different methods (relying on stationarity assumptions on investment growth rates or capital-stock-to-output ratios) and three possible starting dates for investment time series (1950, 1980 and 1995) are considered.

Source: Authors' calculations.

59. As shown by Table 4.3, the sensitivity of MFP growth to initial capital stock estimates reflects the sensitivity of capital services growth, although in a mitigated way due to the weighting (by roughly one third) of capital services growth in explaining economic growth. MFP growth estimates only stand out as inaccurate when initial capital stocks are estimated in 1995 by assuming stationary investment growth rates over the past.

Table 4.3. Sensitivity of MFP growth to initial capital stock estimates

Average annual percentage changes, US private sector, 1998-2019

	USA - Benchmark	Stationarity of investment growth rates			Stationarity of capital-stock-to-output ratios		
		1950	1980	1995	1950	1980	1995
1998-2019	0.6	0.6	0.7	3.0	0.6	0.6	0.7
1998-2006	0.7	0.8	0.8	3.3	0.7	0.7	0.7
2006-2012	1.5	1.6	1.6	3.4	1.6	1.6	1.6
2012-2019	-0.3	-0.3	-0.2	2.5	-0.3	-0.3	-0.3

Note: This table shows the sensitivity of MFP growth to changes in initial capital stock estimates. Two different methods (relying on stationarity assumptions on investment growth rates or capital-stock-to-output ratios) and three possible starting dates for investment time series (1950, 1980 and 1995) are considered.

Source: Authors' calculations.

5. Conclusion

60. The measurement of capital stocks in an economy typically implies estimating initial capital stocks at a given date in the past and then cumulating and depreciating investment flows over time. This paper discussed the sensitivity of capital and MFP measurement to changes in the depreciation and retirement patterns of assets, and to the way initial capital stocks are estimated.

61. In order to capture differences in combined depreciation and retirement patterns across countries, this paper focused on geometric approximations of cohort depreciation patterns. This method allowed comparing the asset depreciation and retirement patterns used by national accountants in the United States and Canada, as well as France, Germany, Italy and the United Kingdom, where functional forms for asset depreciation and retirement differ from those used in Canada and the United States.

62. The sensitivity analysis in this paper has two main characteristics.

First, the distribution of cohort depreciation rates across countries for a given asset was used as a measure of uncertainty. The underlying assumption was that these observations provide different estimates of the same unobserved depreciation rate, and that all differences across countries may be related to measurement errors. Doing so ultimately provided an upper bound of the implied uncertainty on capital and MFP measurement.

Second, the national accounts produced by the US BEA were used as a laboratory to analyse the sensitivity of capital and MFP measurement. Since the composition of investment is relatively similar across advanced economies, the sensitivity of capital and MFP measurement in the United States was considered as a good indication of what would be obtained with the data of other advanced economies.

63. Applying the same geometric cohort depreciation rates in the United States as in Canada, France, Germany and the United Kingdom would reduce the net investment rate and the net capital stock of the US private sector by up to one third. Through an increase in the CFC of the government sector, this would also increase the US GDP by up to 0.5%. This largely reflects the faster depreciation of dwellings and non-residential buildings in the national accounts of Canada, France, Germany and the United Kingdom. Switching to Italian depreciation rates, which are closer to those used in the United States, would have a much more limited impact. Compared to the absolute levels of net capital stocks and CFC, the growth rates of net capital stocks, capital services and MFP appear less sensitive to changes in depreciation and retirement patterns, no matter which country's depreciation rates are used.

64. This paper also assessed the accuracy of two commonly used approaches to estimate initial capital stocks and their impact on capital and MFP measurement. These methods involve stationarity assumptions on either investment growth rates or capital-stock-to-output ratios. While the estimation method of initial capital stocks is innocuous for rapidly depreciating assets, it has a more significant impact for long-lived assets.

65. The US example showed that real-estate assets may exhibit large trends and fluctuations in investment growth. Since the same may be true in other countries, estimating initial capital stocks for real-estate assets by assuming constant investment growth rates over time should be avoided. On the contrary, relying on capital-stock-to-output ratios on a large cross-section of countries was shown to work reasonably well to estimate initial capital stocks for the US private sector. Nevertheless, given the wide dispersion in capital-stock-to-output ratios across countries, this result may not be universally true and relying on the cross-country average of capital-stock-to-output ratios may give less reliable results for other countries than the United States.

66. Overall, the empirical evidence in this paper calls for a more frequent review of the methods used by statistical agencies to estimate asset depreciation and retirement patterns, including for assets that have been capitalised for a long time in national accounts (e.g. buildings, structures, machinery and equipment). The aim of this recommendation is not to standardise depreciation and retirement patterns across countries, but to ensure that differences reflect country-specific factors rather than statistical assumptions.

67. The results of this paper also call for a careful use of stationarity assumptions to estimate initial capital stocks, especially for long-lived assets. Before relying on any stationarity assumption, efforts should be made to extend investment time series as much as possible based on historical vintages of national accounts, and to use the information on capital stocks provided by population censuses, company accounts and administrative sources whenever possible.

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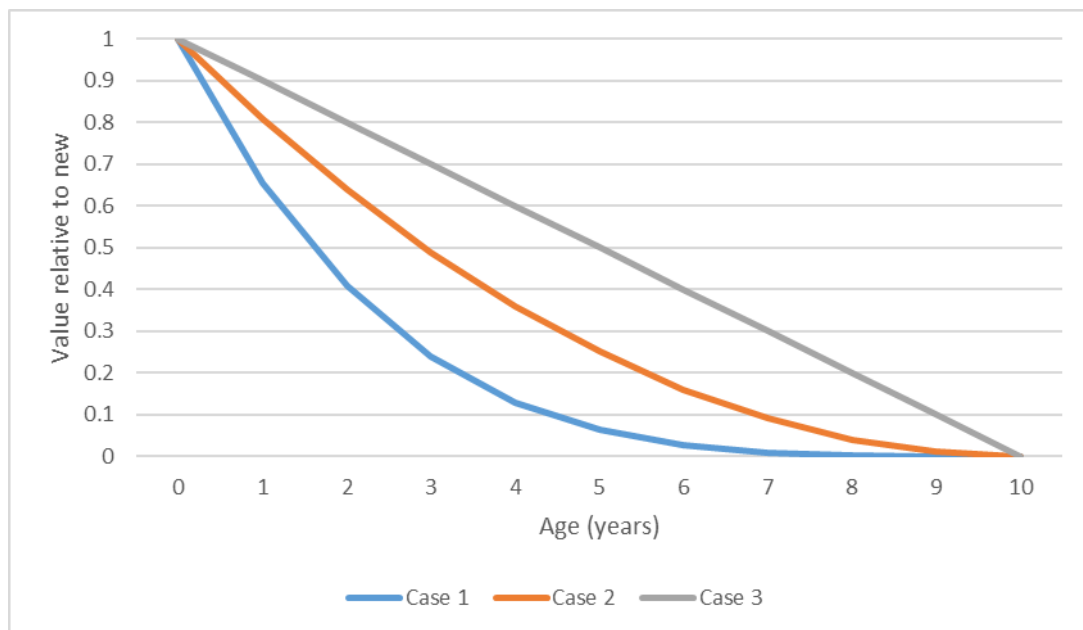
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Annex A. Interpretation and limits of Declining Balance Rates (DBRs)

68. Age-price profiles describe how the value (i.e. the market price) of single assets declines over time due to the shortening of their remaining service life. In this Annex, three different age-price profiles belonging to the same family of (power) functions are considered. Like age-price profiles that can be derived from linear and hyperbolic age-efficiency functions (OECD 2009, Chapter 3), those in cases 1 and 2 are convex to the origin. Case 3 considers a linear age-price profile (Figure A.1).

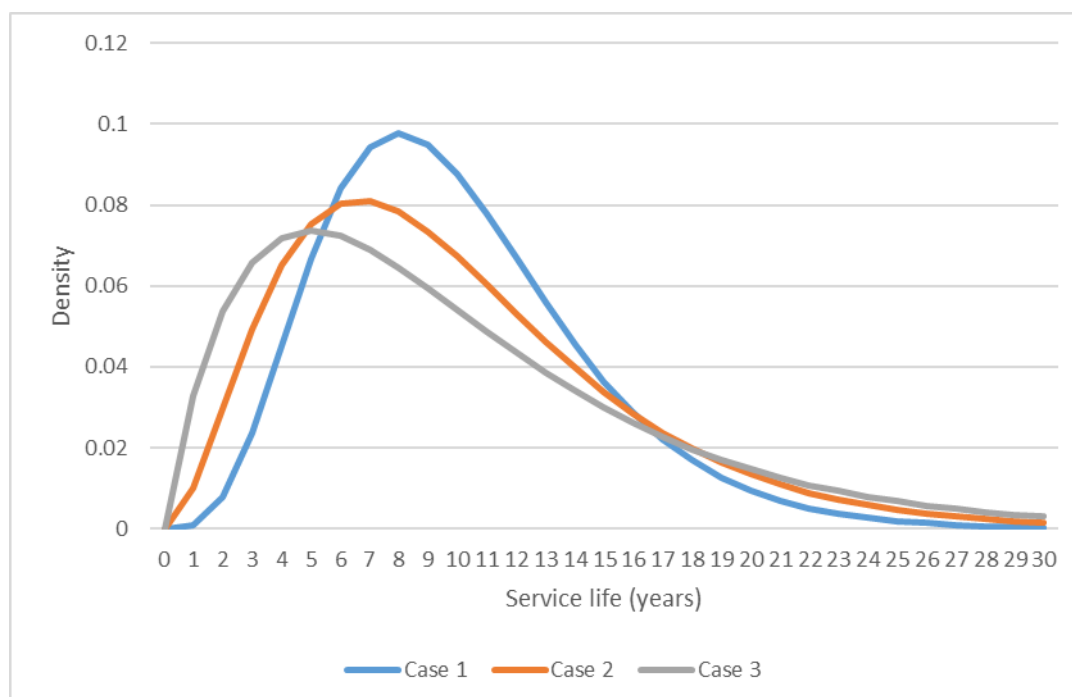
69. Each age-price profile is then combined with a specific retirement profile, belonging to the same family of (gamma) functions. These three retirement profiles are consistent with an asset average service life of 10 years and even though their shapes differ, they are all skewed to the left, in agreement with many asset survival studies (Figure A.2).

Figure A.1. Three individual age-price profiles, each with a service life of 10 years



Note: In this example, all age-price profiles are based on power functions of the type $\left(1 - \frac{s}{L}\right)^{\nu-1}$ where s stands for the age of the asset and L for its service life. The parameter ν is set at 5, 3 and 2 in cases 1, 2 and 3, respectively. All assets shown in Figure A1 have a service life of 10 years.

Figure A.2. Three retirement functions, each with an average service life of 10 years

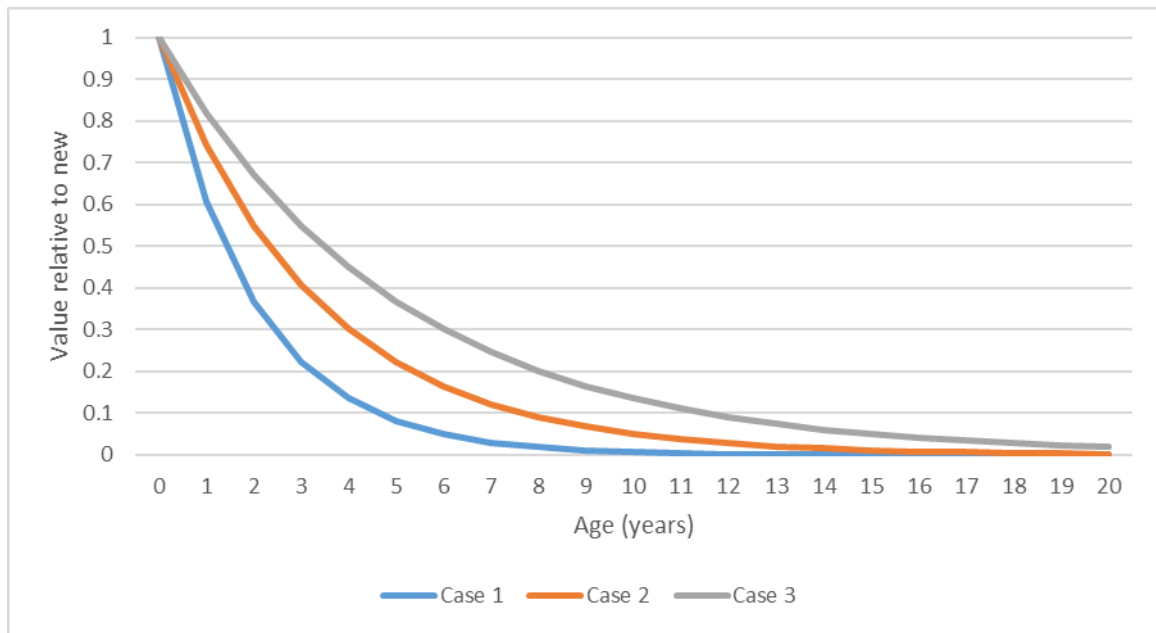


Note: Retirement functions capture the randomness in asset service lives. In this example, Gamma functions with a density $\delta^\nu \cdot L^{\nu-1} \cdot \frac{e^{-\delta L}}{\Gamma(\nu)}$ are used. They are parameterised by ν (same parameter as for age-price profiles) and δ . Their mean is given by the ratio $\frac{\nu}{\delta}$ and corresponds to the asset average service life. It is fixed at 10 years, thus implying that the parameter δ is set at 0.5, 0.3 and 0.2 in cases 1, 2 and 3, respectively.

70. Sliker (2018) demonstrates that the combination of such age-price and retirement profiles leads to exactly geometric depreciation patterns for cohorts of assets. The implied geometric parameters depend on the parameters of the underlying depreciation and retirement functions. Figure A.3 shows that the implied geometric cohort depreciation rates are different in all three cases, even though the average service life of assets remains fixed at 10 years.

71. This example shows that declining balance rates (DBRs), used by Hulten and Wykoff (1981b) to provide an inverse proportional relationship between geometric cohort depreciation rates δ and average asset services lives T ($\delta \equiv \frac{DBR}{T}$), depend on the shape of the underlying depreciation and retirement functions. Therefore, DBRs are country specific, and estimating geometric depreciation rates for a country based on its asset service lives (ASLs) and the DBRs of another country would be misleading. This is further illustrated in Table A.1 showing that assets with similar ASLs in Canada and the United States (e.g. medical buildings) may have very different geometric cohort depreciation rates, and conversely that assets with similar geometric cohort depreciation rates (e.g. construction tractors) may have very different ASLs. This shows the wide heterogeneity of DBRs across countries, including for similar assets.

Figure A.3. Implied geometric cohort depreciation profiles combining age-price and retirement profiles



Note: Sliker (2018) shows that the combination of the depreciation and retirement functions used in Figures A.1 and A.2 leads to exactly geometric functions parameterised by the same parameter δ as in the retirement functions used for Figure A.2.

Table A.1. Comparison of geometric cohort depreciation rates (δ), average service lives (ASLs) and declining balance rates (DBRs) for specific assets in Canada and the United States

Asset SNA code	Country	Asset label	δ	ASL (years)	DBR
N1121	USA BEA	Medical building	0.02	36	0.89
	Statistics Canada	Hospitals, health centres, clinics, nursing homes and other health care buildings	0.06	35	2.17
N1139	USA BEA	Household appliances	0.17	10	1.65
	Statistics Canada	Small electric appliances	0.21	11	2.29
		Major appliances	0.23	10	2.31
N1139	USA BEA	Construction tractors	0.16	8	1.31
	Statistics Canada	Logging, mining and construction machinery and equipment	0.17	13	2.23

Source: BEA, Statistics Canada, and Giandrea et al. (2021).

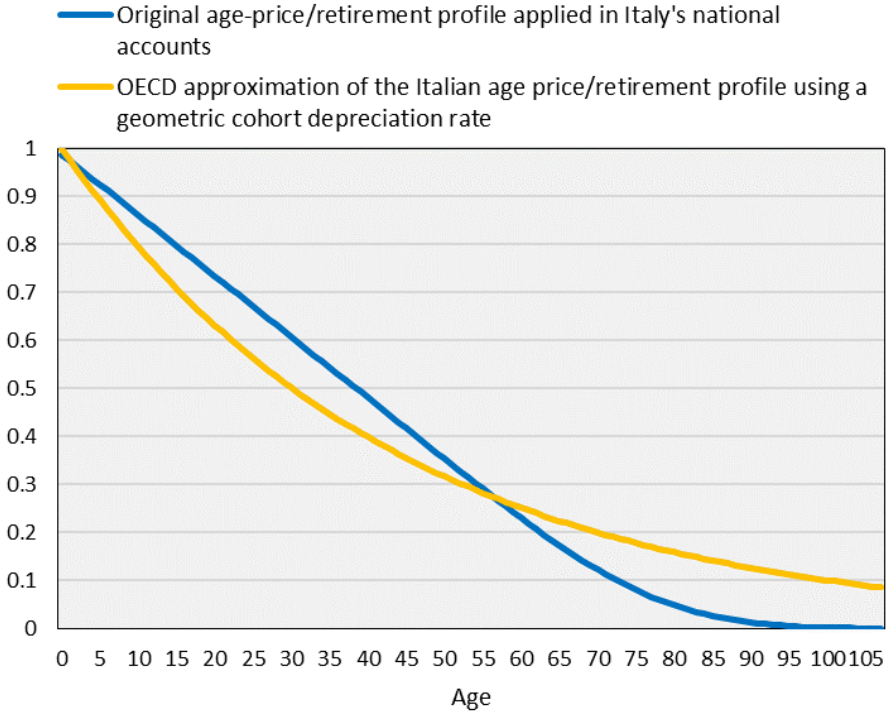
Annex B. Geometric approximations of combined asset depreciation and retirement patterns

72. In France, the depreciation for a cohort of assets is calculated by combining a log-normal retirement distribution with a straight-line depreciation pattern for single assets. In Germany, a straight-line depreciation profile is combined with a Gamma retirement function. In Italy, except for R&D, cohort depreciation is calculated by combining a straight-line depreciation profile with a truncated normal retirement function. In the United Kingdom, except for R&D, the age-price (i.e. depreciation) profile for a single asset is derived from a hyperbolic age-efficiency profile and then combined with a truncated normal retirement function.²⁹ For R&D, both Italy and the United Kingdom assume a geometric cohort depreciation pattern. In all these cases, the combination of depreciation and retirement profiles leads to cohort depreciation profiles that are convex to the origin.

73. Cabannes et al. (2013) estimated the geometric profiles that best approximate the combined retirement and depreciation profiles applied in France. A similar approach is followed for Germany, Italy and the United Kingdom in the present paper. For each asset and industry indexed by i , the combined profile $Z_{i,s}$ that is consistent with the actual assumptions on asset depreciation and retirement is estimated for each country. $Z_{i,s}$ is then approximated with a geometric profile $Z_{i,s}^* = (1 - \delta_i)^s$ and the parameter δ_i is estimated using non-linear least squares.

²⁹ Hyperbolic age-efficiency functions are defined as $g(n) = \frac{T-n}{T-\beta \cdot n}$ where n is the age of the asset and T its maximum service life. As noted by Harper (1982), the shape of these functions is very sensitive to the value of the β parameter. Similarly to the US BLS, the UK ONS takes $\beta = 0.75$ for dwellings, buildings other than dwellings, and other structures, and $\beta = 0.5$ for all other assets except R&D.

Figure B.1. Combined depreciation and retirement pattern and its geometric approximation for dwellings in Italy

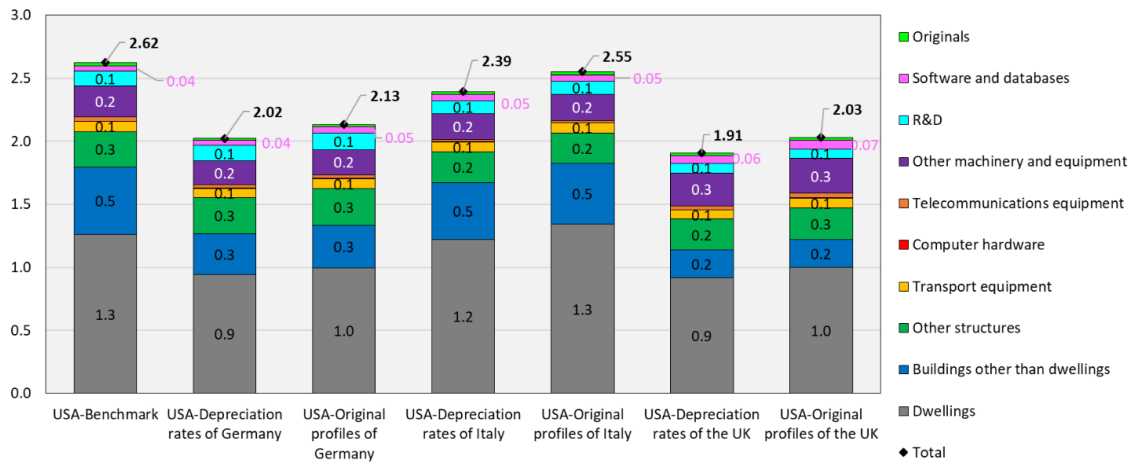


Source: Authors' calculations based on information provided by ISTAT.

74. Figure B.2 shows alternative estimates of US net capital stocks when the cohort depreciation profiles of the BEA are replaced with the original profiles of Germany, Italy and the United Kingdom, and their geometric approximations. The original profiles of Germany, Italy and the United Kingdom, on the one hand, and their geometric approximations, on the other hand, lead to very similar net capital stocks. Since geometric approximations of cohort depreciation rates simplify cross-country comparisons, they are consistently used in this paper.

Figure B.2. Alternative estimates of US net capital stocks when the BEA cohort depreciation profiles are replaced with the original profiles of Germany, Italy and the United Kingdom or their geometric approximations

Ratio of net capital stock to gross value added, US private sector, 2019



Source: Authors' calculations based on information provided by DESTATIS (Germany), ISTAT (Italy) and the ONS (United Kingdom).

Annex C. Cohort depreciation rates and asset correspondence across countries

BEA asset label	OECD asset code	OECD asset label	Geometric cohort depreciation rate					United Kingdom
			United States	Canada	France	Germany	Italy	
RESIDENTIAL ASSETS								
1-to-4-unit structures-new	DWE	Dwellings	0.0114	0.02	0.071	0.035	0.023	0.036
1-to-4-unit structures-additions and alterations	DWE	Dwellings	0.0227	0.04	0.071	0.035	0.023	0.036
1-to-4-unit structures-major replacements	DWE	Dwellings	0.0364	0.04	0.071	0.035	0.023	0.036
5-or-more-unit structures-new	DWE	Dwellings	0.014	0.02	0.071	0.035	0.023	0.036
5-or-more-unit structures-additions and alterations	DWE	Dwellings	0.0284	0.04	0.071	0.035	0.023	0.036
5-or-more-unit structures-major replacements	DWE	Dwellings	0.0455	0.04	0.071	0.035	0.023	0.036
Brokers' commissions and other ownership transfer costs /26/	DWE	Dwellings	0.1375	1	0.071	0.035	0.023	0.036
Manufactured homes	DWE	Dwellings	0.0455	0.081	0.071	0.035	0.023	0.036
Other structures	DWE	Dwellings	0.0227	0.081	0.071	0.035	0.023	0.036
Equipment	DWE	Dwellings	0.15	0.081	0.071	0.035	0.023	0.036
MACHINERY AND EQUIPMENT								
Communications	COM	Telecommunications equipment	0.111	0.228	0.154	0.153	0.282	0.178
Nonelectro medical instruments	OMEW	Other machinery and equipment	0.135	0.301	0.117	0.153	0.138	0.129
Electro medical instruments	OMEW	Other machinery and equipment	0.1834	0.236	0.117	0.153	0.138	0.129
Nonmedical instruments	OMEW	Other machinery and equipment	0.135	0.236	0.117	0.153	0.138	0.129
Photocopy and related equipment	HARD	Computer hardware	0.18	0.242	0.244	0.153	0.261	0.241
Office and accounting equipment	HARD	Computer hardware	0.3119	0.323	0.244	0.153	0.261	0.241
Other fabricated metals	OMEW	Other machinery and equipment	0.0917	0.198	0.117	0.153	0.138	0.129
Steam engines	OMEW	Other machinery and equipment	0.0516	0.086	0.117	0.153	0.138	0.129
Internal combustion engines	OMEW	Other machinery and equipment	0.2063	0.093	0.117	0.153	0.138	0.129
Metalworking machinery	OMEW	Other machinery and equipment	0.121	0.197	0.117	0.153	0.138	0.129
Special industrial machinery	OMEW	Other machinery and equipment	0.102	0.195	0.117	0.153	0.138	0.129
General industrial equipment	OMEW	Other machinery and equipment	0.106	0.182	0.117	0.153	0.138	0.129
Electric transmission and distribution	OMEW	Other machinery and equipment	0.05	0.113	0.117	0.153	0.138	0.129
Light trucks (including utility vehicles)	TRANS	Transport equipment	0.1925	0.235	0.171	0.153	0.172	0.162
Other trucks, buses and truck trailers	TRANS	Transport equipment	0.190	0.238	0.171	0.153	0.172	0.162
Aircraft	TRANS	Transport equipment	0.106	0.138	0.171	0.153	0.098	0.162
Ships and boats	TRANS	Transport equipment	0.0611	0.112	0.171	0.153	0.098	0.162
Railroad equipment	OMEW	Other machinery and equipment	0.0589	0.099	0.117	0.153	0.138	0.129
Household furniture	OMEW	Other machinery and equipment	0.1375	0.25	0.117	0.153	0.137	0.129
Other furniture	OMEW	Other machinery and equipment	0.1179	0.26	0.117	0.153	0.137	0.129
Other agricultural machinery	OMEW	Other machinery and equipment	0.1179	0.178	0.117	0.153	0.138	0.129
Farm tractors	TRANS	Transport equipment	0.1452	0.178	0.171	0.153	0.098	0.162
Other construction machinery	OMEW	Other machinery and equipment	0.155	0.172	0.117	0.153	0.138	0.129
Construction tractors	TRANS	Transport equipment	0.1633	0.172	0.171	0.153	0.098	0.162
Mining and oilfield machinery	OMEW	Other machinery and equipment	0.15	0.172	0.117	0.153	0.138	0.129
Service industry machinery	OMEW	Other machinery and equipment	0.150	0.265	0.117	0.153	0.138	0.129
Household appliances	OMEW	Other machinery and equipment	0.165	0.222	0.117	0.153	0.138	0.129
Other electrical	OMEW	Other machinery and equipment	0.1834	0.115	0.117	0.153	0.138	0.129
Other	OMEW	Other machinery and equipment	0.1473	0.193	0.117	0.153	0.138	0.129

BEA asset label	OECD asset code	OECD asset label	Geometric cohort depreciation rate					United Kingdom
			United States	Canada	France	Germany	Italy	
NON RESIDENTIAL ASSETS								
Office	BOD	Buildings other than dwellings	0.0247	0.068	0.067	0.057	0.039	0.075
Hospitals	BOD	Buildings other than dwellings	0.019	0.062	0.067	0.057	0.039	0.075
Special care	BOD	Buildings other than dwellings	0.0188	0.062	0.067	0.057	0.039	0.075
Medical buildings	BOD	Buildings other than dwellings	0.025	0.062	0.067	0.057	0.039	0.075
Multimerchandise shopping	BOD	Buildings other than dwellings	0.0262	0.093	0.067	0.057	0.039	0.075
Food and beverage establishments	BOD	Buildings other than dwellings	0.026	0.081	0.067	0.057	0.039	0.075
Warehouses	BOD	Buildings other than dwellings	0.0222	0.081	0.067	0.057	0.039	0.075
Mobile structures	BOD	Buildings other than dwellings	0.056	0.062	0.067	0.057	0.039	0.075
Other commercial	BOD	Buildings other than dwellings	0.0262	0.087	0.067	0.057	0.039	0.075
Manufacturing	BOD	Buildings other than dwellings	0.031	0.075	0.067	0.057	0.039	0.075
Electric	OST	Other structures	0.0211	0.055	0.031	0.031	0.039	0.050
Wind and solar	OST	Other structures	0.030	0.065	0.031	0.034	0.039	0.050
Gas	OST	Other structures	0.0237	0.074	0.031	0.032	0.039	0.050
Petroleum pipelines	OST	Other structures	0.024	0.074	0.031	0.032	0.039	0.050
Communication	OST	Other structures	0.0237	0.104	0.031	0.032	0.039	0.050
Petroleum and natural gas	OST	Other structures	0.075	0.117	0.031	0.053	0.039	0.050
Mining	OST	Other structures	0.045	0.159	0.031	0.040	0.039	0.050
Religious	BOD	Buildings other than dwellings	0.019	0.055	0.067	0.057	0.039	0.075
Educational and vocational	BOD	Buildings other than dwellings	0.0188	0.056	0.067	0.057	0.039	0.075
Lodging	BOD	Buildings other than dwellings	0.028	0.081	0.067	0.057	0.039	0.075
Amusement and recreation	BOD	Buildings other than dwellings	0.03	0.081	0.067	0.057	0.039	0.075
Air transportation	OST	Other structures	0.024	0.102	0.031	0.032	0.039	0.050
Other transportation	OST	Other structures	0.0237	0.08	0.031	0.032	0.039	0.050
Other railroad	OST	Other structures	0.018	0.063	0.031	0.029	0.039	0.050
Track replacement	OST	Other structures	0.0249	0.063	0.031	0.032	0.039	0.050
Local transit structures	OST	Other structures	0.024	0.092	0.031	0.032	0.039	0.050
Other land transportation	OST	Other structures	0.0237	0.063	0.031	0.032	0.039	0.050
Farm	BOD	Buildings other than dwellings	0.024	0.089	0.067	0.057	0.039	0.075
Water supply	OST	Other structures	0.0225	0.057	0.031	0.031	0.039	0.050
Sewage and waste disposal	OST	Other structures	0.023	0.062	0.031	0.031	0.039	0.050
Public safety	OST	Other structures	0.0237	0.062	0.031	0.032	0.039	0.050
Highway and conservation and development	OST	Other structures	0.023	0.101	0.031	0.031	0.039	0.050

BEA asset label	OECD asset code	OECD asset label	Geometric cohort depreciation rate					
			United States	Canada	France	Germany	Italy	United Kingdom
INTELLECTUAL PROPERTY PRODUCTS								
Prepackaged software	SOFT	Computer software and databases	0.550	0.550	0.244	0.359	0.325	0.256
Custom software	SOFT	Computer software and databases	0.33	0.33	0.244	0.359	0.325	0.256
Own account software	SOFT	Computer software and databases	0.330	0.330	0.244	0.359	0.325	0.245
Pharmaceutical and medicine manufacturing	RD	Research and development	0.1	0.275	0.1	0.168	0.200	0.287
Chemical manufacturing, ex. pharma and med	RD	Research and development	0.160	0.275	0.160	0.184	0.200	0.287
Semiconductor and other component manufacturing	RD	Research and development	0.25	0.275	0.25	0.208	0.200	0.287
Computers and peripheral equipment manufacturing	RD	Research and development	0.400	0.275	0.400	0.249	0.200	0.287
Communications equipment manufacturing	RD	Research and development	0.27	0.275	0.27	0.214	0.200	0.287
Navigational and other instruments manufacturing	RD	Research and development	0.290	0.275	0.290	0.219	0.200	0.287
Other computer and electronic manufacturing, n.e.c.	RD	Research and development	0.4	0.275	0.4	0.249	0.200	0.287
Motor vehicles and parts manufacturing	RD	Research and development	0.310	0.275	0.310	0.225	0.200	0.287
Aerospace products and parts manufacturing	RD	Research and development	0.22	0.275	0.22	0.200	0.200	0.287
Other manufacturing	RD	Research and development	0.160	0.275	0.160	0.184	0.200	0.287
Scientific research and development services	RD	Research and development	0.16	0.275	0.16	0.184	0.200	0.287
Software publishers	RD	Research and development	0.220	0.275	0.220	0.200	0.200	0.287
Financial and real estate services	RD	Research and development	0.16	0.275	0.16	0.184	0.200	0.287
Computer systems design and related services	RD	Research and development	0.360	0.275	0.360	0.238	0.200	0.287
All other nonmanufacturing, n.e.c.	RD	Research and development	0.16	0.275	0.16	0.184	0.200	0.287
Private universities and colleges	RD	Research and development	0.160	0.275	0.160	0.184	0.200	0.287
Other nonprofit institutions	RD	Research and development	0.16	0.275	0.16	0.184	0.200	0.287
Theatrical movies	ELAO	Entertainment, literary, and artistic originals	0.093	1.000	0.331	0.110	0.172	0.183
Long-lived television programs	ELAO	Entertainment, literary, and artistic originals	0.168	1.000	0.331	0.181	0.172	0.183
Books	ELAO	Entertainment, literary, and artistic originals	0.121	0.121	0.331	0.137	0.172	0.183
Music	ELAO	Entertainment, literary, and artistic originals	0.267	0.267	0.331	0.273	0.172	0.183
Other entertainment originals	ELAO	Entertainment, literary, and artistic originals	0.109	0.109	0.331	0.125	0.172	0.183

Note: This table provides a mapping between the different assets types considered by national accountants at Statistics Canada, INSEE (France), DESTATIS (Germany), ISTAT (Italy), the ONS (United Kingdom) and the BEA (United States), and compares the corresponding geometric cohort depreciation rates.

For Canada, these depreciation rates are averages across industries. They correspond to the “weighted averages of many asset categories” provided by Giandrea et al. (2021). Canadian cohort depreciation rates are not available for books, music, and other entertainment originals. The present sensitivity analysis keeps the US depreciation rates unchanged for these asset types.

For France, the geometric approximations of combined depreciation and retirement patterns provided by Cabannes et al. (2013) are used. French cohort depreciation rates are not available for R&D. Our sensitivity analysis keeps the US depreciation rates unchanged for this asset type.

For Germany, Italy and the United Kingdom, geometric approximations of combined depreciation and retirement patterns are calculated based on the information provided by DESTATIS (Germany), ISTAT (Italy) and the ONS (United Kingdom).

Annex D. Estimation of endogenous rates of return

75. In this paper, endogenous rates of return are computed for 13 aggregate industries belonging to the US private sector (Table D.1).

Table D.1. Industry level at which the internal rates of return are estimated

NAICS code	NAICS label	OECD code	OECD label
11	Agriculture, forestry, fishing, and hunting	VA0	Agriculture, forestry and fishing
21	Mining	VB	Mining and quarrying
22	Utilities	VD+VE	Electricity, gas, steam and air conditioning supply & Water supply; sewerage, waste management and remediation activities
23	Construction	VF	Construction
31-33	Manufacturing	VC	Manufacturing
42 & 44-45	Wholesale trade and retail trade	VG	Wholesale and retail trade; repair of motor vehicles and motorcycles
48-49	Transportation and warehousing	VH	Transportation and storage
51	Information	VJ	Information and communication
52-53	Finance, insurance, real estate, rental, and leasing	VK+VL	Financial and insurance activities & Real estate activities
54-56	Professional and business services	VM+VN	Professional, scientific and technical activities & Administrative and support service activities
61-62	Educational services, health care, and social assistance	VP+VQ	Education & Human health and social work activities
71-72	Arts, entertainment, recreation, accommodation, and food services	VR+VI	Arts, entertainment and recreation & Accommodation and food service activities
81	Other services, except government	VS	Other service activities

Note: OECD codes are industry codes used in the OECD Annual National Accounts database.

76. Estimating the residual income $KInc$ accruing to capital is not straightforward. This aggregate corresponds to the sum of the gross operating surplus (GOS), the capital income

component of mixed income, and taxes less subsidies on production.³⁰ For each industry, the BEA accounts include a single aggregate summing GOS and mixed income. We denote this aggregate by GOSMXI. In addition, the BEA provides for each industry the number of employees, in headcounts and full-time equivalent (FTE) units, and the total number of persons employed (TPE), defined as the sum of the number of self-employed workers and the number of employees in FTE units. No information is available on hours worked by self-employed workers.

77. In order to estimate the labour component of mixed income, it is assumed that self-employed workers receive the same labour compensation as full-time employees working in the same industry.

78. Summing up, the residual income $KInc_{it}$ accruing to capital in each industry i is estimated as follows:

Step 1: The number of self-employed workers (SE_{it}) in each industry i is calculated as the difference between the total number of persons employed (TPE_{it}) and the number of employees in FTE units (EE_{it}):

$$SE_{it} = TPE_{it} - EE_{it}$$

Step 2: A labour compensation (LMX_{it}) is imputed to self-employed workers based on the average labour compensation of full-time employees working in the same industry:

$$LMX_{it} = \frac{COE_{it}}{EE_{it}} * SE_{it}$$

where COE_{it} is the total compensation of employees in industry i in period t .

Step 3: The labour component of self-employed income is subtracted from GOSMXI and taxes less subsidies on production ($D29_D39_{it}$) are added to it:

$$KInc_{it} = GOSMXI_{it} - LMX_{it} + D29_D39_{it}$$

79. Data on taxes less subsidies on production are sourced from the OECD Annual National Accounts database, where they are available by ISIC rev. 4 industry. The correspondence between NAICS and ISIC Rev. 4 shown in Table D.1 is then used. This allows estimating endogenous rates of return for the 13 aggregate industries, which are then used for the calculation of capital services.

³⁰ These are taxes net of subsidies that enterprises incur as a result of engaging in production, independently of the quantity or value of the goods and services produced or sold. They may be payable on the land, fixed assets or labour employed, or certain activities or transactions (e.g. property taxes).